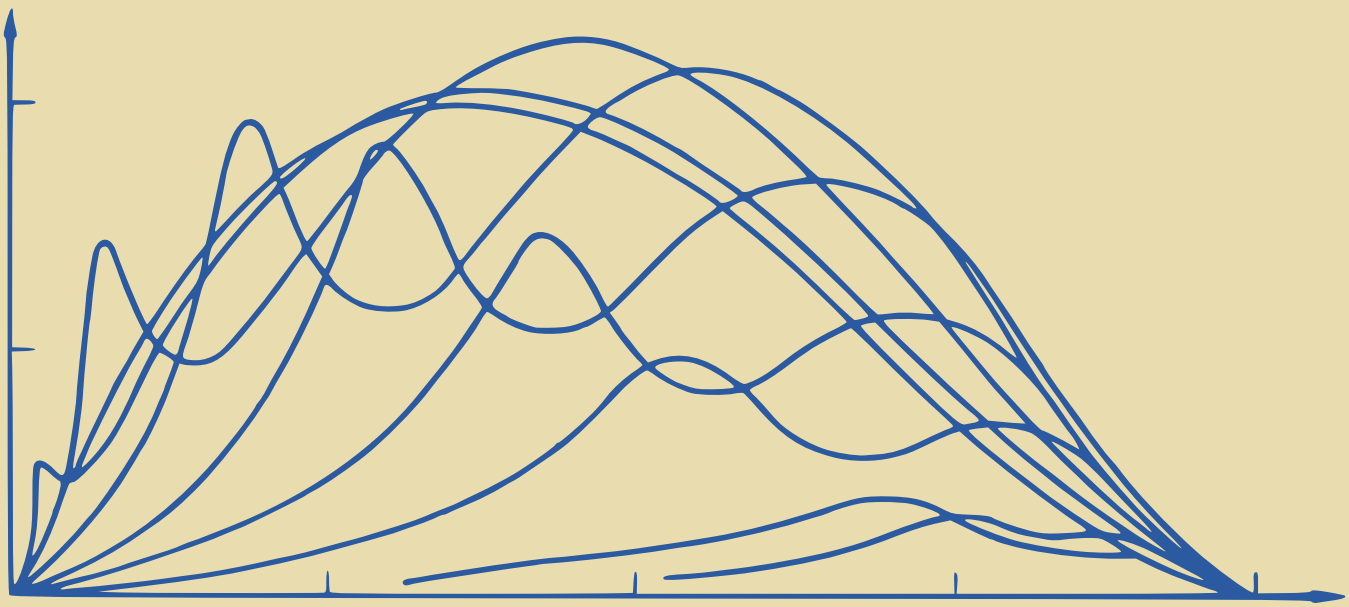
S.A. Kaplan, V.N. Tsytovich

PLASMA ASTRO-PHYSICS



PLASMA ASTROPHYSICS

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PERGAMON PRESS

OXFORD · NEW YORK · TORONTO

SYDNEY · BRAUNSCHWEIG

Pergamon Press Ltd., Headington Hill Hall, Oxford
Pergamon Press Inc., Maxwell House, Fairview Park, Elmsford, New York 10523
Pergamon of Canada Ltd., 207 Queen's Quay West, Toronto 1
Pergamon Press (Aust.) Pty. Ltd., 19a Boundary Street, Rushcutters Bay, N.S.W. 2011, Australia
Vieweg & Sohn GmbH, Burgplatz 1, Braunschweig

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First edition 1973

Library of Congress Cataloging in Publication Data

Kaplan, Samuil Aronovich.
Plasma astrophysics.

(International series of monographs in natural philosophy, v. 59)
Translation of Plazmennaîa astrofizika.
Bibliography: p.
1. Astrophysics. 2. Plasma turbulence. 3. Radio astronomy. I. T͡Sytovich, Vadim Nikolaevich, joint author. II. Title.
QB461.K3513 1973 523.01 73-5785
ISBN 0-08-017190-7

Printed in Hungary

Preface to the Russian Edition

For the last twenty years plasma phenomena have been used for the interpretation of astrophysical effects (Ginzburg, 1946; Shklovskiĭ, 1946; Schlüter and Biermann, 1950; Alfvén, 1950; Spitzer, 1956; Piddington, 1954; Ginzburg and Zheleznyakov, 1959; and so on). However, the necessary prerequisites for a sufficiently reliable use of the results of theoretical and experimental studies of plasma physics have appeared only in the very last years. This is connected with the relative complexity and diversity of plasma phenomena. Only when plasma physics had reached a certain level could it be applied to astrophysics.

Only starting from 1961 or 1962 can one find serious experimental and theoretical studies of the physics of plasma phenomena (Vedenov, Velikhov and Sagdeev, 1961, 1962; Kadomtsev, 1965; Drummond and Pines, 1962; and so on). One can find reviews of the development of experimental studies in the proceedings of various conferences (see the list given in the references). The subsequent years have seen a period of vigorous experimental and theoretical studies stimulat ing whole series of possible practical applications, such as thermonuclear fusion or new ways of accelerating particles. This research has led to the creation of the physics of collective plasma processes in its present form. Qualitative and quantative agreement of the theoretical results with the results of many laboratory studies is the basis for the analysis of plasma phenomena in astrophysics using solidly established facts rather than weakly based hypotheses.

It has been possible to change gradually in plasma physics from an analysis of separate phenomena to the solution of self-consistent problems in which diverse appearances of collective plasma processes can be expressed in terms of a few basic parameters. On the one hand, plasma turbulence spectra were studied (Kadomtsev and Petviashvili, 1963; Pikel'ner and Tsytovich, 1969a; Sharp and Hamberger, 1970) while, on the other hand, several people studied particle acceleration processes which are important in astrophysics (Tsytovich, 1966a; Kadomtsev and Tsytovich, 1970; Tverskoĭ, 1968; Wentzel, 1971) and others studied particle radiation (Gaĭlitis and Tsytovich, 1964a; Kaplan and Tsytovich, 1969a; Tsytovich and Chikhachev, 1969, 1971).

As a result it became possible by combining the solution of self-consistent problems and the results of astrophysical observations to arrive at fairly reliable conclusions about the nature of the plasma processes in the various astrophysical objects. Of course, when the theory and the accumulation of observational data progress such conclusions will be made more precise. The interest in astrophysics is increasing quickly even amongst experimentalists who are involved in laboratory studies of plasmas, and even more amongst theorists for whom astrophysics is a vast field in which to apply theoretical ideas.

The natural interest of astrophysicists in these problems has been hampered by the absence of a book or detailed reviews which presented the modern theory of the very complex plasma phenomena at the right level and in a form which is convenient for a direct application to astrophysics. Thus the appearance of the present book may be thought justified. It is based mainly upon original investigations by the authors and therefore does not pretend to give a complete presentation of plasma phenomena in astrophysics although the authors have endeavoured to cover the most important aspects of the field.

One of the authors (V. N. Tsytovich) is a physicist who has, in particular, studied nonlinear processes in plasmas and plasma turbulence since 1962, working on a number of different aspects of applying the theory to the interpretation of astrophysical phenomena. The other author (S. A. Kaplan) is an astrophysicist who has, in particular, been occupied with applying to theoretical astrophysics new methods in physics. Since 1966 the authors have collaborated in solving a number of astrophysical problems in which collective plasma processes—such as the emission by relativistic electrons and ions, the theory of scattering of electromagnetic radiation in a turbulent plasma, or plasma emission mechanisms of pulsars—play a basic role.

The authors considered the task of writing a book in which fairly complicated topics would be presented, if possible, in an intelligible form without sacrificing the exactness and generality of a qualitative description of the physics of the various phenomena and their mathematical formulation. The book is not intended solely for astrophysicists. The problems of the theory of the interaction of fast particles with a plasma, their radiation, and the behaviour of a plasma in a very strong magnetic field are of interest also for the study of laboratory plasmas.

The first chapter of the book gives at a relatively elementary, but physically rigorous, level the general ideas of the nature of collective plasma processes and of plasma turbulence, using the method of elementary excitations, developed by one of the authors and expounded in more detail in other monographs (Tsytovich, 1970, 1971a, 1972b). This chapter serves essentially as an introduction to the other three chapters which contain the main contents of the book and are devoted to the actual analysis of plasma processes on the Sun (Chapter 2), in objects such as galactic nuclei and quasars (Chapter 3) and in pulsars (Chapter 4). The authors have attempted to write the general theoretical part of these chapters independent of the actual estimates, bearing in mind possible future applications of the theory and probable new astrophysical discoveries. The same purpose is also served by the detailed tables of the probabilities of various processes in the Appendix—where we also indicate methods to apply them—where we have given in a convenient form also the probabilities which were not directly used in the book.

In writing Chapters 2 to 4 the authors felt the need to give a large number of calculations and these chapters contain not only the original analysis of possible interpretations of the observational data but also new theoretical results. Of course, the interpretation of the observational data reflects the point of view of the authors, although other possibilities are also discussed. However, such discussions are probably incomplete and the authors therefore apologize to those research workers whose results for one reason or other—and the restrictions on the size of the book are not the least amongst those—could not be reflected. The whole book was written by the two authors together. A number of new theoretical calcula-

tiions, which were particularly important for the writing of Chapters 3 and 4 and also of the tables, are due to Tsytovich, who also composed the tables of the Appendix. The subject matter of the book has been discussed many times by the authors with their colleagues at the Lebedev Physical Institute, the Gor'kiĭ Radio-Physical Research Institute, the Shternberg Astronomical Institute, the Dubna Institute for Nuclear Studies, the Institute of Cosmic Studies, and the Department of Astrophysics at Leningrad University. To all of them the authors express their deep gratitude. One of the authors (V. N. Tsytovich) had the opportunity to take part in seminars on a number of problems in plasma astrophysics during the time he spent in 1970 at Oxford (and also at Cambridge and Jodrell Bank) and he is very grateful to the participants of these discussions, especially to Professor ter Haar. These discussions have been taken into account in the writing of the book. In particular, the development of the idea of radiative compression as a mechanism for the formation of the pulsed pulsar emission (Tsytovich, Buckee and ter Haar, 1970 a, b) enabled the authors of the present book to work out a more detailed presentation of the properties of an ultra-relativistic plasma in a very strong magnetic field and of the mechanisms of its emission which is the basis of the presentation in Chapter 4.

The topics of the present book are based upon lectures given by V. N. Tsytovich at the Moscow State University (in 1968 to 1971) and in the Institute of Cosmic Studies (in 1971) and by S. A. Kaplan at the Gor'kiĭ State University (in 1968 to 1971).

The authors express their deep gratitude to S. B. Pikel'ner for his careful reading of the book and many useful hints which have been taken into account in the final version of the book, to V. L. Ginzburg, B. B. Kadomtsev, L. M. Ozernoĭ, E. V. Suvorov and V. Yu. Trakhtengerts for reading the manuscript and making a number of important and useful remarks, and also to N. S. Petrukhin, S. I. Kozlova, K. B. Murashkina and M. L. Fedotkina for their help in preparing the manuscript.

The editor V. I. Rydnik has done much to improve the book and the authors express also to him their deep gratitude.

S. A. KAPLAN
V. N. TSYTOVICH

Preface to the English Edition

THE English edition of this book will appear less than a year and a half after the publication of the Russian edition. The development of observational astrophysics proceeds so fast that even now we could have added appreciably to those sections of the book which gave a brief summary of the observational data. We felt, nevertheless, that in accordance with the aim of our book the main additions should refer to the general physical and plasma processes in the universe. The most important ones refer mainly to Chapter 4, dealing with pulsars, where we have given a more detailed treatment of the properties of an ultra-relativistic plasma and its instabilities, of the processes connected with the anomalous electrical conductivity of the circum-pulsar plasma, of the pulsar wind, and of the mechanism to transfer the rotational energy of the pulsar to the energy in the plasma and in the electrical field.

We should like to express our great gratitude to Professor D. ter Haar, who took upon himself the troublesome task to translate and edit the English edition, for many important suggestions and advices which greatly assisted us in improving the book.

S. A. KAPLAN
V. N. TSYTOVICH

Introduction

General discussion of the role of plasma processes under cosmic conditions

It is well known that the Universe is a plasma. Only a very insignificant part of the matter in the Universe is in the solid state and a completely negligible part is in the liquid state. Of course, not every gas is a plasma, but under cosmic conditions even the regions of neutral hydrogen or the atmospheres of cold stars, in which the number of ionized atoms is less than a thousandth of the number of neutral atoms, show plasma properties.

Astrophysicists studied plasmas even when the concept of a plasma had not yet been invented. (Langmuir introduced the term "plasma" in 1929 when describing the properties of matter in gas-discharge tubes.) Indeed, one of the most important stages in the development of astrophysics was when Milne in 1921 developed a theory of stellar atmospheres based upon an analysis of the temperature-dependence of the degree of ionization of a gas (Saha equation) which made it possible to elucidate the main features of stellar spectra. The existence of different regions in the interstellar medium of non-ionized (H I) and ionized (H II) hydrogen, discovered by Strömgren in 1939, had very important consequences for the whole of physics of the interstellar gas and for the theory of stellar evolution. These are only two examples from many.

However, when considering the matter in the Universe to be a plasma, astrophysicists considered only one aspect of its physics until recently. Let us explain this point.

The main problem of astrophysics is the analysis of the spectra of celestial objects in the widest sense of the word. Indeed, if we exclude the interplanetary medium which nowadays is intensively studied experimentally by means of cosmic probes, and also the planets which in the near future (already realized for the Moon, Venus and Mars) will also be studied through direct measurements on their surfaces, all other astrophysical objects (stars, nebulae, galaxies, quasars, pulsars and so on) can only be studied through the analysis and interpretation of their radiative spectra, including the optical and radio-bands, and X-ray, ultraviolet and infrared radiations. Some (albeit relatively meagre) information is also obtained from corpuscular radiation (cosmic rays and neutrinos).

The main task in astrophysics is thus the interpretation of spectra. To do this we must, however, firstly know the theoretical radiation mechanisms, and secondly be able to determine the state in which the systems which emit the electromagnetic waves find themselves. For a long time, especially when there was only optical astronomy, only the emission of light by atoms was considered in astrophysics. The term "astrophysics on the basis of atomic theory" then appeared. The connection between astrophysics and atomic physics was mutually beneficial: not only did astrophysics make a wide use of the results of the

theory and experiments in atomic physics, but it provided itself many data which helped in the understanding of atomic processes (for instance, in the field of the study of metastable states and forbidden transitions). The role of plasma physics was then very small: it was used only for estimating the degree of ionization of the atoms and for determining the free electron concentration.

At the end of the forties and the beginning of the fifties radio-astronomy started to develop vigorously. Atomic physics does not contribute much to the interpretation of radio-spectra: one uses it for the analysis of the observed radio-lines (of hydrogen, hydroxyl, water, formaldehyde, and so on). First of all, we have bremsstrahlung mechanisms: during the acceleration and deceleration of electrons passing near ions radiation is produced which in astrophysics is often called the radiation emitted in free–free transitions, and also magneto-bremsstrahlung is produced (which is called cyclotron radiation in the case of non-relativistic electrons, and synchrotron radiation in the case of relativistic electrons). The role of plasma physics is here already greater: the state of the plasma determines the conditions for the generation of the radiation through those mechanisms and the conditions for the propagation of the radio-waves.

However, radio-astronomy cannot be satisfied by bremsstrahlung mechanisms only. A study of the sporadic radio-emission from the Sun and particularly the discovery in the sixties of powerful radio-sources (radio-galaxies, quasars, pulsars) demanded from physics the "invention" of effective radiation mechanisms which are able to convert speedily different forms of energy into electromagnetic waves. Moreover, it became clear that the acceleration of relativistic particles under cosmic conditions is not a rare phenomenon, but a regularly occurring and often encountered process. Here already one requires far more from plasma physics. We must include in the analysis of the spectra of celestial bodies the most profound plasma properties.

The essential difference between a plasma and a normal gas lies not only in the fact that it consists of ions and electrons and not of neutral atoms alone. It is important to note that due to the long range of the Coulomb forces and the mobility of the light electrons a basic role is played in the plasma by collective processes, that is, oscillations and waves. While in an ordinary gas we can consider merely sound waves which are practically unconnected with electromagnetic radiation, or the very low-frequency Alfvén waves, there are in a plasma several types of waves with a variety of spectra which are in one way or another connected with electromagnetic fields and thus also with electromagnetic radiation. Especially important is the case of the so-called collisionless plasma in which the electron mean free path is much longer than the characteristic dimensions of the collective phenomena. Just in such circumstances do the vibrational and wave properties of the plasma appear fully. If we bear in mind that the spectra of many cosmic objects (especially in the radio-band) are formed in a collisionless plasma, it becomes clear that plasma phenomena must play a large role in the interpretation of these spectra.

We can thus speak of the appearance of "astrophysics on the basis of plasma theory". We must here expect, of course, that the connection between astrophysics and plasma physics will be to their mutual advantage: astrophysics will not only use the data obtained by experimenters and theorists working with a "terrestrial" plasma but itself must find new results which are generally of interest for plasma physics. Even further, while atomic

physics has got from astrophysics little in comparison with what it has given to astrophysics, plasma physics has received from astrophysics much more than it has given. Astrophysics has posed plasma physics new problems, and found for it concrete applications for problems in plasma physics which earlier had been considered to be abstractly theoretical. It seems to us that the great value of astrophysics for plasma physics merits the introduction of the term "plasma astrophysics" as a section of physics and astrophysics which solves its own specific class of problems.

In how far is plasma astrophysics different from laboratory plasma physics?

In laboratory plasma physics studies one is usually concerned with systems of small dimensions. The plasma in laboratory arrangements is therefore almost always transparent for electromagnetic radiation of sufficiently high frequencies. In other words, the laboratory plasma is optically thin for such frequencies. Of course, it generates intensive electromagnetic waves (for instance, near the plasma frequency) but even those can not be absorbed in the plasma itself.

In plasma astrophysics the position is the reverse. The dimensions of the plasma systems are huge, and their optical thickness can be large. This means that an appreciable part of the energy contained in the plasma can change into electromagnetic radiation of relatively high frequencies. In a cosmic plasma high-frequency electromagnetic waves can excite an intensive plasma turbulence or, on the contrary, lead to a stronger dissipation of the turbulence. The first difference between plasma astrophysics and laboratory plasma physics lies thus in the appreciably stronger interaction with the radiation.

Waves in a plasma can accelerate charged particles, transferring to them their own energy. These processes are often observed under laboratory conditions, but it is difficult then to obtain particles with high energies, again because of the limited size of the apparatus. For instance, if the Larmor radius of the particles is larger than the size of the apparatus, the particles leave the acceleration regime and hit the walls and are in general wasted. The dimensions of cosmic plasma systems are so large that even with a relatively low initial energy the charged particles can be accelerated to ultra-relativistic energies. Plasma astrophysics differs thus from laboratory plasma physics also in the appreciably larger role played by fast and relativistic particles (cosmic rays) the acceleration of which in the cosmic plasma is a necessary consequence of the collective processes occurring in this plasma. Cosmic rays often play an important role in many of the phenomena discussed here.

And, finally, there is a third difference. In all laboratory studies of a plasma a large role is played by its inhomogeneity and confinement (by walls or magnetic fields). This inhomogeneity leads to the appearance of new types of waves (such as drift waves) and new forms of instabilities which hamper so much the solution of the basic problem of plasma physics —the construction of thermonuclear devices. On the other hand, the cosmic plasma can be considered to be practically homogeneous: the characteristic lengths over which its parameters change are many orders of magnitude larger than the characteristic lengths of the collective processes. There are also no abrupt boundaries to the cosmic plasma. Theoretical models of a homogeneous plasma correspond thus much better to a cosmic than to a laboratory plasma.

Introduction

The three particular aspects of plasma astrophysics mentioned here make it so specifically different both from general plasma physics and also from astrophysics that, in our opinion, the introduction of the term "plasma astrophysics" is justified.

Plasma turbulence astrophysics

It is difficult to define the whole class of problems which are studied or will be studied by plasma astrophysics. For instance, the many problems connected with the study of the propagation of electromagnetic waves in the cosmic plasma could also be included in plasma astrophysics. One could also include the study of stable and unstable configurations of cosmic objects and even the analysis of the degree of ionization under different circumstances. These problems are far from having been solved with the same degree of completeness and there is no sense in trying to discuss them in the framework of a single book.

We shall restrict ourselves to one problem of plasma astrophysics which seems to us to be of most interest and importance since just in this problem the deepest plasma properties appear to the greatest extent. We are talking here about plasma turbulence astrophysics.

A plasma is a system with a very large number of degrees of freedom. In particular, this leads to the existence of a large number of different kinds of plasma waves with different frequencies and wavenumbers. It is well known that in any system the energy always tends to a more or less uniform distribution over all possible degrees of freedom. This means that if the plasma has got a substantial amount of energy in the form of the excitation of intensive waves of one mode in a narrow frequency range, the interaction between the waves will inevitably lead to a redistribution of the energy over all possible modes and frequencies. The role played by the interactions between the waves is here equivalent to the role played by collisions in a gas.

The state of a plasma in which waves of one or of several modes are excited in a wide range of frequencies and of wavenumbers is called plasma turbulence. The state of excitation is maintained by a source for the turbulence energy.

The cosmic plasma can be both in a quiescent and in a turbulent state. The latter occurs when the plasma is under the influence of a strong external non-stationary action. As examples we mention a powerful radiation flow or its sudden flare-up, fast particle currents, gas explosions, or shock waves. Such processes often happen in the Universe.

We shall not be interested in a quiescent plasma. Such a plasma emits like an ordinary hot gas, that is, there are no powerful sources of radiation. Particle acceleration cannot occur in it either.

We shall also not be interested in all turbulent plasmas. One can divide the turbulence in a plasma into two kinds which are to some extent independent of one another: the high-frequency, peculiarly plasma turbulence, and the low-frequency, the so-called hydrodynamic (or magnetohydrodynamic) turbulence. The last kind of turbulence consists for incompressible, subsonic flow of vortex motion, and for supersonic flow in a distribution of waves (in particular, shock waves) with frequencies much less than the average collision frequency. The dimensions of these motions (wavelengths) may be comparable to the dimensions of the whole system. As a rule, therefore, hydrodynamic turbulence determines in cosmic conditions the structure, form, energy content, and motion of an astrophysical object.

The peculiarly plasma turbulence consists of an excitation in the plasma of a number of plasma waves of different types at frequencies much higher than the collision frequency. The wavelengths of these waves are many orders of magnitude less than the characteristic dimensions of the astrophysical objects. The energy density of these waves can either be much less than the density of the thermal and kinetic energy of the plasma, or of the same order of magnitude as that energy. The peculiarly plasma turbulence therefore turns out to affect the structure and motion of celestial bodies much less than hydrodynamic turbulence. In this case something else is important.

Small-scale plasma turbulence determines the most powerful electromagnetic radiation of the plasma and the acceleration by the plasma of particles to ultra-relativistic energies. That is its main property. An indirect effect of the peculiarly plasma turbulence on the dynamics of celestial bodies and also on hydrodynamic turbulence is connected with the fact that the energy of the accelerated particles turns out to be comparable with other forms of energy, and is also connected with the change in plasma properties when the plasma is in a turbulent state (one has anomalous resistivity, anomalous viscosity and thermal conductivity, and turbulent heating). We shall be concerned with the astrophysics of the peculiarly plasma turbulence which deals with radiation mechanisms which are specific for a plasma, with the interaction between high frequency electromagnetic and plasma waves, with the acceleration of particles by waves in the plasma, and with anomalous dissipation effects.

As a rule, the connection between hydrodynamic and plasma turbulence is a weak one: the corresponding scales of the motions are too different. However, such a connection may also be very important in many cases. Hydrodynamic turbulence with steep fronts (shock-waves) can generate high-frequency plasma turbulence very effectively. The low-frequency acoustic waves may interact with plasma waves. Although we shall not consider hydrodynamic turbulence, we shall take such interactions into account.

The main aim of the present book is an exposition of the problem of the generation of radiation and the problem of particle acceleration in a turbulent plasma. This is what we shall call plasma turbulence astrophysics. Of course, to do this we must also know the physics of plasma turbulence and give an analysis of the excitation and damping of plasma waves and consider the transfer of energy between different kinds of waves.

It is probable that it is not yet possible to state at this moment how large, indeed, is the role played by plasma turbulence in the general picture of all processes occurring in the Universe. It is not excluded that it plays an important role in many old problems. We shall give here only some general considerations about this point.

When interpreting radiative spectra of powerful sources one usually starts, as we mentioned, with the synchrotron hypothesis: one assumes that the radiation occurs because relativistic electrons move in magnetic fields. The concentration and energy spectrum of the electrons, and the intensity and direction of the magnetic field, are then chosen in such a way that the whole picture is internally consistent and that the intensity of the radiation corresponds with observations.

Such an analysis, however, cannot be considered to give a theoretical explanation of the phenomenon. The source of the relativistic electrons and the particular choice of their spectrum and concentration, which are needed for the explanation of the observed intensity

of the radiation, remain problems, among many others, which are not answered. The magnitude and structure of the magnetic field are also assumed and not explained. The example of the pulsars shows that the problem of the source of the fast electrons is a non-trivial one, as there these particles lose energy in a fraction of a second and therefore in that case powerful acceleration mechanisms must be in action.

Of course, we do not dispute the value of the synchrotron radiation mechanism under cosmic conditions. We wish only to show how astrophysicists have usually set out on their analysis of the observational data and how here even in the simplest cases there remain many unsolved problems.

We shall start in the present book from the hypothesis that plasma turbulence will play a large role and will in many cases determine the energy spectrum of the relativistic particles, their concentration, and the powerful radio-emission. We shall see that in the framework of this single hypothesis one can find an explanation of many astrophysical phenomena. The fact that with its aid one can explain both the acceleration of the particles and their radiation is, in our view, an important advantage for it, at least from a heuristic point of view.

In a number of cases plasma emission mechanisms can compete very successfully with synchrotron mechanisms. But even when they are relegated to a second place, the plasma acceleration of particles guarantees a sufficient supply of relativistic electrons for the synchrotron mechanism so that here also the plasma turbulence hypothesis is a step forward.

Apparently plasma turbulence is also important for the analysis of the origin of cosmic rays. Up to now this problem has been solved, as we mentioned, from the point of view of the energy balance. It has, for instance, been established that in supernova explosions sufficient energy is released for the acceleration of as many particles as are necessary to explain the concentration of relativistic electrons in our Galaxy. However, essentially there has been no analysis of the physical mechanisms which would guarantee the acceleration of these particles. The plasma turbulence hypothesis enables us to give such an analysis and, in particular, to explain which concrete conditions (plasma density, radiation level, intensity of the turbulent pulsations, and so on) are necessary in order that this acceleration indeed can take place and in order that one can obtain an almost universal power spectrum for cosmic rays with an index $\gamma = 2{\cdot}5$ to $2{\cdot}7$ (see Chapter 3).

We wish to note one more point in conclusion. Recent discoveries in astronomy, such as quasars and pulsars, are evidence of the fact that the acceleration of some fraction of particles in the cosmic plasma to relativistic energies is a widely extended phenomenon. In this connection it has been stated that here a violation of the second law of thermodynamics takes place. It looks at first sight that here we have not a dissipation of energy which changes it into heat, but that on the contrary a part of the energy is transferred to fast particles, accelerating them even further. It is as if the energy changes to a more organized form. Of course, there is no contradiction with the second law of thermodynamics and the whole paradox can be explained as follows.

If energy is transferred to the plasma, it must in final reckoning be dissipated and be changed into heat. The entropy will then, of course, increase. If the energy dissipation is determined only by collisions the entropy in a dilute plasma increases slowly. The excitation of plasma turbulence steeply increases the energy dissipation and therefore strongly speeds

up the increase in entropy. At the same time the rapid increase in the entropy of the main part of the system makes it possible to decrease the entropy in a small part of the system—of course, in such a way that the total entropy of the system increases. This then leads to the possibility of a rapid acceleration of a fraction of the particles.

In general, the decrease in entropy in a small region at the cost of an appreciable and fast increase in the larger part of the system is a basic feature of any organization under cosmic conditions and, in particular, of life and civilization.

Thus also in plasma turbulence the acceleration of particles is a consequence of the fast dissipation of energy over different types of plasma waves. Plasma turbulence astrophysics can thus have a bearing upon the most general and profound problems in physics and cosmology.

Statement of the problems in plasma astrophysics and contents of the present book

When starting the work on the present book, which is to a large extent based upon the research of the authors themselves, we felt the necessity to discuss the physical basis for modern theories of plasma turbulence. The extensive journal and the as yet sparse monograph literature is on the whole almost inaccessible to readers who are interested not so much in the development of plasma theory rather than in the application of its results to other parts of physics and astrophysics.

We give therefore in the first chapter of the present book as far as possible, without loss of physical rigour, an elementary exposition of the modern theory of turbulent processes in a plasma. It is true that we have put the emphasis on those problems which are of most interest for astrophysicists but the discussion of the basis itself of the physical processes in a turbulent plasma is given in an as far as possible general form. At the end of the chapter is a brief qualitative description of the results of a number of experimental studies of a turbulent plasma which illustrate and underline the theoretical ideas which are the basis also of plasma astrophysics.

In order to make the essential features clear we give in that chapter, as a rule, approximate and translucent derivations of the various relations. However, the final expressions are always written down in their exact form. Plasma theory provides us with many expressions which describe in a general form different processes occurring in a turbulent plasma (Kadomtsev, 1965; Karpman, 1968; Kadomtsev and Karpman, 1971; Tsytovich, 1971a). Many of those expressions can be simplified (for instance, by averaging over angles). In the present book, both in the main text and also in the Appendix, we give usually just those simplified (but not approximated) expressions for the probabilities of different processes. We hope that the use of these formulae will turn out to be more convenient than referring to the original general results which the reader can find in the literature quoted.

When using the results of the theory of a turbulent plasma in astrophysics we must bear in mind the following. When studying a problem in the physics of a "terrestrial" plasma we can split it into parts. For instance, we can evaluate the radiation by electromagnetic waves for a given spectral density of Langmuir turbulence or calculate the particle acceleration assuming all other parameters to be fixed. However, in astrophysics it is rarely possible to split up the general problem. The particle acceleration is not only determined by the

turbulence spectra, but in turn the latter depend on the energy distribution of the accelerated particles. Electromagnetic waves may generate plasma turbulence, but afterwards they will be scattered by the same turbulence.

We must thus in plasma astrophysics state and solve so-called self-consistent problems taking into account the interconnection of all phenomena in a given problem. Of course, this complicates the mathematical description of the corresponding problems considerably, but a qualitative analysis and in many cases also a relatively complete quantitative analysis (within the limits of astrophysical accuracy) is entirely possible.

From this it follows that we must look for a solution of the self-consistent problems of plasma astrophysics not in their general form but bearing in mind actual astrophysical objects and the conditions obtaining in them. It is true that there is a specific "astrophysical" difficulty: we do not observe what is necessary to produce a clear picture of the phenomenon nor the basic characteristics, but that which can be measured with the available means of observation. Often we see in that case secondary details while the basic ones escape us: in the fullest sense of the word "we cannot see the wood for the trees". It is clear that under those conditions many theories turn out to be untenable when more essential data become available. That is the fate of many (if not all) astrophysical problems. However, one may think that if a theory is constructed without internal inconsistencies it has all the same a well-defined astrophysical and physical value even if the phenomenon which this theory aims to describe turns out to be in fact more complicated.

It appears thus to be expedient to formulate the problem as follows: is it possible to construct, starting from the known physical conditions in some astrophysical object, a self-consistent theory without internal contradictions on the basis of plasma turbulence, which describes the phenomena of interest to us and explains the basic observational data? It is possible that it does not succeed to explain all known observational data. If one encounters pronounced contradictions, the theory must be abandoned. However, if there are no such contradictions, it is possible that the lack of explanation of the observational data can be understood in the framework of the same theory after one has been able to accumulate new data and to refine the theory.

The separation of the book into chapters devoted to different problems in plasma astrophysics is determined by the physical properties of the plasma in which the turbulent processes develop.

In the second chapter of the book we consider a non-relativistic plasma in which there is a small admixture of fast epithermal, but non-relativistic particles. The plasma turbulence is here created by the currents of these particles or by large-scale motions such as, for instance, shock waves, or by the fast compression of a region under the action of external forces such as, for example, a magnetic field gradient near neutral lines. In a plasma in which turbulence is developing particle acceleration is also possible, but we are mainly interested in the generation of electromagnetic radiation of electromagnetic radiation at frequencies close to the plasma frequency. The study of these problems has a direct bearing on the interpretation of the sporadic radio-emission of the Sun which is reflected in the title of that chapter.

In the third chapter we consider a non-relativistic plasma in which there is a considerable admixture of ultra-relativistic electrons. The characteristic feature of the turbulence of such a plasma is the generation of high-frequency electromagnetic radiation and the important role

played by particle acceleration mechanisms. In the plasma "reactors" of such a medium power-law particle energy distributions are produced which are so characteristic for cosmic conditions and high-power radiation is generated. Such reactors can exist in such objects as quasars or galactic nuclei, which is reflected in the title of the chapter. A characteristic trait of these objects is also the generation of high-power radio-emission while emitting particles.

A completely relativistic plasma which finds itself in a strong magnetic field has specific properties and the interest in it was stimulated by the discovery of pulsars. Apparently in such a plasma also we find the generation of plasma turbulence reactors, which are considered in the fourth chapter.

Each of these chapters has a similar structure. At the beginning of the chapter we give a summary of the observed data which enable us to determine the nature of the physical conditions in which the plasma turbulence of the objects considered occurs. Then follow a few "theoretical" sections in which we consider qualitatively and sometimes also quantitatively the properties of the plasma turbulence applicable to the conditions of these objects. Finally, in conclusion, we discuss different models for the phenomena considered and discuss briefly different interpretations with their positive and negative aspects. We do not pretend that our interpretation is the best one. The most important thing for us was the construction of an internally self-consistent physical picture, using modern plasma turbulence theory and applying it to a number of observed astrophysical data.

At the end of the book we discuss the old, but as yet not completely solved, problem of the origin of cosmic rays and we also indicate possible ways for a further development of plasma astrophysics.

CHAPTER I

The Physics of Plasma Turbulence

We discuss in this chapter some general problems in the theory of plasma turbulence. Some aspects of the theory were developed with special reference to an analysis of "terrestrial" plasmas, but many of the methods and results can immediately be taken over for plasma astrophysics. However, we shall put most emphasis on problems connected with electromagnetic radiation which plays such an important role in astrophysics, and we shall choose from the general theory only those aspects which are most necessary for the further treatment. We do not consider actual astrophysical objects, although we give a few numerical estimates which are relevant to cosmic conditions.

§ 1. The frequencies and waves of plasma turbulence

Characteristic frequencies of plasma processes

It is well known that the basic quantity which characterizes the collective processes in a plasma is the electron plasma or Langmuir frequency:

$$\omega_{pe} = \sqrt{\left(\frac{4\pi e^2 n_e}{m_e}\right)} = 5{\cdot}65 \times 10^4 n_e^{1/2} \text{ s}^{-1}, \tag{1.1}$$

where e and m_e are the electron charge and mass, while n_e is the electron density in cm^{-3}. We can define also the ion plasma frequency:

$$\omega_{pi} = \sqrt{\left(\frac{4\pi Z^2 e^2 n_i}{m_i}\right)}, \tag{1.2}$$

where Ze is the charge of the ions. In a cosmic plasma the ions are usually protons. In that case $Z = 1$, $m_i = 1840\, m_e$, and as the hydrogen density is much larger than the density of all other elements taken together we can put $n_i \approx n_e$. In that case we have $\omega_{pi} \approx \omega_{pe}(m_e/m_i)^{1/2} \approx \omega_{pe}/43$.

The free electron density varies under cosmic conditions within wide limits, and the plasma frequencies therefore also differ widely. If the fast particle density is small, the majority of the more important collective plasma processes occurs at frequencies close to ω_{pe} and at the same frequencies the strongest plasma emission will also take place. It is thus expedient to

compare ω_{pe} with the frequency bands of the electromagnetic radiation observed in astrophysics.

Stars which are not too hot (with a surface temperature less than 8000°K) have chromospheres—extended external layers with electron densities of $\sim 10^{10}$ to 10^{14} cm^{-3}. In the chromosphere we have thus $\omega_{pe} \approx 6\times10^{9}$ to 6×10^{11} s^{-1} which correspond to the centimetre or millimetre radio-bands, which are easily accessible by modern radio-astronomy. The chromosphere of the Sun belongs to this class of objects and its radio-emission has been studied in great detail.

Above the chromosphere we have the outermost layer of the Sun, the corona. So far we have observed a corona only for the Sun. Here the electron density falls from 3×10^{8} cm^{-3} to much smaller values with increasing height. As an average we can take the estimate $n_e \approx 10^{8}$ to 10^{6} cm^{-3} and therefore $\omega_{pe} \approx 6\times10^{8}$ to 6×10^{7} s^{-1}. These frequencies fall in the well-studied metre band. This is the reason why the solar corona was the first object for plasma astrophysics. Chromospheres and coronae of other stars have just recently become accessible to observations but the conditions in these objects can also be estimated theoretically. Moreover, one must note that it has been possible to observe radio-bursts of stars of the UV Ceti type, and these apparently originate in the coronae of these objects.

A similar density ($n_e \approx 10^{8}$ to 10^{6} cm^{-3}) may occur in quasars and also, perhaps, in the central regions of galaxies, but in those cases radiation at high frequencies plays a more important role due to the abundance of fast particles.

In the interplanetary, the interstellar, and even more the intergalactic, space the electron density is very small. In most of interstellar space $n_e \approx 0{\cdot}02$ cm^{-3}, that is, $\omega_{pe} \approx 10^{4}$ s^{-1}. The wavelength of the corresponding radio-waves is 180 km. Such radiation is very effectively absorbed in the interstellar space itself and it is unlikely that it can be studied by observational means. As an estimate for the electron density in intergalactic space we may put $n_e \approx 10^{-4}$ to 10^{-5} cm^{-3} and, thus, $\omega_{pe} \approx 6\times10^{2}$ to 2×10^{2} s^{-1}.

From the point of view of the astrophysics of plasma turbulence we must thus consider the wide band of plasma frequencies $\omega_{pe} \approx 10^{2}$ to 10^{12} s^{-1}, although, of course, the whole of this range is not equally interesting.

The cosmic medium is a plasma in a magnetic field. It is difficult to find a region of astrophysics where magnetic fields do not play some role. The influence of the magnetic field on plasma turbulence is characterized by the electron gyro-frequency,

$$\omega_{He} = eH/m_e c = 1{\cdot}76\times10^{7}\, H \text{ s}^{-1}, \tag{1.3}$$

and by the ion gyro-frequency,

$$\omega_{Hi} = ZeH/m_i c. \tag{1.4}$$

Here c is the velocity of light and H the strength of the external magnetic field in Oersted. To obtain an estimate of the influence of the magnetic field on the plasma turbulence we must compare ω_{He} and ω_{Hi} with the characteristic frequencies of the turbulence. Let us compare ω_{He} and ω_{pe}.

At the surface of the Sun and of most other stars the magnetic field is less than 100 Oe. The corresponding gyro-frequency is $\omega_{He} \approx 10^{9}$ s^{-1}. There are, however, on the solar surface during active periods regions (sun spots) with a very strong magnetic field—up

to 4×10^3 Oe—and then $\omega_{He} \approx 10^{11}\ s^{-1}$. Moreover, we know that there exist the so-called magnetic stars where the magnetic field strength on a large part of the surface reaches values of up to 10^4 Oe. Here we have for the gyro-frequency $\omega_{He} \approx 2\times10^{11}\ s^{-1}$. One usually assumes that the magnetic field strength in the chromosphere is less than 10^2 Oe and in the corona less than 10 Oe, but it can be higher above spots. The gyro-frequency is always appreciably smaller than the plasma frequency. All the same, we shall see that the influence of the magnetic field on plasma turbulence can turn out to be important in some cases.

The magnetic field does not vary very much in interstellar space. On average $H \approx 3$ to 6×10^{-6} Oe. Hence we have $\omega_{He} \approx 10^2\ s^{-1}$ which is small compared to the Langmuir frequency.

There is also a magnetic field in quasars and in galactic nuclei, but its strength is unknown. According to some estimates we have here in the external regions $H \approx 10^{-4}$ to 10^{-3} Oe. The gyro-frequency is in that case much smaller than the plasma frequency. Near pulsars, on the other hand, the magnetic field can be very large: the field strength might reach values $\sim 10^{12}$ Oe at their surface. The corresponding gyro-frequency, $\omega_{He} \approx 10^{19}\ s^{-1}$, falls even in the X-ray band. Such a plasma has very specific properties.

We have thus mostly in the cosmic plasma the inequality $\omega_{pe} \gg \omega_{He}$. Occasionally one finds that $\omega_{He} \gtrsim \omega_{pe}$. A special case occurs when $\omega_{pe} \ll \omega_{He}$, but this occurs, apparently, only under exceptional conditions (pulsars). We note that the magnetic field affects the magnetic turbulence in the Earth's magnetosphere very strongly, but this problem lies outside the framework of the present book (see, for example, Kennel and Petschek, 1966; Trakhtengerts, 1968, 1970, 1971).

For ion oscillations, except for pulsars, the inequality $\omega_{pi} \gg \omega_{Hi}$ is practically always satisfied under cosmic conditions. Hence, the magnetic field plays a very large role for very-low-frequency plasma turbulence for frequencies less than ω_{Hi}. Of course, these frequencies lie outside the radiation bands observed in astrophysics, but all the same many consequences of such turbulence turn out to be very important.

The next quantities which are characteristic parameters for plasma processes are the thermal velocities of the particles (v_{Te} for the electrons and v_{Ti} for the ions) and a characteristic sound velocity v_s:

$$v_{Te} = \sqrt{\left(\frac{T_e}{m_e}\right)}, \quad v_{Ti} = \sqrt{\left(\frac{T_i}{m_i}\right)}, \quad v_s = \sqrt{\left(\frac{\xi_e T_e + \xi_i T_i}{m_i}\right)}, \tag{1.5}$$

where T_e and T_i are the temperatures of the electron and the ion gas in energy units, $\xi_e = 1$, $\xi_i = 0$ in a collisionless plasma, and $\xi_e = \xi_i = \frac{5}{3}$ in the region of frequent collisions. We must in a turbulent plasma always distinguish between T_e and T_i as the electrons and ions are heated up differently. The case when $T_e = T_i$, which is common in the cosmic medium, will be called an isothermal plasma. One also rather often under cosmic conditions meets with the case when $T_e \gg T_i$—because of the turbulent heating of the electrons—especially in shock-wave fronts and other turbulent regions. It will later be necessary for us to determine v_s.

Under cosmic conditions the electron and ion temperatures are different, but we can distinguish regions of cold plasmas (stellar atmospheres and chromospheres, interstellar

medium, outside regions of quasars) where $T_e \approx 10^4$ °K ($\sim$ 1 eV or 10^{-12} erg), and regions of hot plasmas (solar corona) where $T_e \approx 10^6$ to 10^7 °K ($\sim 10^2$ to 10^3 eV or 10^{-10} to 10^{-9} erg). In the first case the electron thermal velocity $v_{Te} \approx 3\times 10^7$ cm/s, and in the second case $v_{Te} \approx 3\times 10^8$ to 10^9 cm/s. The intermediate case occurs seldom. In dense regions in the interstellar gas the temperature is very low, but the free electron density is also very small so that the plasma properties of the gas sometimes do not come to the fore. Much higher temperatures are also met with: such a plasma can be called near-relativistic ($T_e \approx m_e c^2$) or ultra-relativistic ($T_e \gg m_e c^2$). We shall later consider an ultra-relativistic plasma in detail (Chapter 4), but in the present chapter we shall restrict ourselves only to the case when $T_e \ll m_e c^2$.

An important parameter determining the collective properties of a plasma is the Debye radius

$$d_e = \frac{v_{Te}}{\omega_{pe}} = \sqrt{\left(\frac{T_e}{4\pi e^2 n_e}\right)} = 7\sqrt{\left(\frac{T_e(\text{in °K})}{n_e(\text{in cm}^{-3})}\right)}, \quad d_i = \frac{v_{Ti}}{\omega_{pi}} = \sqrt{\left(\frac{T_i}{T_e}\right)}\cdot d_e. \quad (1.6)$$

It is well known that in a normal plasma violation of quasi-neutrality of the plasma (that is, the equality of positive and negative charge densities) is restricted to volumes of order d_e^3.

The Debye radius is very small in stellar atmospheres ($d_e \approx 10^{-3}$ cm). It is larger in the corona and in quasars ($d_e \approx 0{\cdot}1$ to 1 cm) and even larger in interstellar space ($d_e \approx 10^2$ to 10^3 cm). We shall often deal with the magnitude of the Debye radius in what follows.

We need also the parameter

$$N_D = n_e d_e^3, \quad (1.7)$$

the so-called Debye number. Under cosmic conditions N_D is always several orders of magnitude larger than unity (from 10^2 at the stellar surface to 10^8 in the corona and 10^9 in interstellar space).

The inequality $N_D \gg 1$, on the one hand, means that within the limits of the regions in which quasi-neutrality may be violated there are many free electrons, while, on the other hand, this condition is equivalent to $e^2 n_e^{1/3} \ll T_e$, which means that the average energy of the Coulomb interaction between the charges in the plasma is much less than their kinetic energy.

The next characteristic parameters are the frequencies of collisions of electrons with electrons or ions,

$$\nu_e = \frac{\Lambda_e}{3(2\pi)^{3/2}} \frac{\omega_{pe}}{N_D} \approx \frac{\omega_{pe}}{N_D}, \quad (1.8)$$

or of ions with one another:

$$\nu_i = \frac{\Lambda_i}{3(2\pi)^{3/2}} \left(\frac{T_e}{T_i}\right)^{3/2} \frac{\omega_{pi}}{N_D} \approx \left(\frac{m_e}{m_i}\right)^{1/2} \left(\frac{T_e}{T_i}\right)^{3/2} \nu_e. \quad (1.9)$$

Here Λ_e and Λ_i are the so-called Coulomb logarithms which depend logarithmically on the densities n_e and n_i and on the temperatures T_e and T_i [for instance, $\Lambda_e = \ln(4\pi N_D)$]. These quantities vary between the limits 20 and 50 in the cosmic plasma. Usually one finds

a value $\Lambda \approx 30$. We shall in what follows neglect the logarithmic dependence and use for ν_e and ν_i the right-hand sides of equations (1.8) and 1.9), which are sufficiently accurate for the study of plasma turbulence in astrophysics.

Types (modes) of collective waves in a plasma

The study of different kinds of normal modes in a plasma is possibly the largest and most researched part of present-day plasma physics. The general picture is made complicated by the fact that there are many characteristic frequencies and many possibilities for the polarization of the waves. We cannot dwell upon all peculiar waves in a plasma, but that is not necessary either. For the analysis of plasma turbulence and even more strongly for plasma astrophysics we need only clearly establish the basic characteristics of these waves and the limits within which their parameters change. We shall give below these data both in formulae and in graphs; for a more detailed discussion we refer to the books and reviews by Ginzburg (1970), Stix (1962), Shafranov (1967), Ginzburg and Rukhadze (1970), and Akhiezer, Akhiezer, Polovin, Sitenko and Stepanov (1967).

One can characterize any wave by the following parameters:

(i) The dispersion relation $\omega = \omega(\boldsymbol{k})$, that is, the connection between the wave frequency ω and the wavevector $\boldsymbol{k}$. The wavevector is directed along the normal to the wavefront and its magnitude is equal to the wavenumber $k = 2\pi/\lambda$, where λ is the wavelength.

(ii) The phase velocity $v_p = \omega/k$ and the group velocity $\boldsymbol{v}_g = d\omega/d\boldsymbol{k}$ of the waves. The phase velocity is directed along the wavevector $\boldsymbol{k}$ and the group velocity is the gradient of the function $\omega(\boldsymbol{k})$ and determines the magnitude and the direction of the transport of energy in the wave.

(iii) The decay (or growth) rate γ of a wave. The quantity γ^{-1} characterizes the time during which the wave amplitude decreases (or increases) by a factor e.

(iv) The limits within which the parameters ω, k, v_p, and v_g can lie. These limits are determined, on the one hand, by the conditions which make it possible that there exists a connection between ω and $\boldsymbol{k}$, and, on the other hand, by the necessity that there is a relatively small absorption of the wave energy. If the absorption is strong, the waves practically do not exist.

(v) The polarization of the waves. Waves in a plasma can be either transverse (with the electric field vector perpendicular to the direction of the wave propagation), or longitudinal (with the electric field vector along the vector $\boldsymbol{k}$), or more complicated. The transverse polarization can be linear, circular or elliptic. The distinction between transverse and longitudinal waves is important for the analysis of the interaction and absorption of waves.

We must bear in mind that waves in a turbulent plasma have peculiar properties which are connected with their very strong interaction, and as a result to some extent the uniqueness of the connection between frequency and wavenumber is lost. The dispersion relations have thus only an approximate character. Nevertheless, if the characteristic time for the interactions is much longer than the wave period, we can use for those waves the usual

dispersion relations. To emphasize the physical difference between turbulent waves and normal linear waves, we shall call the former plasmons. For what follows it will be sufficient to restrict ourselves to simplified dispersion relations only.

We shall consider the waves in a plasma in order of decreasing frequency.

1. *High-frequency electromagnetic waves* (indicated by a superscript t). These we shall take to be the normal electromagnetic radiation at frequencies well above all characteristic plasma frequencies ($\omega^t \gg \omega_{pe}$, $\omega^t \gg \omega_{He}$, and so on). For astrophysical problems it is sufficient to use the simplest dispersion relation for quasi-longitudinal propagation:

$$\omega^t(\boldsymbol{k}) = \sqrt{\left(\frac{\omega_{pe}^2}{1 \pm (\omega_{He}/\omega^t)\cos\vartheta} + c^2k^2\right)}. \tag{1.10}$$

Here ϑ is the angle between the wavevector $\boldsymbol{k}$ and the magnetic field $\boldsymbol{H}$. The phase and group velocities differ little from the light velocity c, and $v_p \geqslant c$, and $v_g \leqslant c$. A useful relation is also $v_p v_g = c^2$ which is exact when $\omega_{He} = 0$.

The polarization of these waves depends strongly on the magnitude of the magnetic field (that is, on the ratio of ω_{pe} and ω_{He}) and the direction of wave propagation. The polarization picture is rather complex, but the following points are important. If the wavenumber satisfies the inequality

$$k \ll \sqrt{\left(\frac{\omega_{He}\omega_{pe}}{2|\cos\vartheta|}\right)} \frac{\sin^2\vartheta}{c}, \tag{1.11}$$

two independent high-frequency transverse electromagnetic waves with circular polarization will propagate through the plasma. The electric field vector of one of these waves rotates around the direction of propagation of the wave with an angular velocity equal to the wave frequency in the same direction as the rotation of a positive charge in the given magnetic field. This wave is called the ordinary wave, indicated by an index "o" and its dispersion relation is determined by equation (1.10) with the plus-sign. The rotation of the electric vector of the second wave (index "e") is in the same direction as the rotation of an electron in the given magnetic field, there is the minus-sign in the dispersion relation (1.10), and this wave is called the extraordinary one. The velocities of the ordinary and the extraordinary waves are close to the velocity of light, but the difference between the two velocities can turn out to be important because of the large path traversed by the waves in the cosmic medium.

The extraordinary wave is (for propagation in the direction of $\boldsymbol{H}$) somewhat retarded as compared to the ordinary wave. This leads to a slow rotation of the polarization vector which is obtained when we add the rotating polarization vectors of both waves, that is, it leads to the well-known Faraday effect. The Faraday effect plays an important role in astrophysics as it enables us to reach conclusions about the direction of the magnetic fields from the observational data. This effect is studied in detail in the theory of the propagation of electromagnetic waves in a plasma and we shall not consider it here. We merely give the expression for the magnitude of the angle of rotation of the polarization plane $\psi(R)$ when the wave has traversed a distance R:

$$\psi(R) = \frac{2\omega}{c^2}|v_p^{(o)} - v_p^{(e)}|R = \frac{\omega_{pe}^2\omega_{He}}{c\omega^2} R\cos\vartheta. \tag{1.12}$$

The direction vectors of the phase and group velocities are also not the same when the effect of a magnetic field is taken into account, but the angle between them is small and is of no interest for our further analysis.

A very characteristic property of any wave in a plasma, and thus also of the electromagnetic waves, is its damping rate, that is, the quantity which is the reciprocal of the characteristic time after which the wave intensity is decreased by a factor e. We shall denote the damping rate by the symbol γ with the appropriate index; when there is absorption this quantity is taken to be negative (when the wave is building up its growth rate is positive and is also denoted by the symbol γ). One often uses in physics, and especially in astrophysics, the absorption coefficient μ, a quantity which is the reciprocal of the distance over which the wave intensity decreases by a factor e. Between the damping rate γ and the absorption coefficient μ we have the following relation:

$$\mu = -\gamma/v_g. \tag{1.13}$$

The use of the damping rate instead of the absorption coefficient is convenient if only because the group velocity does not enter into it and this group velocity is different for waves with different frequencies. Moreover, it is convenient to compare γ with the wave frequency and with other parameters of the plasma turbulence.

Of course, there exists a wave only if $\gamma \ll \omega$. The larger the Q-factor of the wave, that is, the quantity ω/γ, the longer it will exist and the more probable is the occurrence of its interacting with other waves.

The basic mechanism for the absorption of high-frequency electromagnetic waves is the damping of the electrons oscillating in the wave when they collide with other electrons or ions. Therefore we find

$$\gamma^t = -\nu_e\left[\frac{\omega_{pe}}{\omega^t \pm \omega_{He}\cos\vartheta}\right]^2 \approx -\left(\frac{\omega_{pe}}{\omega^t}\right)^2 \frac{\omega_{pe}}{N_D}. \tag{1.14}$$

It is, however, well known that there are other mechanisms for the absorption of electromagnetic waves, which we shall study below (conversion of these waves into other kinds of plasma waves, synchrotron absorption, and so on). Here we remind ourselves of only one of them, namely, cyclotron absorption. If the frequency of the electromagnetic wave turns out to be a multiple of the gyrofrequency, that is, if $\omega^t = s\omega_{He}$, where $s = 1, 2, 3, \ldots$ is an integer, strong absorption occurs because of the damping of electrons moving in the wave in the external magnetic field. The damping rate turns out to be a complicated function of the wave direction: in particular, for electromagnetic waves with $s \geqslant 2$, travelling strictly along the magnetic field lines, it vanishes, and this happens also for waves propagating at right angles to the field. We note that for not too large s (for instance, $s = 1$ to 4) and $\vartheta \neq 0$ or $\frac{1}{2}\pi$ this damping is rather large in a hot plasma. The most effective cyclotron absorption occurs for the extraordinary wave with $\omega \approx \omega_{He}$ when the electrons rotate in phase with the wave. This effect is, incidentally, observed in the sporadic radioemission of the Sun.

The frequencies of the electromagnetic waves stretch thus from magnitudes of the order of ω_{pe} (both when $\omega_{pe} > \omega_{He}$ and when $\omega_{pe} < \omega_{He}$) to infinity with possible gaps at the frequencies $s\omega_{He}$.

2. *Transverse plasmons* (indicated by the superscript p). It is convenient and from a physical point of view appropriate to consider separately electromagnetic waves with frequencies very close to ω_{pe}. We shall call those waves transverse plasmons. When $\omega_{He} \ll \omega_{pe}$, their dispersion relation is:

$$\omega^{p}(k) = \sqrt{(\omega_{pe}^2 + c^2k^2)} \approx \omega_{pe} + c^2k^2/2\omega_{pe}. \tag{1.15}$$

The last equality can be used when $k \ll \omega_{pe}/c$.

The phase and group velocities are equal to

$$v_p = \frac{\omega^p}{k} \approx \frac{\omega_{pe}}{k} \gg c, \quad v_g = \frac{c^2k}{\omega_{pe}} = \frac{c^2}{v_p} \ll c. \tag{1.16}$$

Transverse plasmons have the same polarization properties as high-frequency electromagnetic waves, but under conditions of strong plasma turbulence the polarizations of these plasmons are fast mixed so that it is often not necessary to distinguish between the ordinary and the extraordinary wave (however, sometimes this is important, see § 8). The absorption of transverse plasmons when the damping occurs due to collisions can also be calculated, using equation (1.14).

In other words, the main difference between the transverse plasmons and the high-frequency electromagnetic waves consists in the fact that their phase velocity is much larger than the velocity of light and their group velocity much smaller. This means that transverse plasmons "stick" strongly in the regions where plasma turbulence is excited and are thus more strongly absorbed. Very important is also their rather strong non-linear coupling with longitudinal plasmons.

3. *Longitudinal plasmons* (indicated by the superscript l). These are the well-known electrostatic Langmuir plasma waves.† The electric field vector is in this case along the direction of wave propagation. For these waves, as for ordinary sound waves, the perturbation of the electron density consists in compressions and rarefactions. The difference between them and sound waves consists in the fact that this perturbation is transferred not through particle collisions but through electric fields. If $\omega_{pe} \gg \omega_{He}$, the dispersion relation for the longitudinal plasmons has the form

$$\omega^{l}(k) = \sqrt{(\omega_{pe}^2 + 3v_{Te}^2k^2)} \approx \omega_{pe} + \frac{3v_{Te}^2k^2}{2\omega_{pe}}, \tag{1.17}$$

where the second term is always small compared to the first one.

The phase and group velocities are determined by the relations:

$$v_p = \frac{\omega^l}{k} \approx \frac{\omega_{pe}}{k}, \quad v_g = \frac{3v_{Te}^2k}{\omega_{pe}} \approx \frac{3v_{Te}^2}{v_p}. \tag{1.18}$$

The collisional absorption of the longitudinal plasmons is also described by equation (1.14) and as here $\omega^l \approx \omega_{pe}$, we can simply put

$$\gamma^l = -\nu_e = -\frac{\omega_{pe}}{N_D}. \tag{1.19}$$

† The term "plasmon" was introduced for these waves by Pines (1956).

The Langmuir waves are also absorbed through other mechanisms, in particular, by synchrotron and cyclotron absorption. We shall consider synchrotron absorption in § 12, while cyclotron absorption is important relatively rarely, when ω_{pe}/ω_{He} lies close to a relatively small integer.

For longitudinal waves in general and for Langmuir waves in particular there is yet another very effective mechanism for energy absorption, namely, Landau damping. We shall study the physics of this phenomenon in more detail in the next section. Here we give the formula for the Landau damping rate for the case of longitudinal waves in a plasma without a magnetic field:

$$\frac{\gamma^l}{\omega_{pe}} = -\sqrt{\left(\frac{\pi}{8}\right)} \frac{1}{(k d_e)^3} \exp\left\{-\frac{1}{2}\left(3+\frac{1}{(k d_e)^2}\right)\right\}, \tag{1.20}$$

and we note its most important consequences. If the wavelength of the plasmons are comparable with the Debye radius ($k \sim 1/d_e$) they are so strongly damped that it is completely impossible to talk about their existence. However, if $k \ll 1/d_e$, the damping of the longitudinal plasmons is exponentially small. We shall assume that longitudinal plasmons with $k < 1/3d_e$ are practically not Landau damped while plasmons with $k > 1/3d_e$ on the same grounds do practically not exist. From this it also follows that the phase velocity of longitudinal plasmons is larger than $3v_{Te}$ and their group velocity less than v_{Te}.

We must emphasize the fact, which is very important for what follows, that the phase velocity of plasma waves can change in very wide limits, from $3v_{Te}$ (and in some cases even from a value rather close to v_{Te}) to very large values which, in particular, can be much larger than the velocity of light. The group velocity of longitudinal plasmons is as a rule very small, much less than the electron thermal velocity. Longitudinal plasmons therefore always remain in plasma turbulence regions and practically never manage to get outside the limits of these regions. It is important to note that for the same frequency the group velocity of the longitudinal plasmons is much less than the group velocity of the transverse plasmons (of course, this refers only to the case of a non-relativistic plasma: $v_{Te} \ll c$).

4. *Gyro-frequency plasmons* (indicated by a superscript h). We have so far neglected the influence of a magnetic field upon the resonance properties of the plasma and this is not always justified, not even when $\omega_{pe} \gg \omega_{He}$. Moreover, there are cases in astrophysics when ω_{pe} is of the same order as ω_{He} and even when $\omega_{pe} \ll \omega_{He}$. Waves with a characteristic frequency close to ω_{He} will be called gyro-frequency plasmons.

Although the dispersion relations for plasma waves are rather unwieldy when the magnetic field is taken into account, we shall nevertheless write them down (Gershman, 1953):

$$\omega^4 - \omega^2(\omega_{pe}^2+\omega_{He}^2)+\omega_{pe}^2\omega_{He}^2\cos^2\vartheta = \omega_{pe}^2 R\left(\frac{\omega_{He}^2}{\omega^2}, \vartheta\right) v_{Te}^2 k^2, \tag{1.21}$$

where the dimensionless function $R(u, \vartheta)$ of the dimensionless argument $u = \omega_{He}^2/\omega^2$ and of the angle ϑ can be written in the form

$$R(u, \vartheta) = \frac{3}{1-4u}\sin^4\vartheta + \left[1+\frac{5-u}{(1-u)^2}\right]\sin^2\vartheta\cos^2\vartheta + 3(1-u)\cos^4\vartheta. \tag{1.22}$$

We shall restrict ourselves to some particular cases. When $\omega_{pe} \gg \omega_{He}$ we have

$$\omega^l(\boldsymbol{k}) = \sqrt{(\omega_{pe}^2 + \omega_{He}^2 \sin^2\vartheta + 3v_{Te}^2 k^2)}, \tag{1.23}$$

$$\omega^h(\boldsymbol{k}) = \sqrt{\left[\omega_{He}^2 \cos^2\vartheta - \frac{\omega_{He}^4}{\omega_{pe}^2} \sin^2\vartheta \cos^2\vartheta - R\left(\frac{1}{\cos^2\vartheta}, \vartheta\right) v_{Te}^2 k^2\right]}, \tag{1.24}$$

The first of these relations shows that all we said earlier about longitudinal plasmons can be "corrected" when the magnetic field is taken into account by simply replacing ω_{pe}^2 by $\omega_{pe}^2 + \omega_{He}^2 \sin^2\vartheta$.

In a strong magnetic field when $\omega_{He} \gg \omega_{pe}$ we have

$$\omega^h(\boldsymbol{k}) = \sqrt{\left(\omega_{He}^2 + \omega_{pe}^2 \sin^2\vartheta + 4\frac{\omega_{He}^2}{\omega_{pe}^2} v_{Te}^2 k^2 \cot^2\vartheta\right)}, \tag{1.25}$$

$$\omega^l(\boldsymbol{k}) = \sqrt{(\omega_{pe}^2 + 3v_{Te}^2 k^2)}\,|\cos\vartheta|. \tag{1.26}$$

The equations (1.23) to (1.26) have much in common and this enables us in many cases, at least qualitatively, to estimate the effect of a magnetic field on many processes in a turbulent plasma. For instance, we shall see below that an analysis of conservation laws is of great value as it enables us to determine the possibility of various processes. To do that it is necessary only to know the dispersion relations which enable us to evaluate the phase and group velocities. The first can be very different here, but always are larger than $3v_{Te}$ as Landau damping occurs, albeit not to the same extent, for all forms of waves considered here. All group velocities are small (Gershman, 1960).

The collision damping rate of these waves is always of the order $\nu_e = \omega_{pe}/N_D$. The cyclotron damping of gyro-frequency plasmons is exponentially small when $k \ll \omega_{He}/v_{Te}$ and vanishes completely for plasmons propagating strictly along the magnetic field.

All waves with the dispersion relation (1.21) are longitudinal: their electric field is along the wavevector (this is also true for the gyro-frequency plasmons in a strong magnetic field).

5. *Whistlers* (indicated by the superscript w). Waves with frequencies $\omega_{pe} > \omega > \omega_{He}|\cos\vartheta|$ (for the case when $\omega_{pe} > \omega_{He}$) cannot propagate in the plasma. We considered in the preceding subsection waves with frequencies $\omega \approx \omega_{He}|\cos\vartheta|$. When the frequency is even lower there appears a new kind of almost transverse waves with frequencies in the range $\omega_{He}|\cos\vartheta| > \omega > \omega_{Hi}|\cos\vartheta|$. Such waves are called whistlers as they were first observed in the form of low-frequency whistlers in atmospherics and produced interference in radio-links. Such waves are called helicons in applications to solid state plasmas.

Whistlers have properties similar to the high-frequency transverse electromagnetic waves. Instead of two waves there remains here one which corresponds to the extraordinary wave, when the angle between the whistler wavevector and the field direction is sharp ($0 < \vartheta < \pi/2$), and to the ordinary wave when the direction of the whistler is the other way ($\pi/2 < \vartheta < \pi$).

Near the upper limit of the frequencies ($\omega \approx \omega_{He}|\cos\vartheta|$) whistlers go directly over into plasma waves with the dispersion relation (1.24), while near the lower limit ($\omega \approx \omega_{Hi}|\cos\vartheta|$) they go over into magneto-sound waves (see below).

The dispersion relation for whistlers is:

$$\omega^{w}(k) = \frac{\omega_{He}|\cos\vartheta|}{\omega_{pe}^2 + c^2k^2}\, c^2k^2 \approx \frac{\omega_{He}|\cos\vartheta|}{\omega_{pe}^2}\, c^2k^2. \tag{1.27}$$

The last equality is valid when $\omega \ll \omega_{He}$. As the frequency range of the whistlers is restricted it follows that their wavenumbers and phase velocities are also restricted:

$$\left.\begin{aligned} \frac{\omega_{pi}}{c\sqrt{(|\cos\vartheta|)}} < k < \frac{\omega_{pe}}{c}\,; \\ \frac{\omega_{Hi}}{\omega_{pi}}\, c < v_p = \frac{\omega_{He}}{\omega_{pe}^2}|\cos\vartheta|\, c^2k < \frac{\omega_{He}}{\omega_{pe}}\, c. \end{aligned}\right\} \tag{1.28}$$

The phase velocities of whistlers are always less than the velocity of light. Their group velocity is twice their phase velocity (when $\omega_{pe} \gg ck$). It is important that the direction of the group velocity of a whistler lies closer to the magnetic field than the direction of the phase velocity. If we denote the angle between the phase and group velocities by φ, we have $\tan(\vartheta - \varphi) = \frac{1}{2}\tan\vartheta$. In other words, we may assume that whistlers preferentially transfer energy along the direction of the magnetic field (in both directions). We must note, by the way, that a whistler is exactly a transverse wave only when $\vartheta = 0$, when also $\varphi = 0$, while otherwise its polarization properties are complicated. However, when the inequalities (1.28) are satisfied, the electric field components will be at right angles to the external magnetic field. Like all plasma waves whistlers are damped due to electron–ion collisions. Their damping rate is $\gamma^{w}/\omega^{w} = -\omega_{pe}(1 + \cos^2\vartheta)/\omega_{He}|\cos\vartheta|N_D$. The presence of a longitudinal component of the electric field of a whistler leads to Landau damping, described by the relation

$$\gamma^{w} = -\sqrt{\frac{\pi}{8}}\, \frac{v_{Te}c^2k^3}{\omega_{pe}^2} \sin^2\vartheta\,|\cos\vartheta| \exp\left[-\frac{\omega_{He}^2c^4k^2}{2\omega_{pe}^4v_{Te}^2}\right]. \tag{1.29}$$

The damping of whistlers is exponentially small when the wavenumber and frequency satisfy the inequalities

$$k \gg \frac{\omega_{pe}^2 v_{Te}}{\omega_{He}c^2}, \quad \omega > \omega_{He}|\cos\vartheta|\,\frac{v_s^2}{v_A^2}, \tag{1.30}$$

where $v_A = c(\omega_{Hi}/\omega_{pi})$ is the so-called Alfvén velocity. When $\cos\vartheta \approx 1$ exponentially small damping can occur only provided $v_A \gg v_s$. Otherwise, and for frequencies which do not satisfy inequality (1.30), the damping of whistlers is rather small:

$$\frac{\gamma^{w}}{\omega^{w}} = -\frac{v_s}{v_A}\sqrt{\left(\frac{\pi}{8}\,\frac{\omega^{w}\sin^4\vartheta}{\omega_{He}|\cos\vartheta|}\right)}. \tag{1.31}$$

The plasma waves which we have considered so far are usually called high-frequency waves. They are characterized by the fact that in these waves mainly the electrons oscillate. A second group of waves for which both electrons and ions take part in the oscillations are called low-frequency waves. We must, however, not forget that even those waves have frequencies much higher than the collision frequency.

6. *Ion-sound waves* (indicated by a superscript s). It is well known that longitudinal sound waves, consisting of compressions and rarefactions, can propagate in any gas. If the propagation of such waves is coupled with the transfer of momentum from particle to particle during collisions, the frequency of such waves must be less than the collision frequency, that is, $\omega \ll \nu_i \approx (T_e/T_i)^{3/2}\omega_{pi}/N_D$. Such waves exist both in an ordinary gas and in an ionized plasma. But in the latter momentum can also be transferred from particle to particle through the electric field. Then the above-mentioned restrictions on the frequency of the wave are not present and sound waves (the so-called ion-sound waves) are thus possible in a plasma with frequencies much larger than ν_i. The dispersion relation for ion-sound waves is

$$\omega^s(k) = \frac{v_s k}{\sqrt{\{1+(kd_e)^2\}}} + \frac{3v_{Ti}^2 k^2}{2\omega_{pi}}, \tag{1.32}$$

where $v_s = \sqrt{(T_e/m_i)}$, when $T_e \gg T_i$; as we saw earlier, it is necessary for the existence of ion-sound waves that this inequality is satisfied. If $k \ll 1/d_e$, the phase and group velocities of the ion-sound waves are the same and are constant, that is, independent of the wavenumber.

From above the frequency of the ion-sound waves is limited by the condition $\omega^s \lesssim \omega_{pi}$. From below there is also a bound on the frequencies of ion-sound waves. We first of all note that at a frequency $\omega^s \approx \omega_{Hi}$ ion-sound waves experience strong cyclotron absorption involving the ions. Therefore, the ion-sound wave branch more or less breaks off at $\omega^s \approx \omega_{Hi}$. However, at even lower frequencies ion-sound waves again become possible: the so-called slow magneto-sound waves (magnetized sound) with a dispersion relation

$$\omega^{ms}(\boldsymbol{k}) = v_s k\,|\cos\vartheta|. \tag{1.33}$$

Such waves occur only when $v_A = c\omega_{Hi}/\omega_{pi} \gg v_s$. When the frequency is lowered even further acoustic collision waves (superscript cs) appear with a frequency $\omega < \nu_i = (T_e/T_i)^{3/2}\omega_{pi}/N_D$. Their dispersion relation has the form

$$\omega^{cs}(k) = \sqrt{\left(\frac{5}{3}+\frac{T_e}{T_i}\right)} v_{Ti}k, \tag{1.34}$$

as long as $\nu_i > \omega^{cs} > (m_e/m_i)^{1/2}\nu_i (\xi_e = 1, \xi_i = \frac{5}{3}$; see (1.5)). It is possible to maintain here a temperature inequality. At yet lower frequencies the electron and ion temperatures become equal and the wave becomes an ordinary sound wave with a dispersion relation $\omega^{cs}(k) = (10/3)^{1/2}v_{Ti}k$.

All sound waves are damped due to ion collisions. When $\omega^s \gtrsim \nu_i$ the damping rate is equal to $\gamma^s \approx -0{\cdot}6\nu_i(T_i/T_e)$. In the frequency range $\nu_i > \omega^{cs} > (m_e/m_i)^{1/2}\nu_i$ the damping rate also decreases: $\gamma^s \approx -(m_e/m_i)^{1/2}\nu_i$. Finally, at very low frequencies when $\omega^{cs} \ll (m_e/m_i)^{1/2}\nu_i$ we have

$$\gamma^{cs} \approx -\sqrt{\frac{m_i}{m_e}}\frac{(\omega^{cs})^2}{\nu_i}. \tag{1.35}$$

Here the damping decreases fast with decreasing frequency. We note that the damping rate changes continuously when we pass through the "isothermalization" frequency which is $\sim (m_e/m_i)^{1/2}\nu_i$.

The longitudinal ion-sound waves experience a rather strong Landau damping. The Landau damping rate for low-frequency waves consists of two parts, an electron (γ_e^s) and an ion (γ_i^s) part. The Landau damping for ion-sound waves with $\omega^s \gg \omega_{Hi}$ is characterized by the damping rate

$$\left.\begin{aligned} &\gamma^s = \gamma_e^s + \gamma_i^s, \quad \gamma_e^s/\omega^s = -\sqrt{(\pi m_e/8m_i)}, \\ &\frac{\gamma_i^s}{\omega^s} = -\sqrt{\frac{\pi}{8}}\left(\frac{v_s}{v_{Ti}}\right)^3 \exp\left[-v_s^2/2v_{Ti}^2\right] = -\sqrt{\frac{\pi}{8}}\left(\frac{T_e}{T_i}\right)^{3/2} e^{-T_e/2T_i}. \end{aligned}\right\} \tag{1.36}$$

The electron Landau damping always strongly limits the length of ion-sound wave trains which can not be larger than $(8m_i/\pi m_e)^{1/2} \approx 70$ wavelengths, unless, of course, there is a wave build-up. The ion damping is exponentially small, if $v_s \gg v_{Ti}$, that is, when $T_e \gg T_i$.

There is no Landau damping for collisional sound waves ($\omega^{cs} < \nu_i$). It is interesting that when we pass through the frequency $\omega^{cs} \approx \nu_i$, both the Landau damping and the collisional damping are of the same order of magnitude ($\gamma^{cs} \approx -(m_e/m_i)^{1/2}\nu_i$). For very-low-frequency waves only the damping (1.35) remains.

Magnetized sound experiences the same electron Landau damping with a damping rate

$$\frac{\gamma^{ms}}{\omega^{ms}} = -\sqrt{\left(\frac{\pi m_e}{8m_i}\right)}. \tag{1.37}$$

In brief, all ion-sound and ion-plasma waves, with the exception of the very-low-frequency sound waves with $\omega^{cs} \ll (m_e/m_i)^{1/2}\nu_i$ are rather strongly absorbed which in turn may lead to a fast heating of the plasma. We note that as primarily the electrons are heated up, the ion-sound waves when they are dissipated produce the conditions for their own existence, namely, a difference between the electron and ion temperatures.

The limitation on possible wavenumber ranges is here connected not with absorption (as in the case of Langmuir waves) but with the corresponding limitation on frequencies as the phase velocities vary little. As $\omega^s \lesssim \omega_{pi}$, the upper limit for the wavenumbers, $k < \omega_{pi}/v_{Ti} \approx (T_e/T_i)^{1/2} d_e^{-1}$, is somewhat larger than for longitudinal plasmons. The ion-sound waves are strictly limited from below in frequency by ω_{Hi}. Hence $k > \omega_{Hi}/v_s$. Similarly for magnetized sound we have $k < \omega_{Hi}/v_s$.

7. *Alfvén and fast magneto-sound waves* (indicated by the superscripts a and m, respectively). These are waves with frequencies much less than ω_{Hi}. Such waves are known from magnetohydrodynamics when $\omega_{Hi} \ll \nu_i$. However, if $\omega_{Hi} \gg \nu_i$ there exist also similar collisionless waves.

The dispersion relation for Alfvén waves is

$$\omega^a(\boldsymbol{k}) = \frac{v_A k\,|\cos\vartheta|}{\sqrt{(1+v_A^2/c^2)}} \approx v_A k\,|\cos\vartheta|. \tag{1.38}$$

Here $v_s \ll v_A \ll c$, where v_A is the above-mentioned Alfvén velocity,

$$v_A = \frac{\omega_{Hi}}{\omega_{pi}} c = \frac{H}{\sqrt{(4\pi m_i n_i)}}. \tag{1.39}$$

The dispersion relation for the fast magneto-sound waves is for $\omega \ll \omega_{Hi}$ and $v_s \ll v_A \ll c$

$$\omega^m(k) = k\sqrt{(v_A^2 + v_s^2 \sin^2\vartheta)} \approx v_A k, \tag{1.40}$$

and for $|\omega - \omega_{Hi}| \ll \omega_{Hi}$

$$\omega^m(k) = \omega_{Hi} + \frac{v_s^2 k^2 \sin^2\vartheta}{2\omega_{Hi}}. \tag{1.41}$$

The phase velocities for Alfvén and magneto-sound waves are, respectively, equal to $v_A \cos\vartheta$ (when $v_A \ll c$) and v_A, while the group velocities are the same in the two cases and equal to v_A. For Alfvén waves the direction of the group velocity is always the same as the direction of the magnetic field. These waves transfer energy along the magnetic field, like whistlers.

The wavenumbers of Alfvén and magneto-sound waves are rather small:

$$k < \frac{\omega_{Hi}}{v_A} = \frac{\omega_{pi}}{c}. \tag{1.42}$$

Magneto-sound waves undergo, like all longitudinal sound waves, Landau damping but with a smaller relative damping rate:

$$\frac{\gamma^m}{\omega^m} = -\sqrt{\left(\frac{\pi m_e}{8m_i}\right)} \frac{v_s}{v_A} \frac{\sin^2\vartheta}{|\cos\vartheta|}. \tag{1.43}$$

The Landau damping for Alfvén waves is appreciably smaller:

$$\gamma^a = \gamma^m(\omega^a/\omega_{Hi})^2 \ll \gamma^m. \tag{1.44}$$

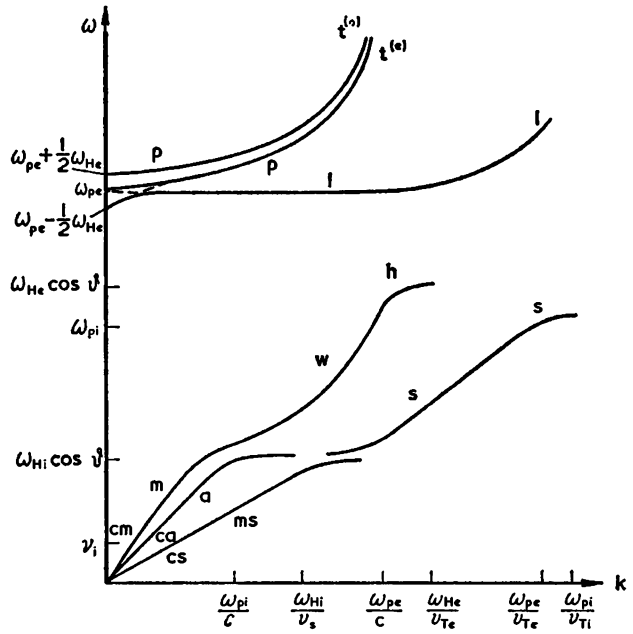

FIG. 1. Qualitative behaviour of the dispersion curves for waves in a plasma which is in a weak magnetic field ($\omega_{pe} \gg \omega_{He}$, but $v_A > v_s$). The curves are labelled by the superscripts in the dispersion relations.

The damping of the Alfvén waves decreases with decreasing frequency. This means that *ceteris paribus* Alfvén waves lose their energy less than all other low-frequency waves. It is true that it is nevertheless impossible to consider Alfvén waves as being altogether undamped in the limit of very small ω^a. First of all, we shall see below that their energy can be transferred to magneto-sound waves and the damping can then occur in the shape of the magneto-sound waves, and, secondly, Alfvén waves can be absorbed during ion–ion collisions with a damping rate of the order of

$$\gamma^a \approx \gamma^{ca} \approx \nu_i \frac{k^2 v_{Ti}^2}{\omega_{Hi}^2}, \qquad \text{when} \qquad v_A < v_{Ti}(m_i/m_e)^{1/4},$$

$$\gamma^a \approx \gamma^{ca} \approx \nu_e \frac{k^2 c^2}{\omega_{pe}^2}, \qquad \text{when} \qquad v_A > v_{Ti}(m_i/m_e)^{1/4},$$

for collisional as well as for collisionless waves.

We have given in Fig. 1 summarizing graphs of all dispersion relations and in Fig. 2 the damping rates for the case when $\omega_{pe} \gg \omega_{He}$.

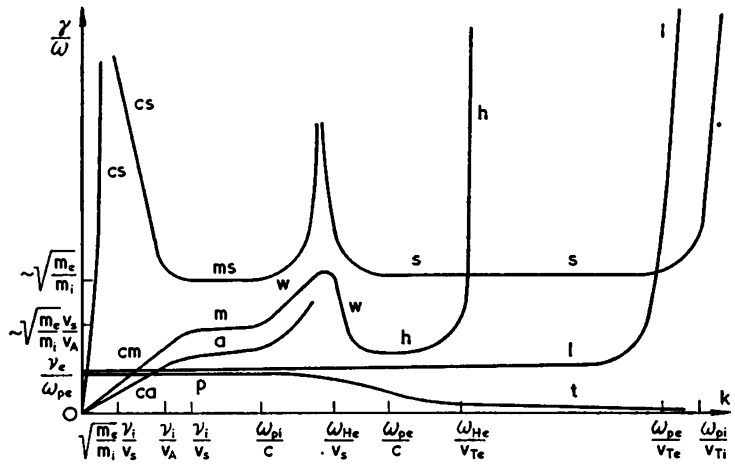

FIG. 2. Qualitative behaviour of the relative damping rates of different waves in a plasma. The curves are labelled by the superscripts in the dispersion relations.

§ 2. Emission and absorption mechanisms for waves in a plasma

For an analysis of plasma turbulence we must first of all know emission and absorption mechanisms for all possible types of waves in a plasma and also the mechanisms for their mutual interactions.

The present section will deal with the first problem, while the next section will deal with the wave interactions. We must, however, first familiarize ourselves with the methods for describing turbulence, namely, spectral densities for the energy and the determination of the number of waves.

ENERGY SPECTRAL DENSITY AND NUMBER OF WAVES IN PLASMA TURBULENCE

We shall denote by W_k with the appropriate superscript (for instance, W_k^l, W_k^t, W_k^a) the amount of energy of waves of a given kind contained in unit volume of coordinate space and in unit volume of wavenumber space. The total energy density of these waves (per unit volume of coordinate space) is thus

$$W = \int W_k \, d^3k = \int W_k \, dk_x \, dk_y \, dk_z . \tag{2.1}$$

Instead of the quantity W_k we also introduce the number of waves per unit phase volume:

$$N_k = \frac{(2\pi)^3 W_k}{\omega(k)} . \tag{2.2}$$

The factor $(2\pi)^3$ arises from the normalization condition in phase space. The number of waves is the phase energy density of these waves (that is, W_k) divided by the energy of one wave (that is, in quantum language, by the quantity $\hbar\omega(k)$, where $\hbar$ is Dirac's constant).

The study of waves in a plasma is part of classical physics so that the appearance of Dirac's constant should, generally speaking, not be necessary. This means that Dirac's constant should fall out of all final expressions, even if we carried it along in all intermediate steps. In order not to have to carry this useless quantity along we shall take the energy to be simply equal to ω, and not to $\hbar\omega$, and the wave momentum to be equal to k, and not to $\hbar k$, as it is in actual fact. If we bear in mind that one really should have instead of (2.2) the formula $N_k = (2\pi^3)W_k/\hbar\omega$, one sees easily that N_k is a dimensionless quantity and, indeed, determines the number of waves per unit volume of coordinate space and per unit volume of wavenumber space.

We shall often assume that the plasma turbulence is isotropic, that is, we shall assume that the quantity W_k depends only on the magnitude of the wavenumber. We can then in (2.1) integrate over the directions of the wavevector which leads to

$$W = \int W_k \, dk = 4\pi \int W_k k^2 \, dk. \tag{2.3}$$

We shall call the quantity W_k the energy spectral density of the plasma waves. This is one of the main characteristics of plasma turbulence. From (2.3) and (2.2) we have

$$W_k = \frac{k^2\omega(k)}{2\pi^2} N_k . \tag{2.4}$$

The spectral density of the energy in (2.3) and (2.4) is taken per unit wavenumber. One could just as well have taken it per unit frequency:

$$W_\omega = W_k \frac{dk}{d\omega} = \frac{W_k}{v_g(k)} , \tag{2.5}$$

where $v_g(k)$ is the group velocity of the waves. In astrophysics one normally uses the energy spectral density per unit frequency. For high-frequency electromagnetic waves when $v_g(k) = c$, it is equally convenient to use W_k or W_ω. The position is different in the case of

plasma waves. We saw § 1 that, for instance, in the case of longitudinal plasmons, the wavenumbers change over a much larger range than the frequencies. Even though the frequency and the wavenumber are uniquely connected it is nevertheless simpler to bear in mind the limitations on k rather than those on ω. It is thus more convenient to use in plasma turbulence the quantity W_k rather than W_ω although, as we emphasize once again, they are uniquely connected.

It is normal in observational astrophysics to measure fluxes of electromagnetic radiation per unit frequency, F_ω. If the solid angle Ω is known under which the source of this radiation is seen, we can also define the spectral intensity $I_\omega = F_\omega/\Omega$ which is the amount of energy emitted per unit time by a unit area per unit frequency into a unit solid angle in the given direction. We shall in what follows evaluate this quantity and compare it with observations, assuming that Ω is known.

It is clear that for isotropic electromagnetic radiation we have

$$W_\omega^t = \int I_\omega \frac{d^2\Omega}{v_g^t} = \frac{4\pi I_\omega}{v_g^t}. \tag{2.6}$$

From this we get a relation which we shall often use in what follows:

$$I_\omega = W_k^t/4\pi. \tag{2.7}$$

All plasma waves, and thus also the electromagnetic waves, are always emitted and absorbed in any plasma in thermodynamic equilibrium. Their distribution over frequencies and wavenumbers is described by the universal Planck formula (in accordance with what we have said earlier the Planck constant will not occur in the formulae). For the number of waves per unit phase volume we have

$$N_k = \frac{1}{e^{\omega/T}-1} \approx \frac{T}{\omega(k)}, \tag{2.8}$$

where T is the temperature of the equilibrium plasma in energy units, and where we have assumed that $T \gg \omega$.

One often uses in astrophysics the concept of the effective temperature of the radio-emission, $T_{\text{eff}}(\lambda)$. This characteristic quantity can also be extended to all waves of plasma turbulence. We can according to (2.4) write in the isotropic case:

$$T_{\text{eff}} = \frac{2\pi^2}{k^2} W_k. \tag{2.9}$$

If the turbulence in the plasma is well developed we have $T_{\text{eff}} \gg T$ for those waves for which this turbulence is excited.

For high-frequency electromagnetic waves equation (2.9) becomes

$$T_{\text{eff}}(\lambda) = \pi\lambda^2 I_\omega, \tag{2.10}$$

where $\lambda = 2\pi/k$ is the wavelength. We have here taken into account the fact that electromagnetic waves have two polarization states.

We shall use many of the characteristic quantities of plasma turbulence which we have introduced here. For more exact calculations we need to know W_k, but in many cases we can for rough estimates limit ourselves simply to W.

The energy density of the plasma waves can be compared with the thermal energy density of the gas. If $W \ll n_e T_e$, we call the turbulence weak and this case will be the topic of our studies. We call the case when $W \approx n_e T_e$ strong turbulence. In that case there is such an intensive interaction between different kinds of waves that after a time of the order of $1/\omega_{pe}$, which is neglibigle for astrophysics, a fast dissipation of the energy of the plasma waves (turbulent heating) has taken place which brings the plasma again into a state with $W \ll n_e T_e$. Unless we make a statement to the contrary we shall as a rule assume that this latter condition is satisfied.

Spontaneous emission mechanisms for waves in a plasma

If the appropriate conditions are fulfilled all kinds of waves in the plasma can be emitted by one mechanism or another.

For the time being we leave aside atomic mechanisms, that is, the emission and absorption of electromagnetic quanta during quantum transitions both between bound states in atoms and between bound and free states. These mechanisms have a certain value for plasma astrophysics in a number of particular cases which we shall, however, not discuss here (Kaplan, Kleĭman, and Oĭringel', 1972).

Well-known are the so-called bremsstrahlung mechanisms, that is, the production of electromagnetic waves when electrons are decelerated or accelerated. The acceleration and deceleration of heavy ions turns out to be ineffective in cosmic conditions. Two bremsstrahlung emission mechanisms are important in astrophysics: the acceleration and deceleration of electrons when they collide with ions and their motion in curved, helical orbits in an external magnetic field (cyclotron and synchrotron emission by relativistic electrons).

Strictly speaking, bremsstrahlung mechanisms correspond also to the scattering of electromagnetic waves by electrons. The scattered wave is emitted as a consequence of the acceleration or deceleration of an electron which is acted upon by the field of the incoming wave. We must distinguish the Thomson effect where the frequencies of the incident and the scattered waves are the same and only the direction of the wavevector is changed, and the Compton effect where the frequency of the scattered wave is different from (larger than or smaller than) that of the incident wave. Other plasma waves are also emitted through these mechanisms with some degree of efficiency.

However, another mechanism which differs from the bremsstrahlung one, turns out to be more effective for the emission of waves in a plasma. Unfortunately, this emission mechanism (the so-called Cherenkov effect†) cannot usually directly generate electromagnetic waves under the conditions of a cosmic plasma. In such a plasma only waves with a phase velocity less than or equal to the particle velocities and thus less than the velocity of light can be emitted. On the other hand, the particles need not be accelerated for the emission

† Vavilov and Cherenkov discovered this emission mechanism experimentally and Tamm and Frank (1937) explained it theoretically. This mechanism is called (in the U.S.S.R.) the Vavilov–Cherenkov effect. However, it is now traditional to speak of "Cherenkov radiation" and "Cherenkov emission mechanism".

through this mechanism: particles moving rectilinearly with a constant velocity also radiate —it is thus a non-bremsstrahlung mechanism. The main point of this mechanism lies in the following. A moving charge polarizes the plasma attracting charges of different sign and repelling charges of the same sign. If the phase velocity of the propagation of the perturbations of the charge density in the plasma is less than the velocity with which the external charge moves, the plasma polarization can in no way move beyond the charge and "tear itself away" from it. This leads to the emission of the energy used to polarize the plasma.

From this it follows that a particle moving with a velocity v can emit all waves with phase velocities $v_p \leqslant v$. Only those waves are emitted for which the component of the particle velocity along the direction of propagation of the wave is equal to the wave phase velocity –emission condition of Tamm and Frank (1937):

$$v_p = v \cos \vartheta, \quad \omega = (\boldsymbol{k} \cdot \boldsymbol{v}). \tag{2.11}$$

In that case the particle remains for a long time in resonance with the wave.

Ceteris paribus, the power of the Cherenkov emission mechanism per unit frequency is comparable with the bremsstrahlung mechanism power, that is, comparable with the power emitted during the acceleration or deceleration of a particle moving with the same speed as the particle emitting the Cherenkov radiation. The advantage of the latter lies, however, in the fact that the emission lasts as long as the particle moves and not only during the periods of the rare collisions between particles or of the very rare cases when quanta are scattered by electrons. Moreover, both electrons and ions produce Cherenkov radiation with the same power. For the excitation of plasma turbulence and for many processes occurring in a plasma (for instance, particle acceleration) this non-bremsstrahlung mechanism is therefore especially important, the more so as there are many waves in a plasma with a small phase velocity.

Plasma waves which have been excited by one of the emission mechanisms considered here can through non-linear interactions—which we shall study in the next section—transform into transverse electromagnetic waves of various frequencies. In final reckoning therefore the Cherenkov mechanism also leads to the production of electromagnetic radiation.

We shall in what follows consider in detail the production of electromagnetic waves in plasma turbulence regions. We shall call those processes, which compete with the normal bremsstrahlung processes for the emission of electromagnetic waves, plasma mechanisms for (electromagnetic) radiation. Doppler shifts can lead to appreciable changes in frequency.

Different cases of the emission of different types of waves by various mechanisms will be considered when we study actual astrophysical objects but we shall discuss now some general properties. We introduce the concept of the probability for the emission of a single wave per unit time per unit wavenumber volume, u_k. In the general case this quantity will depend upon the direction of the emission. If possible, we shall in what follows average the emission probability over the angular variables. In that case, and also generally for isotropic emission, we can express the emission power P_k, that is, the amount of energy emitted by a particle per unit time per unit wavenumber, as follows in terms of the probability u_k:

$$P_k = \frac{k^2 \omega(k)}{2\pi^2} u_k . \tag{2.12}$$

This formula can be used to determine the emission power through its probability or, the other way round, to find the probability, if we know the emission power. We shall give a few examples of such two-fold use of this relation. However, we first note that we can give an order of magnitude estimate for the emission probability (more exact formulae can be found in the Appendix):

$$u_k = 2\pi^2 \frac{e^2 v^2}{[\omega(k)]^2}, \tag{2.13}$$

where v is a characteristic velocity of the radiating particle and $\omega(k)$ the radiation frequency.

The estimate (2.13) needs some explanation. In general the emission probability is determined by several different parameters. For instance, the probability for dipole emission depends on the acceleration of the particle, the probability for Cherenkov emission on the particle velocity, the magneto-brems emission probability on the number of the harmonic, and so on. In each case one can find an exact expression for the probability u_k by radiation theory methods. However, for rough estimates or for an understanding of the physical meaning of complicated relations it is often desirable to have a simple estimate which gives the right order of magnitude. One can, for instance, obtain equation (2.13) by writing down (2.12) for dipole emission and assuming that the acceleration is equal to $v\omega(k)$. Furthermore, (2.13) is also approximately valid for other emission mechanisms such as synchrotron or Cherenkov emission. One must only take into account the averaging over geometric and time factors which reflect the specific aspects of the emission mechanism concerned. We shall show below how this can be done in actual examples. In what follows we shall often compare exact expressions, which are not always so translucent, with expression (2.13).

We shall now give expressions for u_k for some emission mechanisms which are very important for plasma astrophysics.

1. *Thermal bremsstrahlung in electron–ion collisions.* This is a well-known mechanism for the emission of electromagnetic as well as plasma waves in a wide range of frequencies and wavenumbers. The exact expression for the averaged emission probability, taken for a single thermal electron, has the following form:

$$u_k = \frac{\Lambda_e}{3\sqrt{(2\pi)}} \frac{e^2 v_{Te}^2}{\omega^2} \frac{\omega_{pe}}{\omega N_D} = \frac{2\pi}{3} \left(\frac{e v_{Te}}{\omega}\right)^2 \frac{\nu_e}{\omega}. \tag{2.14}$$

Here Λ_e is the Coulomb logarithm. Expression (2.14) differs from (2.13) by the averaging factor ν_e/ω: indeed, the length of time an electron emits at a frequency ω is of the order $1/\omega$ while the characteristic time between collisions is of the order $1/\nu_e$. From this it follows that, when time-averaging the emission probability (2.13) which refers only to the immediate moment of the acceleration or deceleration, one should multiply by a factor ν_e/ω. We must emphasize once again that (2.14) determines the probability for the emission not only of electromagnetic waves but also of any other kind of waves in the plasma.

2. *Cyclotron emission at the first harmonic,* $\omega = \omega_{He}$. If a non-relativistic electron moves along a helical path in a magnetic field—with velocity components $v_\parallel$ and $v_\perp$—it radiates at a frequency $\omega = \omega_{He}/(1 - v_\parallel/v_p)$. In a plasma with thermal electrons this leads to the

appearance of an emission band centred around the frequency ω_{He} and with a width $\Delta\omega \approx 2\omega_{He}(v_{\parallel}/v_p) \approx 2\omega_{He}(v_{Te}/v_p)$. After averaging the cyclotron emission probability of thermal electrons we have

$$u_k = \pi^2 \frac{e^2 v_{Te}^2}{\omega_{He}^2} \frac{v_p}{v_{Te}}. \tag{2.15}$$

The agreement between (2.15) and (2.13) is clear. The factor v_p/v_{Te} is connected with the fact that the radiation is concentrated in a narrow range of frequencies.

3. *Synchrotron emission by relativistic electrons.* It is well known that a wide spectrum of electromagnetic waves is emitted through the synchrotron mechanism with the maximum emission at a frequency

$$\omega = 0{\cdot}3\omega_{He}\left(\frac{\varepsilon}{m_e c^2}\right)^2, \tag{2.16}$$

where ε is the energy of the relativistic electron. As in the case of other mechanisms, synchrotron emission can also take place in the form of plasma waves. We shall consider synchrotron radiation in detail in § 12 and here just give the expression for the probability of the emission of electromagnetic waves at the maximum, that is, at the frequency (2.16):

$$u_k = 1{\cdot}6\pi \frac{e^2c^2}{\omega^2} \frac{\omega_{He}}{\omega} \approx 210 \frac{e^2c^2}{\omega_{He}^2}\left(\frac{m_e c^2}{\varepsilon}\right)^6. \tag{2.17}$$

The occurrence of the factor ω_{He}/ω is connected with the following fact. The synchrotron emission of an ultra-relativistic electron is at any given time concentrated in a narrow solid angle $\sim (m_e c^2/\varepsilon)^2$, directed along the instantaneous particle velocity. When averaging the emission coefficient over all angles, that is, over $d^2\Omega/4\pi$, we must multiply the emission probability by $(m_e c^2/\varepsilon)^2 \approx \omega_{He}/\omega$, that is, the relative solid angle occupied by the instantaneous electron emission.

4. *Thomson scattering by free electrons.* An electromagnetic wave incident upon an electron will make it oscillate with its own frequency ω and this leads to a re-emission of an electromagnetic wave with the same frequency, but in a different direction. Other plasma waves are scattered in the same way and, furthermore, during such a scattering process they can change from one mode of plasma waves to another mode with the same frequency. For instance, when a longitudinal plasmon is scattered it can change into a transverse wave or an ion-sound wave can change into a whistler. We shall consider such a transformation in detail in the next section.

The probability for Thomson scattering of longitudinal and transverse plasmons and of high-frequency electromagnetic waves is determined by the relation

$$u_k = \frac{2(2\pi)^3}{3} \frac{e^4}{m_e^2} \frac{W_\omega}{\omega^3} = \frac{\pi}{3} \frac{\omega_{pe}^4}{n_e^2 \omega^3} W_\omega, \tag{2.18}$$

where W_ω is the spectral density of the scattered radiation, and $\omega = \omega(k)$ is the frequency of the incident and of the scattered radiation.

One can reduce equation (2.18) also to (2.13). Indeed, let E be the electric field of the incident wave. The electron then obtains in this field a velocity $v = eE/m_e\omega$. The field E is connected with W_ω through the relation $W_\omega = E^2/4\pi\omega$. The square of the velocity of the electron which is affected by the incident wave is thus:

$$v^2 = \frac{e^2}{m_e}\frac{4\pi W_\omega}{\omega}. \tag{2.19}$$

Substituting (2.19) into (2.13) and adding a factor $\frac{2}{3}$ which takes into account the averaging over directions, we get (2.18). The derivation which we have given here of the probability for Thomson scattering enables us to elucidate the meaning of non-linear interactions between waves in a plasma.

5. *Compton scattering by relativistic electrons.* Strictly speaking "Compton effect" is the name for the quantum change in frequency when scattering occurs by a free electron, which may be at rest. However, in astrophysics quantum scattering effects are unimportant as long as the frequency of the scattered wave satisfies the inequality

$$\omega' \ll \frac{2m_ec^2}{\hbar}\frac{m_ec^2}{\varepsilon}. \tag{2.20}$$

In astrophysics one now uses traditionally the term Compton effect for the scattering of electromagnetic waves by relativistic electrons with a change in frequency due to the Doppler effect. The frequency of the scattered wave may here be either much larger or much smaller than the frequency of the incident wave. In the first case

$$\omega = \frac{\omega'}{1-(v/c)} \approx 2\omega'\left(\frac{\varepsilon}{m_ec^2}\right)^2: \tag{2.21}$$

energy is taken from the scattering electron, while in the second case

$$\omega = \omega'\left(1-\frac{v}{c}\right) \approx \frac{\omega'}{2}\left(\frac{m_ec^2}{\varepsilon}\right)^2: \tag{2.22}$$

the incident wave gives energy to the electron.

One often calls the case (2.22) the direct, and the case (2.21) the inverse Compton effect. However, neither this nomenclature, nor the distinction between a "direct" and an "inverse" effect are justified from the point of view of physical contents. Nevertheless, we retain here the nomenclature Compton scattering for all cases of an appreciable change in frequency due to the Doppler effect, but we shall not use the terms "direct" and "inverse".

Compton scattering can occur for all forms of plasma waves, and the fact that there occurs then a considerable change in frequency means that now the conditions for the transformation of one kind of plasma wave into another are easily satisfied. For instance, ion-sound waves can change into Langmuir waves when scattered by relativistic electrons, or directly into electromagnetic waves, and vice versa.

We shall now write down an expression for the probability of Compton scattering by relativistic electrons. The exact expressions are rather complicated as for a given frequency

ω' of the incident radiation in the Compton process a wide emission spectrum within a frequency range of the order (2.21) to (2.22) is generated.

We shall study the Compton scattering in detail in § 13. Here we give only the formula for the probability of Compton scattering of electromagnetic waves:

$$u_k = \frac{2(2\pi)^3}{3}\frac{e^4}{m_e^2}\frac{W'_{\omega'}}{\omega^2\omega'}\frac{\omega'}{\omega} = \frac{2(2\pi)^3}{3}\frac{e^4}{m_e^2}\frac{W'_{\omega'}}{\omega^3}. \tag{2.23}$$

The difference between (2.23) and (2.18) is connected with the fact that we must take into account that the scattered radiation is concentrated into a solid angle $(m_e c^2/\varepsilon)^2 \approx \omega'/\omega$, as was done in (2.17) through the addition of a factor ω_{He}/ω.

6. *Cherenkov radiation.* The probability for emission through this mechanism depends strongly on the angular variables and on the ratio of the particle velocity and the phase velocity of the wave. We give in tabular form in the Appendix the probabilities for the emission of various waves, averaged over the angles. Here we give only the probability for the emission of longitudinal plasmons averaged over angles:

$$u_k = 2\pi^2\frac{e^2\omega_{pe}}{k^3 v} = 2\pi^2\frac{e^2v^2}{\omega_{pe}^2}\left(\frac{\omega_{pe}}{kv}\right)^3 = 2\pi^2\frac{e^2v^2}{\omega_{pe}^2}\left(\frac{v_p}{v}\right)^3. \tag{2.24}$$

We must bear in mind that the condition for Cherenkov emission, $\omega_{pe} = (\boldsymbol{k}\cdot\boldsymbol{v}) \leqslant kv$, must necessarily be satisfied. The maximum probability occurs for emission exactly along the direction of motion of the particle when $\omega_{pe} = kv$. We shall consider the case of Cherenkov emission of Alfvén waves in § 12.

Stimulated emission and absorption of waves in a plasma

It is well known that spontaneous processes are always accompanied by stimulated (or induced) processes, leading either to wave absorption or to stimulated wave emission. We shall illustrate stimulated processes through the example of Cherenkov emission. Absorption is here nothing but Landau damping which we discussed in the preceding section. If there is a plasma wave and a charge moving with a velocity somewhat smaller than the phase velocity of the wave and along the wavevector of the wave, the charge will stay rather a long time near, say, the maximum of the wave amplitude. Gradually the charge will begin to lag behind, but the electromagnetic field of the wave will, so to speak, "push" it, giving it its energy. If the charge moves somewhat faster than the phase velocity of the waves, the charge will "push" the wave, increasing its amplitude. If the number of slower charges is higher than that of faster charges (in identical velocity ranges) the total effect will be an absorption of wave energy, and that is Landau damping. The increase in the wave amplitude corresponds to stimulated emission which differs from spontaneous emission through the fact that in the plasma there are waves with the same parameters as those of the emitted waves; these stimulate the emission processes in direct proportion to the number of waves per unit phase volume which are already present in the plasma.

In the well-known Einstein coefficients treatment it is necessary to introduce together with the probability u_k for spontaneous emission also probabilities for induced emission

and induced absorption which are equal to $N_k u_k$, where N_k is again the number of waves per unit phase volume. The balance between induced emission and absorption depends on the level occupations, that is, in our case on the momentum distribution function of the radiating particles:

$$n = \int f_p \frac{d^3\boldsymbol{p}}{(2\pi)^3} = \int f_p \frac{dp_x\, dp_y\, dp_z}{(2\pi)^3}, \tag{2.25}$$

where f_p is the density of radiating particles per unit phase volume and n the particle density per unit coordinate space volume (cm^{-3}). If, as often is the case, f_p depends only on the absolute magnitude of the momentum, p, and not on its direction, we can introduce a momentum magnitude distribution function f_p or an energy distribution function f_ε. We have then

$$f_p = \frac{4\pi p^2}{(2\pi)^3} f_p, \quad n = \int f_p\, dp. \tag{2.26}$$

For ultra-relativistic particles the energy is $\varepsilon = cp$, and we have

$$f_\varepsilon = \frac{4\pi\varepsilon^2}{(2\pi c)^3} f_p, \quad n = \int f_\varepsilon\, d\varepsilon. \tag{2.27}$$

We shall now give some important definitions. The quantity

$$J_k = \int P_k f_p \frac{d^3\boldsymbol{p}}{(2\pi)^3} = \frac{k^2\omega}{(2\pi)^2} \int u_k f_p \frac{d^3p}{(2\pi)^3} \tag{2.28}$$

shall be called the coefficient for spontaneous wave emission per unit volume and unit wavenumber.

In the general case the emission coefficient will be per unit phase volume, $d^3\boldsymbol{k}/(2\pi)^3$. We then have (note that $\int J_k\, dk = \int J_k\, d^3\boldsymbol{k}/(2\pi)^3$)

$$J_k = J_{k,\vartheta} = \omega(\boldsymbol{k}) \int u_k f_p \frac{d^3\boldsymbol{p}}{(2\pi)^3}. \tag{2.29}$$

If the emission is isotropic, which is often the case in plasma astrophysics, at least to a first approximation, the quantities (2.28) and (2.29) are connected through the simple relation:

$$J_k = J_{k,\vartheta} = \frac{2\pi^2}{k^2} J_k. \tag{2.30}$$

If we take induced processes into account we must replace (2.29) by the equation

$$Q_k = Q_{k,\vartheta} = \omega(\boldsymbol{k}) \int u_k [f_p + N_k(f_p - f_{p-k})] \frac{d^3\boldsymbol{p}}{(2\pi)^3}, \tag{2.31}$$

where $Q_{k,\vartheta}$ is the total power (the balance between emission and absorption). We have here taken into account the momentum conservation law when particles emit or absorb waves: $\boldsymbol{p}' = \boldsymbol{p} - \boldsymbol{k}$.

Since in a plasma we have usually for the waves and particles $k \ll p$, we get from (2.31)

$$Q_{k,\vartheta} = J_{k,\vartheta} + \omega N_k \int u_k \left(\boldsymbol{k} \cdot \frac{\partial f_p}{\partial \boldsymbol{p}}\right) \frac{d^3\boldsymbol{p}}{(2\pi)^3}. \tag{2.32}$$

If everywhere in the particle phase volume $(\boldsymbol{k} \cdot [\partial f_p/\partial \boldsymbol{p}])$ is negative, as is the case, for instance, for a Maxwell distribution, the second term on the right-hand side of (2.32) is also negative and describes absorption or, as one often expresses it in astrophysics, reabsorption of waves. However, if the distribution function is anisotropic or if there are other deviations from equilibrium functions which as a rule may be very small, conditions may appear when in some element of the phase volume $(\boldsymbol{k} \cdot [\partial f_p/\partial \boldsymbol{p}]) > 0$. It can then occur that also the whole of the second term on the right-hand side of (2.32) becomes positive. The stimulated emission will then dominate and amplification occurs, that is, a maser effect for the waves concerned.

Stimulated absorption and stimulated emission are described by decay rates and growth rates, respectively, which are determined by the formula

$$\gamma(\boldsymbol{k}) = \int u_k \left(\boldsymbol{k} \cdot \frac{\partial f_p}{\partial \boldsymbol{p}}\right) \frac{d^3\boldsymbol{p}}{(2\pi)^3}. \tag{2.33}$$

If the particle distribution function depends only on their energy, we have for Cherenkov emission:

$$\left(\boldsymbol{k} \cdot \frac{\partial f_p}{\partial \boldsymbol{p}}\right) = (\boldsymbol{k} \cdot \boldsymbol{v}) \frac{\partial f_p}{\partial \varepsilon} = \omega \frac{\partial f_p}{\partial \varepsilon}. \tag{2.34}$$

Strictly speaking, equations (2.33) and (2.34) can be used only when the effect of a magnetic field on the process of emission of waves by particles can be neglected. In the particular case of Cherenkov radiation it is necessary for this that the wavelength is much smaller than the Larmor radius. However, it turns out that when we substitute (2.34) into (2.33) we can use (2.33) also in the more general case, provided we understand by u_k the emission probability averaged over all angles and particle trajectories.

Knowing u_k and the particle momentum distribution function we can easily calculate both the absorption coefficient and the growth rate. In particular, in a plasma with a Maxwellian velocity distribution for the electrons and for the ions (possibly with $T_e \neq T_i$) we get for the damping rate in the electron gas:

$$\frac{\gamma(\boldsymbol{k})}{\omega(\boldsymbol{k})} = -\int \frac{u_k f_p}{T_e} \frac{d^3\boldsymbol{p}}{(2\pi)^3} = -\frac{2\pi^2}{k^2\omega} \frac{J_k}{T_e}, \tag{2.35}$$

and a similar expression for the ion gas. If we now substitute into this expression the formula for the emission probability given earlier, we obtain the damping rates given in § 1. In particular, by substituting the Cherenkov emission probability into (2.35) and using condition (2.11) and a Maxwellian distribution function we obtain the equations describing the Landau damping.

For each actual case of a non-Maxwellian velocity distribution we must proceed separately. However, one can obtain a few general estimates. For instance, let the main plasma

be penetrated by a weak beam of electrons moving with an average velocity v_0 and a velocity spread $\Delta v_0 \ll v_0$; The total electron density in the beam n_* will be much less than the electron density n_e in the main plasma. If $v_0 \gg v_{Te}$ there is a "hump" in the distribution function for the velocity component along the beam. Inside the "hump" $\partial f_p/\partial \varepsilon > 0$ and conditions are produced there which are favourable for the stimulated excitation of plasma waves provided that there are waves with phase velocities larger than $v_0 - \Delta v_0$. The largest value of the growth rate will occur for waves moving along the beam with phase velocities between $v_0 - \Delta v_0$ and v_0.

Let us estimate this for Langmuir waves. Substituting (2.24) and (2.34) into (2.33) we find

$$\frac{\gamma^1(k)}{\omega_{pe}} = 2\pi^2 \frac{e^2\omega_{pe}}{k^3} \int \frac{1}{v} \frac{\partial f_p}{\partial \varepsilon} \frac{d^3p}{(2\pi)^3}. \tag{2.36}$$

An estimate for the integral in (2.36) is $n_*/m_e(\Delta v_0)^2 v_0$, where $m_e(\Delta v_0)^2$ is the energy spread of the beam particles (the beam "temperature"). We then get from (2.36), if we also use the fact that $\omega_{pe}/k \approx v_0$:

$$\frac{\gamma^1}{\omega_{pe}} \approx \frac{\pi}{2} \left(\frac{v_0}{\Delta v_0}\right)^2 \frac{n_*}{n_e}. \tag{2.37}$$

This important formula for the growth rate due to the instability of an electron beam will often be met with in what follows. We can similarly evaluate growth rates of other waves.

Let us turn to equation (2.32). We shall assume that the power $Q_k = (k^2/2\pi^2)Q_{k,\vartheta}$ is used to increase the number of waves in the appropriate phase volume. We then have

$$Q_k = \frac{dW_k}{dt} = J_k + \gamma(k) W_k. \tag{2.38}$$

The solutions of this equation are different for the cases of growth and of damping of waves. Let us first consider the damping case ($\gamma < 0$).

We shall assume that in the plasma with a temperature T there is an initial level of the spectral density of the plasma wave energy, $W_k(0)$. As a result of emission and absorption processes the energy spectral density is after a time t changed to the quantity

$$W_k(t) = \left[\frac{J_k}{|\gamma(k)|} - W_k(0)\right](1 - e^{-|\gamma(k)|t}) + W_k(0), \tag{2.39}$$

which follows from equation (2.38) written down for the isotropic case. In an optically thick plasma after a sufficiently long time ($|\gamma| t \gg 1$) a "constant" thermal level of the plasma wave energy density is established (compare with (2.9)):

$$W_k = \frac{J_k}{|\gamma(k)|} = \frac{k^2 T}{2\pi^2}, \tag{2.40}$$

which is independent of the initial level.

The total energy density of thermal plasma waves is given by the equation

$$W_T = \int_0^{k_{max}} W_k \, dk = \frac{k_{max}^3 T}{6\pi^2}, \tag{2.41}$$

where k_{max} is the maximum allowable wavenumber. For Langmuir waves k_{max} is determined from the condition $\gamma^l \approx \omega_{pe}$, that is, $k_{max} \approx 1/d_e$. We then have

$$W_T^l \approx \frac{T_e}{6\pi^2 d_e^3} \approx \frac{n_e T_e}{6\pi^2 N_D}. \tag{2.42}$$

Similarly we get for ion-sound waves: $k_{max} \approx \omega_{pi}/v_{Ti}$ and we get for the density of their thermal energy:

$$W_T^s \approx \frac{T_i}{6\pi^2 d_i^3} \approx W_T^l \left(\frac{T_e}{T_i}\right)^{1/2}. \tag{2.43}$$

For whistlers and Alfvén waves we have very small values of k_{max} so that the density of thermal plasma waves of those types is small.

In the growth case ($\gamma > 0$) the energy density of the plasma waves increases exponentially with time. The solution of (2.38) has the following form:

$$W_k = \frac{J_k}{\gamma(k)}(e^{\gamma t} - 1) + W_k(0)\, e^{\gamma t}. \tag{2.44}$$

It is, however, valid only as long as we neglect non-linear interactions. It is, on the other hand, important that here the energy density of the plasma waves can be by far greater than the thermal level.

§ 3. Non-linear wave interactions in a plasma

Wave interactions in a plasma have many consequences which are important for plasma astrophysics. For instance, they lead to a transfer of energy from unobservable to detectable spectral ranges. The easiest waves to excite in a plasma are the Langmuir waves and usually they have the largest energy density. However, they do not emerge from that part of the plasma where they originate. Non-linear interactions transfer their energy to transverse waves which can freely leave the plasma. Through wave interactions in the plasma energy can be transferred from one range of wavenumbers to another, where plasma waves can interact strongly with fast particles and thereby a small number of particles can be accelerated to very high energies.

We shall in the present section describe the physical basis of the theory of wave interactions in a plasma and we shall give some formulae which we shall use in the subsequent analysis. In later chapters we shall consider other relations which are relevant for non-linear wave interactions.

When expounding the theory we shall restrict ourselves to a qualitative, non-rigorous description and to writing down the appropriate formulae in their simplest form, which is, however, sufficient for our purpose. The reader can find details of the theory and methods for detailed calculations in the many books and reviews which have appeared (Vedenov, Velikhov and Sagdeev, 1961, 1962; Vedenov, 1967; Kadomtsev, 1965; Galeev, Karpman and Sagdeev, 1965; Gorbunov and Silin, 1965; Kovrizhnykh, 1966; Tsytovich and Shvartsburg, 1967; Tsytovich, 1970, 1971a, 1972b; Kaplan and Tsytovich, 1969a; Pustovalov

and Silin, 1972; Kadomtsev and Petviashvili, 1963; Gailitis and Tsytovich, 1964a). We emphasize once more that the exposition in the present section must be considered to be only a first introduction to a more rigorous analysis which can be found in the literature cited.

Non-linear scattering of waves by thermal ions and electrons in a plasma

There are two basic kinds of plasma wave interactions. The physics of both phenomena is simple although the detailed formulae in both cases turn out to be very cumbersome. However, bearing in mind when these relations can be used in astrophysics and with what accuracy, we give here an elementary derivation of the simplified (that is, averaged over angles) expressions for the probabilities for wave interactions, which is fully sufficient for further calculations.

The first kind of non-linear interactions is, essentially, Thomson or Compton scattering. We saw already in the preceding section that the wave frequency and its direction change when scattering by a free electron occurs. We also noted then that there was a possibility for a change in the type of wave. This is just the interaction between waves of the same or of different types in a plasma. The peculiar circumstances consist in the fact that one cannot consider the thermal electrons in a plasma to be free when one is dealing with the scattering of low-frequency waves: in a volume of size $\lambda^3 \approx (2\pi/k)^3$ there are always many plasma electrons. If waves in the optical band with wavelengths much less than the inter-electron distance are scattered, they will force each electron to oscillate independent of all others. When plasma waves with, say, a frequency ω_{pe} are scattered, each wave can make many electrons to oscillate coherently. When we consider the transformation of plasma waves we must therefore consider their scattering by fluctuations in the electron density. In turn the fluctuations are caused by the motion of the thermal particles in the plasma themselves. These considerations make it possible to draw the following intuitive picture (for a more rigorous basis see Tsytovich, 1970).

Let us consider one, randomly chosen, thermal, charged plasma particle. It creates around itself a polarization cloud. If it is an ion, it attracts neighbouring electrons and repels ions. As a result a "coat" of electrons with an excess negative charge is formed around an ion. The size of this "coat" is of the order of the Debye radius. It moves with the ion. If a plasma wave is incident upon the "coat" the wave is scattered and forms other plasma waves (which may be electromagnetic waves). This process is called non-linear scattering. The scattering by density fluctuations turns out to be equivalent to the scattering by the charges "dressed in the coat". We must here bear in mind that in the polarization region (the "coat") there are rather strong electric fields coupling the electrons and the ion considered so that the displacement of the electrons in the scattering process will immediately affect the motion of the ion. The change in the wave energy occurs thus in the scattering process at the expense of the ion energy. A thermal electron creates a similar "coat", but with an excess positive charge. The scattering picture now, however, turns out to be more complicated. An incident plasma wave (for instance, a Langmuir wave) moves about not only the "coat", but also the "central" electron. Both the "coat" and the electron emit secondary (scattered) waves but as their charges are of different sign, the two emissions are in antiphase with respect to one another. As a result the two scattered waves often cancel one another which is equivalent

to a vanishing of the scattering. In most cases the scattering of Langmuir waves by an electron and its "coat" is thus much weaker than the scattering by the "coat" of an ion. The cancellation is smaller if either the electron energy is of the order of relativistic energies (which occurs most strikingly in the conversion of longitudinal into transverse waves; see Gaĭlitis and Tsytovich, 1964a) or the phase velocities of the plasma waves are so large that the antiphase behaviour of the excitations of the "coat" and the electron is violated. One can show (Tsytovich, 1970) that the latter condition is satisfied, if $v_p > v_{Te}(m_i/m_e)^{1/2}$. In actual fact, however, if we are dealing with scattering involving a change in k of the same order as k, scattering by electrons will be important only when

$$v_p \lesssim \left(3\frac{m_i}{m_e}\right)^{1/5} v_{Te} \approx 6v_{Te}. \tag{3.1}$$

If the phase velocity is larger, the non-linear scattering by ions is more important.

We shall therefore consider in a little more detail the scattering of plasma waves by the "coat" of thermal ions. Let the frequency and wavevector of the incident wave be equal to ω' and $\boldsymbol{k}'$ and those of the scattered wave ω and $\boldsymbol{k}$. As the type of the scattered wave may be different from that of the incident wave, the dispersion relations $\omega'(\boldsymbol{k}')$ and $\omega(\boldsymbol{k})$ can also differ. During the scattering by the "coat", and hence by the ion, the ion loses part of its energy, which we write as $\varepsilon-\varepsilon'$, and part of its momentum, $\boldsymbol{p}-\boldsymbol{p}'$.

It is clear that in the scattering process the conservation laws for the total energy and the total momentum of wave and particle must be satisfied. We write these conditions in the form

$$\varepsilon-\varepsilon' = \omega(\boldsymbol{k})-\omega'(\boldsymbol{k}'), \quad \boldsymbol{k}-\boldsymbol{k}' = \boldsymbol{p}-\boldsymbol{p}'. \tag{3.2}$$

Bearing in mind that the momentum of the wave $\boldsymbol{k}$ is much smaller than the particle momentum $\boldsymbol{p}$ we have

$$\begin{aligned} \omega-\omega' &= c\,[\sqrt{(p^2+m^2c^2)}-\sqrt{(p'^2+m^2c^2)}] \\ &= c\,[\sqrt{(p^2+m^2c^2)}-\sqrt{\{(\boldsymbol{p}-\boldsymbol{k}+\boldsymbol{k}')^2+m^2c^2\}}] \\ &\approx ([\boldsymbol{k}-\boldsymbol{k}']\cdot\boldsymbol{v}). \end{aligned} \tag{3.3}$$

This condition which we shall simply call a conservation law enables us to reach at once several important conclusions. When we analyse the scattering by thermal ions we must put v equal to the ion thermal velocity v_{Ti}. We then have $\omega-\omega' \approx |\boldsymbol{k}-\boldsymbol{k}'|\,v_{Ti}$. If the direction of the wavevector changes appreciably during the scattering, $|\boldsymbol{k}-\boldsymbol{k}'|$ will be comparable with the absolute magnitude k and

$$\omega-\omega' \approx kv_{Ti} \approx \omega\frac{v_{Ti}}{v_p}. \tag{3.4}$$

Since for undamped waves we have always $v_p \gg v_{Ti}$ the change in the wave frequency in the non-linear scattering process will be small. This conclusion is valid for all cases of non-linear scattering by thermal ions, including the cases when the type of the wave is changed.

Knowing the change in frequency we can determine the difference in the absolute magnitudes of the wavevectors $\boldsymbol{k}$ and $\boldsymbol{k}'$. This can be done particularly simply if during the

scattering the type of wave does not change. We have then

$$\omega(k)-\omega'(k') \approx (k-k')\frac{\partial\omega}{\partial k} \approx (k-k')v_g. \tag{3.5}$$

Comparing (3.4) and (3.5) we find

$$\frac{k-k'}{k} \approx \frac{v_{Ti}}{v_g}. \tag{3.6}$$

This quantity can be either much smaller than unity or comparable with unity. We shall call the first case differential scattering as far as the wavenumber is concerned, and the second case integral scattering. In the first case not only the frequency, but also the absolute magnitude of the wavenumber changes little in each scattering process. However, the direction of the wavevector may change considerably. Indeed, condition (3.6) does not restrict the rotation of the vector $\boldsymbol{k}$ to any angle with respect to $\boldsymbol{k}'$. We have thus always integral scattering as far as the angles are concerned. However, it is also possible to have the case where we have integral scattering with respect to the wavenumber, when in a single scattering process the value of the wavenumber also changes appreciably. For scattering by electrons we always have integral scattering as we must instead of (3.6) write down an expression in which v_{Te} enters instead of v_{Ti}, and for Langmuir waves v_g is always less than v_{Te}.

In plasma astrophysics we meet with both differential and integral scattering by ions. However, the first case is the simpler one and we shall illustrate the nature of the relations obtained by evaluating the coefficients for the differential scattering by ions and only afterwards give numerical formulae for the case of integral scattering.

For the most important case of non-linear scattering of a longitudinal plasmon by the "coat" of a thermal ion when the scattered wave remains a longitudinal plasmon we have

$$\frac{\omega-\omega'}{\omega} \leqslant \sqrt{\left(\frac{m_e T_i}{m_i T_e}\right)}\frac{v_{Te}}{v_p} = \frac{1}{43}\sqrt{\frac{T_i}{T_e}}\frac{v_{Te}}{v_p} < \frac{1}{130}; \tag{3.7}$$

$$\frac{k-k'}{k} \lesssim \frac{k_*}{k} = \frac{v_p v_{Ti}}{3v_{Te}^2} = \frac{1}{3}\sqrt{\left(\frac{m_e T_i}{m_i T_e}\right)}\frac{v_p}{v_{Te}} = \frac{1}{130}\sqrt{\frac{T_i}{T_e}}\frac{v_p}{v_{Te}}. \tag{3.8}$$

From this it follows that in an isothermal plasma ($T_e = T_i$) the scattering of plasmons with a phase velocity $v_p < 130\, v_{Te}$ by thermal ions is always differential scattering. In a non-isothermal plasma with $T_e \gg T_i$ this condition comprises an even larger range of wavenumbers.

If the longitudinal plasmon during the scattering by the "coat" of a thermal ion changes into a transverse plasmon, the change in frequency is again small. Equation (3.4) remains valid here. It is true that the wavenumber of a transverse plasmon is much smaller than the wavenumber of a longitudinal plasmon, as follows from the equation

$$\omega^p \approx \omega_{pe} + \frac{c^2(k_p)^2}{2\omega_{pe}} \approx \omega_{pe} + \frac{3}{2}\frac{v_{Te}^2 k^2}{\omega_{pe}} \approx \omega^l. \tag{3.9}$$

However, we can here also introduce the concept of differential scattering with respect to wavenumbers, provided we express all wavenumbers, both those of the longitudinal and those of the transverse plasmons in the same scale.

Let us, say, choose as the basic scale that of the wavenumbers of the longitudinal plasmons. The dispersion relation of the longitudinal plasmons then remains as before: $\omega^{l} = \omega_{pe} + 3v_{Te}^2 k^2/2\omega_{pe}$, while the dispersion relation of the transverse plasmons takes the same form: $\omega^{p} = \omega_{pe} + 3v_{Te}^2 k^2/2\omega_{pe}$ where now the "reduced" wavenumber of a transverse plasmon is $k = (c/\sqrt{3}v_{Te})k_p$. In such a unified scale for the wavenumbers of the longitudinal and the transverse plasmons the conditions that the transfer is differential remain as before, that is, equations (3.7) and (3.8). We must also bear in mind that here the phase velocities of the transverse waves must also be written in a unified scale. We shall often use a unified wavenumber scale for different modes but, of course, the final results will always be reduced to the true scale.

We now turn to the calculation of the probability for the non-linear scattering by the "coat" of thermal ions. It follows from what we have said earlier about the physical essence of this mechanism that in the case of differential scattering this effect is simply Thomson scattering by the charge of the ionic "coat" which numerically is equal to the charge of the ion. The motion of the "coat" of one ion is independent of the motion of the "coat" of another ion so that we can consider each "coat" to be a "free charge" as far as the scattering is concerned. The effective mass of this charge is equal to m_e. For evaluating the probability for the differential scattering by the ion "coat" we can thus simply use equation (2.18).

Let us first consider the case of the differential scattering of longitudinal plasmons when $\omega \approx \omega_{pe}$. Replacing W_ω by W_k^l/v_g we get the following expression for the scattering probability averaged over the angles:

$$u_k = \frac{(2\pi)^3}{9(1+T_e/T_i)^2}\frac{e^4}{m_e^2}\frac{W_k^l}{\omega_{pe}^2 v_{Te}^2 k} = \frac{\pi}{18(1+T_e/T_i)^2}\frac{\omega_{pe}^2 W_h^l}{n_e^2 v_{Te}^2 k}. \tag{3.10}$$

We have used here the fact that the wavenumbers of the scattered and of the incident waves are almost equal. The factor $(1+T_e/T_i)^2$ occurs when the screening of the charges in a non-isothermal plasma is taken into account more rigorously.

As the emission probability is now proportional to W_k, that is, in final reckoning to the number of incident waves, it is expedient to define a scattering probability $u_{k,k'}$ per unit phase volume of both the scattered and the incident wave, that is,

$$u_k = \int u_{k,k'} N_{k'} \frac{d^3k'}{(2\pi)^3}. \tag{3.11}$$

In the case of differential scattering $u_{k,k'}$ has a steep maximum for $\boldsymbol{k} \approx \boldsymbol{k}'$.

Here and henceforth we give scattering probabilities averaged over the angles. Such expressions are simpler and physically translucent. However, in the averaging of the probabilities some special features of the process disappear. In particular, the complete, unaveraged, scattering probabilities contain delta-function factors which take explicitly the energy and momentum conservation (3.2) into account. When averaged, that is, integrated over the angles, these delta-functions disappear. We must therefore when applying (3.10) and other similar expressions always bear in mind that the conditions (3.2) must be satisfied. Moreover, it has turned out to be important in some cases that the scattering is anisotropic. Exact, non-averaged expressions are, for instance, given in the book by Tsytovich (1971a).

As, according to (3.10), u_k is independent of the particle momentum, we have from (2.28) for the coefficient for spontaneous emission per unit volume:

$$J_k(\mathrm{l}+\mathrm{i} \rightleftharpoons \mathrm{l}'+\mathrm{i}') = \frac{\omega_{\mathrm{pe}}^3 W_k^{\mathrm{l}} k}{36\pi(1+T_\mathrm{e}/T_\mathrm{i})^2 n_\mathrm{e} v_{T\mathrm{e}}^2} = \frac{m_\mathrm{e}\omega_{\mathrm{pe}}^3}{36\pi(1+T_\mathrm{e}/T_\mathrm{i})^2} \frac{W_k^{\mathrm{l}} k}{n_\mathrm{e} T_\mathrm{e}}. \tag{3.12}$$

We have used here the fact that in a hydrogen plasma the ion and electron densities are equal ($n_\mathrm{e} = n_\mathrm{i}$) and also put $k' = k$, as we are dealing with differential scattering. The notation $\mathrm{l}+\mathrm{i} \rightleftharpoons \mathrm{l}'+\mathrm{i}'$ indicates symbolically the scattering of a longitudinal plasmon by an ion which turns it into another longitudinal plasmon while changing the ion energy.

In a similar fashion transverse plasmons are also created when a longitudinal plasmon is spontaneously scattered by the "coat" of an ion. In terms of the unified wavenumber scale the probability for the differential scattering of longitudinal into transverse plasmons, averaged over the angles, does not differ from the same probability for the scattering of longitudinal into longitudinal plasmons. A difference appears, however, when we analyse the angular dependence of the scattering probability and this is clearly connected with the different polarizations of longitudinal and transverse plasmons. The probability for the scattering process $\mathrm{l}+\mathrm{i} \rightleftharpoons \mathrm{l}'+\mathrm{i}'$ is proportional to $(\boldsymbol{k}\cdot\boldsymbol{k}')^2$ while the probabilities for the scattering processes $\mathrm{l}+\mathrm{i} \rightleftharpoons \mathrm{p}'+\mathrm{i}'$ and $\mathrm{p}+\mathrm{i} \rightleftharpoons \mathrm{l}'+\mathrm{i}'$ are proportional to $|[\boldsymbol{k} \wedge \boldsymbol{k}']|^2$. This difference disappears in the averaging.

When we consider the spontaneous conversion of longitudinal plasmons into transverse ones in a differential scattering process we can thus use equations (3.10) and (3.12) in the unified wavenumber scale. To change to the true wavenumber scale for the transverse plasmons we multiply (3.10) by $k_\mathrm{p}^2\omega_{\mathrm{pe}}/2\pi^2 = 3v_{T\mathrm{e}}^2 k^2 \omega_{\mathrm{pe}}/2\pi^2 c^2$; we then get

$$J_{k_\mathrm{p}}(\mathrm{l}+\mathrm{i} \rightleftharpoons \mathrm{p}'+\mathrm{i}') = \frac{\omega_{\mathrm{pe}}^3}{12\pi(1+T_\mathrm{e}/T_\mathrm{i})^2} \frac{W_\mathrm{k}^{\mathrm{l}} k}{n_\mathrm{e} c^2}, \tag{3.13}$$

where $k = (c/\sqrt{3}\cdot v_{T\mathrm{e}})k_\mathrm{p}$.

We can evaluate in a similar way the spontaneous conversion of all plasma waves in differential scattering processes. We must then always first of all analyse the conservation laws from which follows that in all scattering by thermal ions the change in frequency is small. Therefore only the following transformations are possible: $\mathrm{l} \rightleftharpoons \mathrm{l}'$, $\mathrm{p} \rightleftharpoons \mathrm{p}'$, $\mathrm{l} \rightleftharpoons \mathrm{p}$, $\mathrm{w} \rightleftharpoons \mathrm{s}$, $\mathrm{a} \rightleftharpoons \mathrm{ms}$, and $\mathrm{a} \rightleftharpoons \mathrm{m}$.

As in the usual radiation processes, to each scattering process, including non-linear scattering, there correspond stimulated scattering mechanisms, the balance of which can lead either to absorption or to the amplification of emission.

In scattering "absorption" is simply the inverse process. If in the spontaneous scattering considered above the wave $\boldsymbol{k}'$ changes into the wave $\boldsymbol{k}$, in the absorption process the wave $\boldsymbol{k}$ changes to the wave $\boldsymbol{k}'$. Induced emission is a stimulated process when the change from the wave $\boldsymbol{k}'$ to the wave $\boldsymbol{k}$ is amplified when there are in the medium waves $\boldsymbol{k}$. When analysing induced processes we can use the same relations as we had in § 2 when we considered induced emission and absorption processes.

Instead of (2.33) we now have

$$\gamma(k) = \int u_{k,k'} N_{k'} \left([\boldsymbol{k}-\boldsymbol{k}'] \cdot \frac{\partial f_p}{\partial \boldsymbol{p}} \right) \frac{d^3p\, d^3k'}{(2\pi)^6}, \tag{3.14}$$

where we have used the definition (3.11) of $u_{k,k'}$ and the fact that the change in the particle momentum in the scattering process is equal to $\boldsymbol{k}-\boldsymbol{k}'$. Instead of (2.34) we have for a Maxwell ion distribution

$$\left([\boldsymbol{k}-\boldsymbol{k}']\cdot\frac{\partial f_p}{\partial \boldsymbol{p}}\right) = -\frac{\omega(k)-\omega'(k')}{T_i} f_p . \tag{3.15}$$

From equations (3.14) and (3.15) we get at once a basic feature of induced scattering processes. If in a scattering process the frequency is increased ($\omega > \omega'$), absorption occurs ($\gamma < 0$). If the frequency decreases ($\omega < \omega'$), induced wave emission occurs ($\gamma > 0$). In other words, in an induced non-linear scattering process by thermal particles the energy transfer always goes in the direction in which the frequency is smaller. This conclusion is valid when the type of waves is not changed or when it is changed, and both for differential and for integral scattering processes.

Only spontaneous scattering can increase the wave frequency; induced scattering by thermal particles always leads to an energy transfer to the region of smaller frequencies. This conclusion depends, however, greatly upon the assumption of a Maxwellian particle velocity distribution. If there are in the plasma a sufficiently large number of epithermal, fast particles, the sign of the induced energy transfer may change.

In each scattering process in which the frequency is decreased some, though not much, energy, $\omega-\omega'$, is, lost, that is, absorbed by the ion. This can be interpreted as Landau damping for virtual waves with frequency $\omega-\omega'$ and wavevector $\boldsymbol{k}-\boldsymbol{k}'$. In an equilibrium plasma Landau damping always dominates over the amplification in Cherenkov radiation. The change in sign of the energy transfer corresponds to the excitation of virtual waves and the transfer of energy to them by the particles. It is clear that such an excitation is possible only in a non-equilibrium plasma.

The decay or growth rates, determined by equation (3.14), can be evaluated both for differential and for integral scattering. Let us first consider the case of differential energy transfer along the longitudinal plasmon spectrum.

Using the fact that $\bar{u}_{k,k'}$ has a steep maximum at $\boldsymbol{k} \approx \boldsymbol{k}'$ for the case of differential scattering we can obtain from equations (3.10) and (3.11) the quantity $\bar{u}_{k,k'}$ which is an average over the wavevector directions. For this quantity we have the equation

$$\int \bar{u}_{k,k'}\, d^3\boldsymbol{k}' \approx \frac{\pi\omega_{pe}^3}{18(1+T_e/T_i)^2 n_e^2 v_{Te}^2 k} . \tag{3.16}$$

We can now also easily evaluate $\gamma^l(k)$. Using the energy conservation law (3.3) we find

$$\gamma^l(1+\mathrm{i} \rightleftharpoons \mathrm{l}'+\mathrm{i}') = \frac{n_i}{6m_i v_{Te}^2}\int \bar{u}_{k,k'}\left[\left(\frac{k^2}{k'}+k'\right)\frac{\partial W_{k'}^l}{\partial k'}+\left(\frac{k^2}{k'}-1\right)W_{k'}^l\right] d^3\boldsymbol{k}' \approx \alpha^l \frac{\partial W_k^l}{\partial k}, \tag{3.17}$$

as $k \approx k'$. We have introduced here a notation which is very important for our subsequent discussion: the differential transfer coefficient referring to the longitudinal plasmon spectrum:

$$\alpha^l(1+\mathrm{i} \rightleftharpoons \mathrm{l}'+\mathrm{i}') = \alpha^l = \frac{\pi\omega_{pe}^3}{27(1+T_e/T_i)^2 n_e m_i v_{Te}^4} . \tag{3.18}$$

Equation (3.17) means that the growth rate of non-linear scattering is determined by the difference in population level of plasma wave energy densities in neighbouring spectral intervals. The condition $\gamma^l > 0$ when $\partial W_k/\partial k > 0$ corresponds to an overpopulation of states with larger energy (frequency or wavenumber). Figure 3 illustrates the process of evolution of a packet of Langmuir waves due to differential transfers leading to its slipping to smaller values of plasmon wavenumbers.

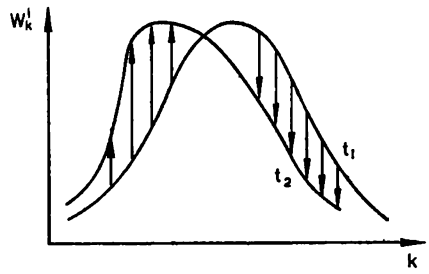

FIG. 3. Evolution in time of a Langmuir wave spectrum ($t_2 > t_1$).

One can similarly also calculate the induced transformation of longitudinal into transverse plasmons and vice versa. The basic property is here also conserved: the energy transfer goes in the direction of decreasing frequency. In other words, the longitudinal plasmon absorbed in the scattering has a larger frequency than the emitted transverse plasmon, while the absorbed transverse plasmon has a larger frequency than the emitted longitudinal one.

The calculation of the damping or growth rate for such a conversion is similar to the calculation for the case of transfer in the spectrum of longitudinal plasmons. A difference consists in that now $\omega - \omega' = \pm(3v_{Te}^2/\omega_{pe})\, k^2$, since $k_p < k_l$. If we use a unified "longitudinal" wavenumber scale, we have for the induced conversion process involving longitudinal and transverse plasmons:

$$\gamma^p(l+i \rightleftharpoons p+i') = \alpha^l \frac{1}{k} \frac{\partial}{\partial k}(kW_k^l), \tag{3.19}$$

$$\gamma^l(p+i \rightleftharpoons l+i') = \alpha^l k \frac{\partial}{\partial k}\left(\frac{W_k^p}{k}\right). \tag{3.20}$$

The superscripts "p" and "l" of the quantities γ indicate that the corresponding γ describe a change in the number of transverse and of longitudinal plasmons.

It is very important that the coefficient of non-linear transfer in this case is again the same as (3.18), as long as we use a unified wavenumber scale. What we said earlier about the connection between the direction of the transfer and the overpopulation in the spectral energy densities remains valid also here (however, we must here require overpopulation of kW_k^l or W_k^p/k, respectively).

Transverse plasmons and electromagnetic waves can remain transverse electromagnetic waves when scattered non-linearly by the polarization "coats" of ions. The damping or growth rate of the induced scattering is here (in the characteristic electromagnetic wave wavenumber

scale):

$$\gamma^{\mathrm{p}}(\mathrm{p}+\mathrm{i} \rightleftarrows \mathrm{p}'+\mathrm{i}') = \frac{\pi\omega_{\mathrm{pe}}^3}{3(1+T_{\mathrm{e}}/T_{\mathrm{i}})^2 n_{\mathrm{e}} m_{\mathrm{i}} c^4} \frac{\partial W_{k_{\mathrm{p}}}^{\mathrm{p}}}{\partial k_{\mathrm{p}}}. \tag{3.21}$$

Strictly speaking, equation (3.21) is valid only for the case of scattering of transverse plasmons with a frequency $\omega \approx \omega_{\mathrm{pe}}$. We shall consider the case of high-frequency waves with $\omega \gg \omega_{\mathrm{pe}}$ in § 5.

We shall now discuss the integral effect of non-linear scattering. We shall distinguish two cases. Consider the case where in the scattering process involving one kind of waves the wavenumber changes little, but the wavevector is rotated over a large angle. We shall call this integral scattering with respect to the angles although with respect to the wavenumbers it remains differential scattering. The second case will be when both the direction of the wavevector and the wavenumber change appreciably.

We emphasize that non-linear scattering with respect to the angles always is integral. This fact is just the reason why we are able to assume that plasma turbulence is approximately isotropic in most cases which are of practical importance.

Only after an isotropization of the plasma waves does the energy transfer along the spectrum from high to low wavenumbers begin. There is then also a redistribution of the wavevector directions; however, such a redistribution does not mean anything in isotropic turbulence.

As long as in the transfer process the condition $|k-k'| \ll k'$ is satisfied the scattering remains differential. However, when the wavenumber decreases further, that is, when

$$k' < k_* = \frac{\omega_{\mathrm{pe}} v_{T\mathrm{i}}}{3v_{T\mathrm{e}}^2} = \frac{1}{130}\frac{1}{d_{\mathrm{e}}}\sqrt{\frac{T_{\mathrm{i}}}{T_{\mathrm{e}}}}, \tag{3.22}$$

the scattering becomes integral also with respect to a change in wavenumber. The quantity k_* is another important parameter of plasma turbulence.

The wavenumbers decrease as before but not gradually, but steeply, in a jump. As a result integral transfer leads to a peculiar condensation of plasmons around the zero value of wavenumbers—somewhat analogous to a Bose condensation although the purely classical effect of non-linear scattering has nothing whatever in common with the purely quantal effect of the Bose–Einstein condensation.

When evaluating the spontaneous and induced non-linear integral scattering we must use the expression (3.11) for u_k. The exact formulae are rather unwieldy and the averaged expressions are given in the Appendix. Using them we find for the coefficient for the spontaneous emission when longitudinal plasmons are scattered by ions:

$$J_k(\mathrm{l}+\mathrm{i} \rightleftarrows \mathrm{l}'+\mathrm{i}') = \frac{k^2\omega_{\mathrm{pe}}}{2\pi^2}\int u_{k,k'} N_{k'} f_p \frac{d^3k'\, d^3p}{(2\pi)^3}$$

$$= \frac{\omega_{\mathrm{pe}}^2}{3(2\pi)^{3/2}(1+T_{\mathrm{e}}/T_{\mathrm{i}})^2 n_{\mathrm{e}} v_{T\mathrm{i}}}\left[\int_0^k \left(k^2+\frac{2}{5}k'^2\right) W_{k'}^{\mathrm{l}} \frac{dk'}{k'} + k\int_k^\infty \left(k'^2+\frac{2}{5}k^2\right) W_{k'}^{\mathrm{l}} \frac{dk'}{k'^2}\right]. \tag{3.23}$$

For the damping or growth rate of the induced scattering we find from (3.14)

$$\gamma^{\mathrm{l}}(\mathrm{l}+\mathrm{i} \rightleftarrows \mathrm{l}'+\mathrm{i}') = \frac{(2\pi)^{1/2}T_{\mathrm{e}}T_{\mathrm{i}}}{8(T_{\mathrm{e}}+T_{\mathrm{i}})^2 n_{\mathrm{e}}m_{\mathrm{e}}v_{T\mathrm{i}}}\left[\int_k^\infty (k'^2-k^2)\left(k'^2+\frac{2}{5}k^2\right)W^{\mathrm{l}}_{k'}\frac{dk'}{k'^3}\right.$$

$$\left.-\frac{1}{k^3}\int_0^k (k^2-k'^2)\left(k^2+\frac{2}{5}k'^2\right)W^{\mathrm{l}}_{k'}\,dk'\right]. \tag{3.24}$$

We have already mentioned that the scattering by electrons is always integral. The corresponding formulae which describe the scattering by electrons are similar to (3.23) and (3.24), and, for instance, instead of (3.24) we have now

$$\gamma^{\mathrm{l}}(\mathrm{l}+\mathrm{e} \rightleftarrows \mathrm{l}'+\mathrm{e}') = \frac{6\sqrt{2\pi}v_{T\mathrm{e}}}{5m_{\mathrm{e}}n_{\mathrm{e}}\omega_{\mathrm{pe}}^2}\left[k^2\int_k^\infty (k'^2-k^2)\left(\frac{1}{3}k'^2+\frac{4}{7}k^2\right)W^{\mathrm{l}}_{k'}\frac{dk'}{k'^3}\right.$$

$$\left.-\frac{1}{k^3}\int_0^k (k^2-k'^2)\left(\frac{1}{3}k^2+\frac{4}{7}k'^2\right)W^{\mathrm{l}}_{k'}k'^2\,dk'\right]. \tag{3.24'}$$

Equation (3.24') is valid for the case when the wavenumbers k and k', although very different, are not small in absolute magnitude. If one of them is small, cancellations occur and the quantity γ^{l} turns out to be small. We give in § 8 the corresponding expressions for the growth rate γ^{l} and the criterion for when k' is small.

The concentration of plasmons in the small wavenumber phase region prevents another form of non-linear interactions, namely, decay interactions which we shall consider now.

Decay and fusion of waves in a plasma

A second form of non-linear interactions between waves in a plasma also leads to a transfer of energy both between waves of the same mode and between different modes. This process is usually called a decay interaction.

We can describe the physics of the decay interaction as follows. Consider a free electron which is simultaneously in the electric field of two waves with different wavevectors $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ and frequencies ω_1 and ω_2. Each of these waves affects the electron at its own frequency so that its total motion can be described by the following equation:

$$m_{\mathrm{e}}\frac{d^2\boldsymbol{r}}{dt^2} = e\boldsymbol{E}_1\cos\left[\omega_1 t-(\boldsymbol{k}_1\cdot\boldsymbol{r})\right]+e\boldsymbol{E}_2\cos\left[\omega_2 t-(\boldsymbol{k}_2\cdot\boldsymbol{r})\right], \tag{3.25}$$

where $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ are the amplitudes of the fields of the two waves. When solving this equation by successive approximations we get in the expression for the wavevector $\boldsymbol{r}$ terms proportional to $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$, which describe the normal scattering, and terms quadratically dependent on the field strengths. Of special interest are the terms involving the product of the strengths $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$. The motion corresponding to them has frequencies $\omega_1+\omega_2$ and

$\omega_1-\omega_2$. It is clear that that part of the motion of the excited electron also proceeds with successive accelerations and decelerations and leads to the emission of waves at the frequencies $\omega_1+\omega_2$ and $\omega_1-\omega_2$. This is just the decay interaction or the fusion of waves, a non-linear effect which is explained by the non-linearity of the electron motion in the field of the two waves, (3.25).

We shall not solve equation (3.25). One sees, however, easily that the order of magnitude of the terms in the expansion of the electron velocity $v = dr/dt$ which are proportional to the product E_1E_2 and symmetric in ω_1 and ω_2 and in k_1 and k_2 is given by

$$v \approx \frac{e^2}{m_e^2}\left(\frac{k_1}{\omega_1}+\frac{k_2}{\omega_2}\right)\frac{E_1E_2}{\omega_1\omega_2}. \tag{3.26}$$

As there is no reason to prefer one of the waves ω_1 or ω_2 it is always necessary to have a symmetric expression for v.

We can now proceed further as follows. When evaluating the emission probability in scattering (equation (2.18)) we compared it with the simplified expression (2.13) in which we took for v the velocity of the electron which was excited by the incident wave, which in that case was equal to $v = eE/m_e\omega$.

It is clear that the emission probability for the fusion of two waves can be estimated in a similar way provided we now take for v the velocity of the electron which is excited at once by the two waves with frequencies ω_1 and ω_2. If we express in v^2 the quantities E_1^2 and E_2^2 in terms of $W_{\omega_1}^{(1)}$ and $W_{\omega_2}^{(2)}$ we get instead of (2.19)

$$v^2 = \frac{e^4}{m_e^4}\left(\frac{k_1}{\omega_1}+\frac{k_2}{\omega_2}\right)^2\frac{(4\pi)^2W_{\omega_1}^{(1)}W_{\omega_2}^{(2)}}{\omega_1\omega_2}. \tag{3.27}$$

Substituting (3.27) into (2.13) we get the following estimate for the emission probability of a wave $\boldsymbol{k}$ when waves $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ fuse:

$$u_k \approx \frac{e^6}{m_e^4}\left(\frac{k_1}{\omega_1}+\frac{k_2}{\omega_2}\right)^2\frac{2(2\pi)^7W_{\omega_1}^{(1)}W_{\omega_2}^{(2)}}{\omega^2\omega_1\omega_2}\frac{n_e}{kk_1k_2}, \tag{3.28}$$

where we have added the factor $(2\pi)^3n_e/k_1k_2k$ which is symmetric in the wavenumbers and which takes into account that the emission process is coherent. Indeed, since in the production of the radiation not separate electrons excited by the wavenumbers k_1 and k_2 but groups of electrons in a volume which, roughly speaking, is of a size $\lambda^3 = (2\pi/k)^3$ take part, one must multiply the emission probability by the coherence factor $n_e\lambda^3$ or, because of the symmetrization, by $(2\pi)^3n_e/kk_1k_2$.

We give in the Appendix exact expressions for the decay probabilities, averaged over the angles. Here we consider merely the most important cases which we need for what follows.

Of course, the energy and momentum conservation laws involving all waves participating in the process of decay or fusion must be satisfied:

$$\omega = \omega_1+\omega_2, \quad \boldsymbol{k} = \boldsymbol{k}_1+\boldsymbol{k}_2. \tag{3.29}$$

It follows from the conservation laws that when two longitudinal Langmuir waves fuse one can only get a transverse (and not a longitudinal) electromagnetic wave with frequency

$\omega \approx 2\omega_{pe}$ (we assume that $\omega_{pe} \gg \omega_{He}$). The momentum conservation law imposes limitations upon the wavenumbers of the interacting plasmons. Let us suppose that the wavenumber of one of the plasmons is much larger than $k = \sqrt{3}\omega_{pe}/c$, the wavenumber of the transverse wave formed in the fusion. It is then clear that the second longitudinal plasmon must also have a wavenumber close to the wavenumber of the first plasmon and that the two wavevectors must be almost antiparallel—only in that case can their sum give a small wavevector for the transverse wave. Thus, if $k_1 \gg \sqrt{3}\omega_{pe}/c$, we must have $\boldsymbol{k}_1 \approx -\boldsymbol{k}_2$.

If, however, the wavenumber of one of the longitudinal plasmons is much smaller than $\sqrt{3}\omega_{pe}/c$, the wavenumber of the second longitudinal plasmon must be close to the wavenumber of the transverse electromagnetic wave, that is, if $k_1 \ll \sqrt{3}\omega_{pe}/c$, we have $k_2 \approx k \approx \sqrt{3}\omega_{pe}/c$. We stress that the resulting transverse wave propagates in the direction of the larger of the wavevectors of the longitudinal plasmons.

The emission coefficient of a transverse wave with a given frequency ω, close to $2\omega_{pe}$, is determined by the product of the spectral energy densities of the longitudinal plasmons $W^l_{k_1} W^l_{k_2}$ with values of the wavenumbers k_1 and k_2 determined by the conservation laws (3.29). The expressions for the emission coefficient per unit wavenumber or unit frequency has a rather complex form. We write down at once the expression for the total (that is, integrated over frequencies) emission power, generated when longitudinal plasmons fuse:

$$J(l_1+l_2 \to t) = \int J_\omega \, d\omega$$

$$\approx \frac{\pi\sqrt{3}}{3} \frac{\omega_{pe}^2}{n_e m_e c^3} \left[W^l_{\sqrt{3}\omega_{pe}/c} \int_0^{\sqrt{3}\omega_{pe}/c} W^l_{k_1} \, dk_1 + \frac{12}{5} \int_{\sqrt{3}\omega_{pe}/c}^{k_D} (W^l_{k_1})^2 \left(\frac{\omega_{pe}}{ck_1}\right)^2 dk_1 \right], \tag{3.30}$$

where $W^l_{\sqrt{3}\omega_{pe}/c}$ is the value of the spectral density of the longitudinal plasmons for $k = \sqrt{3}\omega_{pe}/c$ and $k_D \approx 1/d_e$ is the maximum wavenumber for longitudinal plasmons. The expression given here is convenient for applications, but it is approximate since in the range $0 < k_1 < \sqrt{3}\omega_{pe}/c$ we have used the limiting form of the integrand which is valid for $k_1 \ll \sqrt{3}\omega_{pe}/c$ and in the range $k_1 > \sqrt{3}\omega_{pe}/c$ the limiting form corresponding to $k \gg \omega_{pe}/c$.

Equation (3.30) shows that the largest emission power is generated when longitudinal plasmons with $k_1 \approx k_2 \approx \sqrt{3}\omega_{pe}/c$ fuse. The largest part of the energy of the transverse radiation is concentrated in a relatively narrow range of frequencies:

$$\frac{\Delta\omega^t}{\omega^t} \approx 3\frac{v_{Te}^2 k_1^2}{\omega_{pe}^2} \approx 9\frac{v_{Te}^2}{c^2}. \tag{3.31}$$

Combining (3.30) and (3.31) we get easily an estimate for the spectral power of the radiation: $J_\omega \approx J/\Delta\omega^t$.

The fusion of two longitudinal plasmons into a transverse wave corresponds to yet another mechanism for the absorption of Langmuir waves. The damping rates for the two wavenumber intervals are, respectively,

$$\gamma^l(l_1+l_2 \to t) = -\beta^l \frac{W^l_{k_1}}{k_1^2} = -\frac{\pi\sqrt{3}}{5} \frac{\omega_{pe}^4}{n_e m_e c^5} \frac{W^l_{k_1}}{k_1^2}, \quad \sqrt{3}\frac{\omega_{pe}}{c} \ll k_1 < k_D, \tag{3.32}$$

and

$$\gamma^{l}(l_1+l_2 \to t) = -\frac{\pi}{4\sqrt{3}} \frac{\omega_{pe}^2}{n_e m_e c^3} W^{l}_{\sqrt{3}\omega_{pe}/c}, \quad 0 < k_1 \ll \sqrt{3}\frac{\omega_{pe}}{c}. \tag{3.33}$$

In the latter case the damping rate is independent of the wavenumber, as in the case of collisional damping, so that we can proceed in the same way when we take both absorption mechanisms, the collisional and the radiative ones, into account provided $k_1 \ll \omega_{pe}/c$.

The inverse decay of an electromagnetic wave with frequency close to $2\omega_{pe}$ into two longitudinal plasmons leads to its absorption. This process is possible only when the frequency of the wave satisfies the inequality

$$k_D \approx \frac{\omega_{pe}}{v_{Te}} > k_1 = \sqrt{\left(\frac{\omega_{pe}(\omega-2\omega_{pe})}{3v_{Te}^2}\right)}. \tag{3.34}$$

The absorption rate of electromagnetic waves due to this decay is

$$\gamma^{t}(t \to l_1+l_2) = -\frac{\pi}{30} \frac{\omega_{pe}^3}{n_e m_e c^2} \frac{W^{l}_{k_1}}{v_{Te}^2 k_1}, \tag{3.35}$$

where k_1 ($\gg \omega_{pe}/c$) is determined by (3.34).

We shall often in what follows use another two important decay and fusion processes:

A longitudinal plasmon in a non-isothermal plasma, with $T_e \gg T_i$ can decay into another longitudinal plasmon and an ion-sound wave. In the inverse process when longitudinal plasmons fuse with ion-sound waves one also produces longitudinal plasmons. The change in frequency of the longitudinal plasmons in those processes is less than ω_{pi}, that is, it is very small and the wavenumbers of the longitudinal plasmons therefore also change little. The balance between the decay and fusion processes of longitudinal plasmons and ion-sound waves is thus described by a differential transfer coefficient as in the case of the differential scattering of longitudinal plasmons by ions. We have, when $k > k_*(T_e/T_i)^{1/2}$:

$$\gamma^{l}(l \to l' \pm s) = \alpha^{ls} \frac{\partial W_k^{l}}{\partial k} = \frac{\pi\omega_{pe}^3}{27 n_e m_i v_{Te}^4} \frac{\partial W_k^{l}}{\partial k}. \tag{3.36}$$

We note that α^{ls} in (3.36) differs from the α^{l} in (3.18) by the replacement of the factor $(1+T_e/T_i)^2$ by unity. This means the following. In an isothermal plasma ion-sound waves are not excited and therefore in such a plasma the transfer of energy along the spectrum of the longitudinal plasmons is realized through the non-linear scattering by thermal ions. If the electron temperature begins to exceed the ion temperature the efficiency of the non-linear transfer decreases. However, when $T_e \gtrsim 3T_i$ a decay energy transfer becomes possible for the longitudinal plasmons and the efficiency of this transfer will be even larger (by a factor 4) than in an isothermal plasma. Otherwise the energy transfer of longitudinal plasmons through a decay interaction has the same properties as the energy transfer through non-linear scattering by ions.

A second decay process which is important for plasma astrophysics is the fusion and decay of high-frequency electromagnetic waves with $\omega \gg \omega_{pe}$ as a result of which longitudinal plasmons are created. If the condition $k_l \ll k_t = \omega/c$ is satisfied this process also has a differential character (when we take into account the balance between fusions and decays).

It is clear that processes in which a transverse electromagnetic wave decays into a longitudinal plasmon and another transverse wave also occur in a non-turbulent plasma. In that case the process leads to a gradual transfer of the energy of the electromagnetic waves into the low-frequency region of the spectrum with a rate

$$\gamma^{t}(t \to t' \pm l) = \frac{2\pi^2}{3} \frac{\omega_{pe}^4}{n_e m_e c^3} \frac{\partial}{\partial \omega} \left(\frac{I_\omega}{\omega} \right) = \alpha^{t} \frac{\partial}{\partial \omega} \left(\frac{I_\omega}{\omega} \right), \tag{3.37}$$

where we remember that I_ω is the spectral intensity of the high-frequency radiation. This process plays, apparently, a large role in the determination of the level of the energy of the electromagnetic radiation in powerful cosmic sources.

Under cosmic conditions also other kinds of plasma waves, such as whistlers or Alfvén waves, will take part in decay interactions. We shall consider a few cases in later chapters and give the probabilities for other processes in the Appendix.

Decay processes are not limited to three-plasmon interactions. Four-plasmon interactions when one plasmon decays into three other ones or when two plasmons change into two other ones are also possible. There is in that case an even larger number of variations but especially important are those interactions which are forbidden in the three-plasmon form. For instance, two longitudinal plasmons can not change into one longitudinal plasmon in a three-plasmon interaction but they can, on the other hand, change into two other longitudinal plasmons through a four-plasmon interaction.

The evaluation of four-plasmon interactions is very cumbersome but one can obtain order of magnitude estimates by the same method as for three-plasmon interactions. A rough estimate for the emission coefficient for the four-plasmon interaction of Langmuir waves gives

$$J_k(l+l' \rightleftharpoons l_1+l_1') \approx 0{\cdot}1 \frac{\omega_{pe}^3}{n_e^2 m_e^2 v_{Te}^6} (W_k^l)^3. \tag{3.38}$$

Strictly speaking, equation (3.38) is valid only when the wavenumbers of all four plasmons are about the same. If the wavenumbers of two of the plasmons are much smaller than those of the other two (for instance, $k \ll k_1$) we have instead of (3.38)

$$J_k(l+l' \rightleftharpoons l_1+l_1') \approx 0{\cdot}1 \frac{\omega_{pe}^3}{n_e^2 m_e^2 v_{Te}^6} \frac{k}{k_1} W_k^l (W_{k_1}^l)^2. \tag{3.39}$$

If we express all wavenumbers in a unified scale the emission coefficient for the four-plasmon interaction between longitudinal and transverse plasmons is of the same order of magnitude.

If the turbulence is weak, $W \ll n_e T_e$, the role of interactions which are more complicated than three- or four-plasmon processes is unimportant. The relative importance of non-linear scattering and of decay interactions differs under different conditions in the plasma, and even in the same plasma for different wavenumbers.

We considered two basic mechanisms—the non-linear scattering and decay processes—which lead to energy transfer between plasma waves both involving one type of waves and involving different types. In a plasma with an admixture of fast, epithermal particles, these can also take part in the energy transfer. This case is considered in Chapter 3. It is well known (see, for instance, Denisov, 1957; Moiseev, 1965; and also the book by Zheleznyakov,

1970a; or the review by Golant and Piliya, 1972) that in a smoothly inhomogeneous medium a mutual transformation of waves is also possible. The probability for such a transformation is usually proportional to $(kR)^{-1}$ where k is the wavenumber and R a characteristic dimension of the inhomogeneity. In some cases the probability for such a transformation may be larger, of order $(kR)^{-2/3}$. All the same, however, the cosmic plasma is so uniform over distances of the order of the wavelength that one can in practice in the great majority of applications neglect this effect.

It is important to emphasize the following. Non-linear energy transfer processes involving different plasma waves are an intrinsic property of a plasma, independent in what way and in what amount the plasma receives energy from external sources. If, therefore, we give merely the power of the turbulence production determined by the external sources which produce the turbulence, and use the coefficients we have given here for the energy transfer between different kinds of waves, we can—at least in principle—construct the whole of the plasma turbulence spectrum. The next section will deal with the main propositions of plasma turbulence theory.

§ 4. Nature, spectrum, and excitation of plasma turbulence

Under cosmic conditions as under laboratory conditions there are always sources which excite different forms of plasma waves. These waves are, on the other hand, also absorbed in the plasma. If the power of the sources is small, which is practically always the case when only spontaneous emission occurs, the energy density of the plasma waves is of the order of the level of thermal fluctuations, that is, at least N_D times smaller than the thermal energy of the plasma particles.

Plasma waves which were created spontaneously are usually absorbed at the same frequencies and wavenumbers as those at which they were excited. There is practically no energy transfer along the spectrum or conversion from one type to another.

However, if the power of the plasma wave emission sources is large, in particular, when there are induced emission mechanisms (excitation due to instabilities) the level of the plasma wave energy increases fast and there arises a well-defined non-linear interaction. The energy transferred by the source in exciting one kind of waves in a narrow range of wavenumbers is redistributed over the whole of the allowed phase space both of the original plasma wave mode and also transferred to other plasma wave modes, that is, plasma turbulence is created.

It is clear that it has much in common with ordinary hydrodynamic turbulence. We shall therefore first briefly recapitulate the properties of large-scale dynamic turbulence.

It is well known that in incompressible liquids (or in gases, provided the velocities of the motions are much smaller than the sound velocity) turbulence is the combination of vortices of different sizes. The redistribution of energy between them is also connected with the non-linearity of the hydrodynamic equations. When studying the turbulence of incompressible liquids one often states the problem as follows. One introduces the concept of the wavenumber of a vortex k which is connected with its size λ through the usual relation $k = 2\pi/\lambda$; we emphasize that we are talking about the motion of vortices, not of waves, even though we use the term "wavenumber". In the overwhelming majority of concrete problems one can

assume that the external sources of turbulence transfer energy with a power Q to the vortices of the largest size λ_0 (or $k_0 = 2\pi/\lambda_0$). After that the energy is gradually transferred to smaller size vortices. So to speak, an energy flux is set up through the whole vortex hierarchy from smaller to larger k. In the very small size vortices, $k_\nu \approx 2\pi/\lambda_\nu$, the energy is dissipated due to viscosity. The magnitude of the energy flux through the vortex hierarchy is equal to the power of the turbulence sources and completely determines the universal turbulence energy spectrum in the wavenumber range $k_0 \ll k \ll k_\nu$, the so-called inertial range. The quantity k_ν depends on the viscosity coefficients, while k_0 is determined by the geometry of the medium and the structure of the turbulence sources. The spectral energy density of the universal Kolmogorov spectrum is in the inertial range of wavenumbers equal to

$$W_k = 1{\cdot}3\frac{Q^{2/3}\varrho^{1/3}}{k^{5/3}}, \tag{4.1}$$

where ϱ is the liquid density. Increasing the power of the sources increases the turbulent energy density—and changes the inertial range—but does not change the nature of the spectrum in that range.

Separating the phase regions of the wavenumber spectrum into those in which the pulsations are formed and those in which they are destroyed is a necessary element of any turbulence, including plasma turbulence. Indeed, in the region where the plasma waves are generated their absorption is always weaker than their excitation and the intensity of the waves would become infinite unless there were a way in which the wave could leave the medium —and this is impossible for many plasma waves because of their low group velocity—or a mechanism of energy transfer to another part of the spectrum where these waves are destroyed.

A distinctive feature of plasma turbulence is the presence of eigenfrequencies in a plasma. This makes it impossible to construct a theory simply on the basis of dimensionality, but, on the other hand, it gives us the possibility to advance further in the calculation of nonlinear interactions and to develop in that way a more complete mathematical theory. It is not superfluous to remind ourselves that it has not been possible to make much progress in the mathematical theory of the turbulence of an incompressible fluid. The mathematical simplification of plasma turbulence theory is in principle connected with the following property. Plasma waves have a small relative damping rate (γ/ω), that is, a wave performs many oscillations before it is damped or before it transfers its energy to another wave. For a vortex in an incompressible fluid, $\gamma \approx \omega$: it does not make many revolutions before it is damped or before it transfers its energy to other vortices. The presence of the small parameter γ/ω makes it possible to construct a mathematical theory of plasma turbulence.

Notwithstanding this difference dynamical and plasma turbulence have much in common: in a wide range of wavenumbers their spectra can be determined by a single parameter Q. This, incidentally, is important for plasma astrophysics where it is always desirable to have as few fixed parameters as possible.

It is true that in the case of plasma turbulence Q can be considered the only parameter only if the turbulence is isotropic. A similar limitation occurs also in the theory of dynmical turbulence, but in that case the isotropy of the vortices is natural and is confirmed by measurements. The question of the isotropy of plasma turbulence is rather complicated.

If there is no strong external magnetic field the plasma waves are rather quickly made isotropic by the non-linear interactions themselves so that in that range of wavenumbers where the energy is transferred from one wave to another—which is equivalent to the inertial range in dynamical turbulence—one may assume plasma turbulence to be isotropic. In a strong external magnetic field anisotropy arises and is maintained naturally, especially if we take into account that a number of waves transfer energy preferentially along the field (§ 1). However, even in this case non-linear interactions will often tend to broaden the angular spectrum of the plasma waves and it seldom stays very narrow. However, in some cases in a magnetic field non-linear interactions decrease the angular spectrum. At any rate for a first estimate in astrophysics one can assume that the anisotropy does not introduce important corrections. One can, of course, also formulate the theory taking anisotropy into account.

We shall see that the introduction of only one parameter Q makes it possible to derive without further assumptions all consequences for the plasma waves, including the electromagnetic energy flux. A comparison with observations gives us the whole picture of plasma turbulence and enables us to estimate the necessary value of Q.

The nature of plasma turbulence is also determined by the mechanism of the dissipation of the energy of the excited plasma waves. We shall distinguish three types of turbulence:

(a) The energy of the plasma waves is predominantly dissipated into heat both through electron-ion collisions and due to Landau damping. This case we shall call heating turbulence;
(b) An appreciable (although, perhaps, not the major) part of the plasma wave energy is lost through accelerating particles: this is acceleration plasma turbulence.
(c) A considerable part of the energy of all kinds of plasma waves is transformed into electromagnetic waves, both transverse plasmons and high-frequency radiation. We call this case radiation plasma turbulence.

We must emphasize that in the last two cases a large part of the energy is lost, all the same, in heating the plasma. Even in the process of transferring energy along the spectrum thermal dissipation occurs continuously. For instance, in non-linear scattering the frequency is gradually decreased, and thus also the wave energy. The excess energy is transferred to the scattering particle.

All three kinds of turbulence are of interest for plasma astrophysics, but the latter two are particularly important. We note that they are interconnected as the acceleration of particles increases the possibility of the transformation of plasma waves into high-frequency electromagnetic waves which more easily leave the region of turbulence generation.

The type of turbulence depends on the value of Q and also on how long it has existed. In the course of time one kind of turbulence changes to another. For instance, the larger the build-up of accelerated particles, the stronger the chance that the acceleration turbulence can change into radiation turbulence.

Let us now turn to an analysis of the plasma turbulence spectra. In general, it is essential for many problems in plasma astrophysics to know the complete expressions for the spectral energy densities. However, one cannot always easily obtain them and in many cases one is limited to only a qualitative picture of the behaviour of W_k and a quantitative estimate of

only the total energy of the plasma turbulence. We shall therefore restrict ourselves to discuss the general methods and to describe two or three of the simplest, but most important cases, referring the reader for details about other spectra to the literature (Kadomtsev, 1965; Tsytovich, 1971a).

The problem of determining the plasma turbulence spectrum can be reduced to finding the spectral function W_k which in the general case may depend on seven arguments: the time t, three spatial coordinates $\boldsymbol{r}$ and three wavevector components $\boldsymbol{k}$. However, in the great majority of plasma astrophysics problems it is not necessary to go into such a detailed study and one can therefore restrict oneself to a smaller number of arguments.

Of most interest is the case of stationary turbulence when the spectral function changes more slowly than, say, on the scale of the characteristic time for energy transfer along the spectrum. One can almost always assume that the system changes fastest along one of the coordinates. However, due to the fact that the group velocities of plasma waves are small, the change in W_k with (even one) coordinate often turns out to be unimportant.

As we noted already, the wavevectors are more or less isotropically distributed in a well-developed turbulence. We can thus assume that W_k depends only on the wavenumber k. In the other limiting case, that of one-dimensional turbulence, W_k also depends on one quantity. For astrophysical problems one can thus usually assume W_k to be a function of a single argument, the wavenumber. One can then always determine a region of space, a time interval, and a solid angle of the wavevector directions in which one can neglect the dependence of W_k on other parameters.

The main problem in plasma turbulence theory is the establishment of the balance equations for W_k which describe the kinematic change in W_k due to the propagation of plasmons in space and time, that is, dW_k/dt, where d/dt is a total derivative, the absorption and generation of plasmons and their non-linear transformations.

The total change of the function W_k in time consists of the sum of three terms which we shall now write down assuming for the sake of simplicity that the turbulence is one-dimensional both in coordinate space (along the x-axis) and in wavenumber space. We have then (for instance, for longitudinal plasmons):

$$\frac{dW_k^l}{dt} = \frac{\partial W_k^l}{\partial t} + v_g \frac{\partial W_k^l}{\partial x} - \frac{\partial \omega(k)}{\partial x} \frac{\partial W_k^l}{\partial k} . \tag{4.2}$$

The first term on the right-hand side of (4.2) is determined by the explicit time-dependence of the spectral density and the second term reflects the transfer of plasma waves in space. We bear in mind that the group velocities of practically all plasma waves, including, for instance, also the transverse plasmons, are very small and one therefore seldom needs to take that term into account. The criterion for taking it into account is putting v_g/R, where R is a characteristic size of the system, equal to the growth or decay rate of a plasma wave.

We must consider the third term in more detail. It is well known that the frequency of a monochromatic wave does not change in the geometric optics approximation during propagation in an inhomogeneous medium. For instance, for a longitudinal plasmon $\omega^2 = \omega_{pe}^2 + 3v_{Te}^2 k^2 =$ constant along the whole path. In an inhomogeneous medium, however, both $\omega_{pe}^2 \propto n_e$ and $v_{Te}^2 \propto T_e$ change, so that the wavenumber of a longitudinal plasmon must change. Assuming that the medium is inhomogeneous only in the electron density

we have from the law of conservation of ω^2:

$$\frac{\partial k}{\partial x} = -\frac{\omega_{pe}}{3v_{Te}^2 k}\frac{\partial \omega_{pe}}{\partial x} = -\frac{1}{v_g}\frac{\partial \omega_{pe}}{\partial x}. \tag{4.3}$$

We must thus take into account that plasma waves leave a given wavenumber range—and as a result W_k^l decreases—because of the inhomogeneity of the medium in which they propagate. The two last terms in (4.2) determine simply the total change in W_k^l with respect to the coordinate, deriving both from an explicit x-dependence and from an implicit dependence through a change of the wavenumber. The sign of the third term on the right-hand side of (4.2) is determined by the fact that the wavenumber, and thus ω, increases when the electron concentration decreases.

If the spectral density of the plasma turbulence energy is evaluated per unit frequency, the third term will, of course, be absent:

$$\frac{dW_\omega}{dt} = \frac{\partial W_\omega}{\partial t} + v_g \frac{\partial W_\omega}{\partial x}. \tag{4.4}$$

The effect which we have just described where waves leave a given range of wavenumbers due to the inhomogeneity of the plasma turns out to be important for some astrophysical problems.

The kinetic equations of plasma turbulence are expressions where on the left-hand side we have dW_k/dt and on the right-hand side the change in W_k due to the emission and absorption of these waves through all possible mechanisms (such as bremsstrahlung, Landau damping, or Cherenkov radiation), transfer of energy along the spectrum, and transformation of waves from one mode into another.

In a multi-mode turbulence there is a set of such equations with coupling terms describing the transfer of energy from one mode to another. Moreover, in the complete set of equations for plasma turbulence we must include also kinetic equations for the momentum distribution functions of the plasma particles.

In many cases we can neglect some of the equations; for instance, we can assume that the particle distribution function of the main plasma is Maxwellian. Very often we meet with the case when only one or two kinds of plasma waves are excited to a large energy density. In those cases the problem is considerably simplified. In later chapters which are devoted to a consideration of concrete objects we shall solve a few of such self-consistent problems, but here we shall consider a few of the simplest examples when we may assume the particle distribution function to be given and the turbulence excited of only one or two kinds of waves.

Langmuir wave (longitudinal plasmon) turbulence

From the point of view of plasma astrophysics Langmuir wave turbulence is one of the most important examples of turbulence in a plasma. Moreover, the analysis of longitudinal plasmon turbulence can be performed relatively simply.

We shall not at once write down the complete equations for this problem. It is clearer and simpler to consider them "by parts".

We assume that some sources, which we shall not particularize, excite Langmuir waves in a small wavenumber interval Δk_g around the value k_g; this quantity we shall call the wavenumber of turbulence generation. We assume further that the phase velocity of these waves is larger than $6v_{Te}$, so that we can neglect the non-linear scattering by electrons, but less than $130v_{Te}$, so that we have differential scattering by the ions.

After having been excited the Langmuir waves can (i) be absorbed due to collisions; (ii) be transferred to the region of lower wavenumbers; (iii) change into transverse plasmons; (iv) fuse with other longitudinal plasmons, forming electromagnetic waves with frequencies $\sim 2\omega_{pe}$; (v) undergo four-plasmon interactions; (vi) decay into another longitudinal plasmon and a low-frequency wave; or (vii) be absorbed by a fast particle and accelerate it.

Of course, in the framework of one problem one need not consider all these possibilities. Let us restrict ourselves. We shall assume that the medium is transparent for transverse plasmons, so that their energy density will be small; we can then neglect the induced conversion of longitudinal into transverse plasmons (the opposite case will be considered in § 8) and the spontaneous conversion is always less than the damping due to collisions. We shall also assume that the medium is also "transparent" for fast particles; the loss through accelerating them is then also unimportant.

The non-linear scattering and the four-plasmon interactions only have comparable efficiency in a narrow wavenumber range around some value k_s. Outside that interval one must take into account either only the non-linear scattering (when $k > k_s$) or, basically, only the four-plasmon interaction (when $k < k_s$). Moreover, the magnitude of k_s must be compared with the wavenumber k_* given by (3.22).

If $k_s < k_* < k_g$ the turbulence spectrum consists of intervals: there is differential energy transfer through non-linear scattering by ions ($k_* < k < k_g$); integral energy transfer through scattering by ions ($k_s < k < k_*$); and a region where four-plasmon interactions are important ($0 < k < k_s$). If $k_s < k_*$, the integral transfer through scattering by ions is not of great importance. In induced energy transfer the wavenumber of longitudinal plasmons can only decrease: If in the plasma waves are excited with $k < k_g$ there will thus be no energy transfer to that part of the spectrum where $k > k_g$. However, spontaneous energy transfer to that part of the spectrum is possible. The spontaneous non-linear scattering by ions is here unimportant, as in each scattering process the change in wavenumber is small, but non-linear scattering by electrons, although strongly weakened by cancelling Compton scattering, could by virtue of its integral nature "promote" some level of energy of plasma waves in the region above k.

We shall consider each range of wavenumbers separately. In the range $k_g > k > k_*$ (we assume that $k_* > k_s$) the main processes are: non-linear induced energy transfer along the spectrum (with a decay rate given by (3.17)), loss of plasmons through fusion and the formation of electromagnetic waves with $\omega \approx 2\omega_{pe}$ which will, by assumption, leave the turbulence region (decay rate (3.32)), and damping through collisions. The basic kinetic equation for the longitudinal plasmon turbulence thus has the following form in this interval:

$$\frac{dW_k^1}{dt} = Q_k^1 - \nu_e W_k^1 - \beta^1 \left(\frac{W_k^1}{k}\right)^2 + \alpha^1 W_k^1 \frac{\partial W^1}{\partial k}. \tag{4.5}$$

Here Q_k^l is the spectral power of the turbulence sources. We assume this quantity as given. The wavenumber dependence of Q_k^l is usually rather complicated—we shall show below how to take it into account—but it is at this moment of no interest. It is sufficient to assume that

$$Q_k^l \neq 0 \quad \text{when} \quad k_g \leqslant k \leqslant k_g + \Delta k_g; \qquad Q_k^l = 0, \quad \text{otherwise.} \tag{4.6}$$

Let us start the study of equation (4.5) with the stationary case, $dW_k^l/dt = 0$. In the spectral region where the longitudinal plasmons are generated, $k_g \leqslant k \leqslant k_g + \Delta k_g$, their absorption must be small, because otherwise they would not be generated. We can in that region therefore drop the second and third terms on the right-hand side of (4.5) and retain only the excitation of plasma waves and their departure from this region through non-linear interactions. Since the departure is in the direction of lower wavenumbers, we put $W_k^l = 0$ for $k > k_g + \Delta k_g$, neglecting for the moment spontaneous scattering into the large-k region.

We then find, solving (4.5):

$$W_k^l = \sqrt{\left(\frac{2}{\alpha^l} \int_k^{k_g + \Delta k_g} Q_k^l \, dk\right)}. \tag{4.7}$$

We have used here the fact that α^l is independent of the wavenumber.

When $k \approx k_g$ the spectral density of the longitudinal plasmon energy reaches a first maximum:

$$W_{k_g}^l = \left[\frac{2}{\alpha^l} \int_{k_g}^{k_g + \Delta k_g} Q_k^l \, dk\right]^{1/2} = [2Q^l/\alpha^l]^{1/2} = \sqrt{\left(\frac{54 m_i}{\pi m_e}\right)\left(1 + \frac{T_e}{T_i}\right)\left(\frac{Q^l}{n_e T_e \omega_{pe}}\right)^{1/2} \frac{v_{Te}}{\omega_{pe}} n_e T_e}$$

$$= \sqrt{\frac{6}{\pi}\left(1 + \frac{T_e}{T_i}\right)\left(\frac{Q^l}{n_e T_e \omega_{pe}}\right)^{1/2} \frac{n_e T_e}{k_*}} \approx 3\left(\frac{Q^l}{n_e T_e \omega_{pe}}\right)^{1/2} \frac{n_e T_e}{k_*}. \tag{4.8}$$

In the last equation we put $T_e = T_i$. Although a non-isothermal plasma is quite likely to occur under cosmic conditions, we noted in § 3 that then the transfer is determined not by the non-linear scattering, but by the decay process $l \rightarrow l' \pm s$ and the factor $1 + T_e/T_i$ anyhow does not differ too much from unity.

In the region of lower wavenumbers, $k < k_g$ there is no longer generation of longitudinal plasmons but absorption remains which will diminish W_k^l while there is a further transfer along the wavenumber spectrum. If this absorption is weak the spectrum W_k^l will not change much when k is decreased to $k_* = (130 d_e)^{-1}$ and remains close to (4.8). This simple solution, obtained by Liperovskiĭ and Tsytovich (1970), will often be used in what follows.

Electron-ion collisions lead to a slant of the curve of the spectral density of longitudinal plasmons, that is, to a linear decrease in W_k^l, after the first maximum is reached. Indeed, retaining in (4.5) the second and fourth terms on the right-hand side we have

$$\frac{\partial W_k^l}{\partial k} = \frac{108}{\pi} m_i v_{Te} \omega_{pe} = \frac{\nu_e}{\alpha^l}. \tag{4.9}$$

It is interesting that the magnitude of this slope is independent of the parameters of the turbulence excitation. If $k_g\,\partial W_k^l/\partial k$ is much less than the spectral density of the flat spectrum, that is, if

$$\frac{Q^l}{k_g^2} \gg \frac{54}{\pi}\,\frac{m_i v_{Te}\omega_{pe}}{N_D}, \tag{4.10}$$

collisional damping of the longitudinal plasmon turbulence is unimportant. In the opposite case it is practically impossible for the turbulence to develop.

The absorption of longitudinal plasmons through their fusion and the formation of transverse waves with frequency $\sim 2\omega_{pe}$ becomes important only for small wavenumbers—the effect is proportional to k^{-2}. Because of the different k-dependence one can distinguish the regions of collisional and radiation damping. For the latter we retain on the right-hand side of (4.5) the third and fourth terms (Tsytovich 1969b):

$$W_k^l = W_{k_g}^l \exp\left[-\frac{\beta^l}{\alpha^l}\left(\frac{1}{k}-\frac{1}{k_g}\right)\right] = W_{k_g}^l \exp\left[-k_r\left(\frac{1}{k}-\frac{1}{k_g}\right)\right], \tag{4.11}$$

where $W_{k_g}^l$ is determined by the flat spectrum and the radiation damping parameter is

$$k_r = \frac{\beta^l}{\alpha^l} = \frac{108\sqrt{3}}{5}\,\frac{m_i}{m_e}\left(\frac{v_{Te}}{c}\right)^4 \frac{\omega_{pe}}{c}. \tag{4.12}$$

When $k < k_r$ the spectrum abruptly breaks off and decreases exponentially. In that case the larger part of the energy of the longitudinal plasmon turbulence disappears not in heat, but in radiation.

The example considered here is probably one of the most effective mechanisms for generating radiant energy at frequencies $\sim 2\omega_{pe}$. It is true that the conditions for the existence of the radiation spectrum (4.11) are rather rigid. First of all, this fusion of longitudinal plasmons occurs when their wavenumbers are large, that is, $k_r \gtrsim \sqrt{3}\omega_{pe}/c$. Comparing this condition with (4.12) we have

$$\frac{108}{5}\,\frac{m_i}{m_e}\left(\frac{v_{Te}}{c}\right)^4 \gg 1. \tag{4.13}$$

Secondly, the fusion must be important in the region of differential transfer. Putting $k_r \gg k_*$ we find a second condition which is practically the same as (4.13). Both criteria are satisfied only in a hot plasma. In particular, it follows from (4.13) that for an isothermal hydrogen plasma we have

$$T_e \approx T_i \gg 3\times 10^7\ ^\circ\mathrm{K}. \tag{4.14}$$

Such a hot plasma does not exist in the solar corona, but it is possible in other objects. However, if (4.13) is not satisfied, this does not mean that the turbulence can not be radiative. It simply indicates that radiative losses can not absorb the plasmon energy during their transformation down to the region of integral transfer.

Finally, the effect of radiation turbulence, considered here, is only possible when one can neglect the inverse process of the decay of electromagnetic waves into two longitudinal plasmons. This means that the size of the turbulence region must be rather small. The

criterion for the size of the plasma turbulence region has the following form:

$$R < v_g^t/\gamma^t(k_r) \approx c/\gamma^t(k_r),$$

where $\gamma^t(k_r)$ is the damping rate of electromagnetic waves due to their decay into longitudinal plasmons, determined by equation (3.35) for $k_1 = k_r$.

In actual astrophysical problems the plasma turbulence has a flat spectrum breaking off at $k = k_r$ provided the plasma temperature is high and the size of the turbulent region not too large. At lower temperatures or relatively larger sizes of this region, the flat spectrum is pulled out into the region of large phase velocities until other mechanisms for transferring energy along the spectrum are included. When the collisions play a larger role the slope of the spectrum also becomes appreciable.

We now turn to the region of small wavenumbers and assume to begin with that $k_s < k_*$. We shall not write down the kinetic equations and restrict ourselves to a semi-quantitative description of the physical phenomena accompanying plasma turbulence in that part of the spectrum.

Since the integral transfer only becomes effective when $k \approx k_*$ the larger part of the longitudinal plasmon energy begins to change fast to a region of wavenumbers close to zero. The quantity W_k^l will increase. It is clear that then the four-plasmon interactions also become effective, as they are proportional to $(W_k^l)^3$. They, so to speak, "repel" the plasmons which pile up in a very small phase volume near $k \approx 0$ until some spectrum is established with a maximum at a non-vanishing wavenumber k_0 which can be said to correspond to the basic scale-length.

The spectrum decreases both for $k < k_0$ and for $k > k_0$. Numerical calculations and analytical asymptotic solutions show that the spectra of longitudinal plasmons here are described by the formulae (Pikel'ner and Tsytovich, 1969a; Zhidkov, Makhan'kov, Tsytovich and Khen, 1970; Liperovskiĭ and Tsytovich, 1970)

$$W_k^l \sim k^2; \quad \text{when} \quad 0 < k < k_0; \quad W_k^l \approx k^{-\nu}, \quad \text{when} \quad k_0 < k < k_*, \quad 2{\cdot}84 \leqslant \nu \leqslant 4, \quad (4.15)$$

where the index ν is close to 2·84 for large Q and increases to 4 when Q decreases. It is difficult to determine from elementary estimates the basic scale k_0 and the magnitude of the spectral density W_{k_0} in the maximum around k_0. The point is that the simple formulae (3.38) and (3.39) characterize only the order of magnitude of the four-plasmon interaction. When the spectrum changes, this quantity can change appreciably. In other words, the four-plasmon interaction is, so to speak, "adjusted" to the spectrum obtained and an estimate of W_{k_0} must thus be obtained by solving a complicated self-consistent problem, although one can give simple qualitative estimates for k_0.

One can relatively simply estimate the total energy density of the longitudinal plasmons in the spectral region considered. As from this region almost all the turbulence energy is transferred, it is "killed" here, changing either into heat through electron–ion collisions (heating turbulence), or into electromagnetic radiation with $\omega \approx 2\omega_{pe}$ (radiation turbulence). It is clear that in the first case we have

$$W^l \approx \int_0^{k_*} W_k^l \, dk \approx \frac{Q^l}{\nu_e} \approx N_D \frac{Q^l}{\omega_{pe}}. \quad (4.16)$$

As a rule a larger amount of plasma wave energy accumulates in this region of the spectrum than in the region of larger wavenumbers. We can thus assume that (4.16) gives us an estimate for the total density of the longitudinal plasmon turbulence energy. Hence:

$$\frac{W^l}{n_e T_e} \approx \frac{Q^l}{n_e T_e \omega_{pe}} N_D. \tag{4.17}$$

Earlier we considered radiation turbulence for the case when $k_r > k_*$. If that inequality is not satisfied, fusion of longitudinal plasmons with $k \ll \sqrt{3}\omega_{pe}/c$ dominates as their number is now larger than that of plasmons with larger wavenumbers. We assume that such fusion processes remove longitudinal plasmons faster from the turbulent spectrum than collisional damping (Tsytovich, 1971a). It follows from (3.33) that for this the following condition is necessary:

$$\nu_e \ll \frac{\pi}{4\sqrt{3}} \frac{\omega_{pe}^2}{n_e m_e c^3} W^l_{\sqrt{3}\omega_{pe}/c}. \tag{4.18}$$

Substituting instead of $W^l_{\sqrt{3}\omega_{pe}/c}$ the spectrum (4.8), we get

$$\frac{Q^l}{n_e T_e \omega_{pe}} > \frac{1}{18\pi} \frac{m_e}{m_i} \left(\frac{c}{v_{Te}}\right)^6 \frac{1}{N_D^2}. \tag{4.19}$$

It is easy to satisfy condition (4.19).

We obtain the total energy of the radiation turbulence of longitudinal plasmons by dividing Q^l by the decay rate (3.33):

$$\frac{W^l}{n_e T_e} \approx \frac{1}{3} \sqrt{\left(\frac{m_e}{\pi m_i}\right)} \frac{c^3}{v_{Te}^3} \left(\frac{Q^l}{n_e T_e \omega_{pe}}\right)^{1/2}. \tag{4.20}$$

The basic scale of radiation turbulence, in the neighbourhood of which the spectral density of the plasma wave energy is largest, is determined by the equation

$$\frac{k_0}{k_*} \approx \left[\frac{54 m_i}{m_e} \left(\frac{v_{Te}}{c}\right)^6\right]^{1/2(\nu-1)}. \tag{4.21}$$

In the case of turbulence with collisional damping (turbulent heating) the wavenumber of the basic scale is even smaller as in radiation turbulence the fusion of plasmons prevents them from going to small wavenumbers. In collisional turbulence k_0 depends, though weakly, on the power of the turbulence sources, approximately as $Q^{-1/2(\nu-1)}$.

We now turn to the large wavenumber region, that is, $k > k_g + \Delta k_g$. Part of the longitudinal plasmons with $k < k_g$ can be transferred also to this region of the spectrum but only through spontaneous non-linear scattering by electrons which makes it possible to increase the wavenumber appreciably. Indeed, we get from the conservation laws

$$\omega - \omega' = \frac{3v_{Te}^2}{2\omega_{pe}} (k^2 - k'^2) \approx |\boldsymbol{k} - \boldsymbol{k}'| v_{Te}, \tag{4.22}$$

whence follows the possibility $k' \approx k + \omega_{pe}/3v_{Te} \approx (3d_e)^{-1}$. In other words, non-linear spontaneous scattering by electrons can in a single scattering process transfer a plasmon into a region of wavenumbers comparable to the reciprocal Debye radius.

The evaluation of the probability of such a process is complicated as it is necessary to take into account the already mentioned cancellation of non-linear and Compton scattering. We give immediately the final formula for the longitudinal plasmon emission coefficient for the spontaneous scattering by electrons of plasmons with smaller wavenumbers ($k_* \ll k' \ll k$):

$$J_k(\mathrm{l}+\mathrm{e} \rightleftharpoons \mathrm{l}'+\mathrm{e}') = \frac{4v_{Te}k}{15(2\pi)^{3/2}n_e} \int W^{\mathrm{l}}_{k'} k'^2 \, dk'. \tag{4.23}$$

Of course, apart from the transfer of longitudinal plasmons from smaller to larger wavenumbers, the opposite process also exists, namely, the absorption of those plasmons, the damping rate of which is determined from (3.24) with $k_* \ll k' \ll k$:

$$\gamma^{\mathrm{l}}(\mathrm{l}+\mathrm{e} \rightleftharpoons \mathrm{l}'+\mathrm{e}') = -\frac{2\sqrt{(2\pi)}}{5} \frac{v_{Te}k}{n_e m_e \omega_{pe}^2} \int W^{\mathrm{l}}_{k'} k'^2 \, dk'. \tag{4.24}$$

One can easily evaluate the integrals in (4.23) and (4.24) for the spectrum (4.8). Bearing in mind that when $k > k_g + \Delta k_g$ the spectral density of the plasmon energy is small, we restrict ourselves in the integration to $k \approx k_g$. The behaviour of W^{l}_k for small k is unimportant because of the factor k'^2 in the integrand. We find for the emission coefficient:

$$J_k(\mathrm{l}+\mathrm{e} \rightleftharpoons \mathrm{l}'+\mathrm{e}') = \frac{4v_{Te}k_g^3 k}{45(2\pi)^{3/2}n_e} \sqrt{\frac{2Q^{\mathrm{l}}}{\alpha^{\mathrm{l}}}} = \frac{4}{5} \sqrt{\frac{m_i}{3m_e}} \left(\frac{v_{Te}k_g}{\omega_{pe}}\right)^4 \left(\frac{Q^{\mathrm{l}}}{n_e T_e \omega_{pe}}\right)^{1/2} \frac{m_e \omega_{pe}^3}{2\pi^2} \frac{k}{k_g}, \tag{4.25}$$

and for the damping rate

$$\gamma^{\mathrm{l}}(\mathrm{l}+\mathrm{e} \rightleftharpoons \mathrm{l}'+\mathrm{e}') = -\frac{4}{5} \sqrt{\frac{3m_i}{m_e}} \left(\frac{v_{Te}k_g}{\omega_{pe}}\right)^4 \left(\frac{Q^{\mathrm{l}}}{n_e T_e \omega_{pe}}\right)^{1/2} \frac{k}{k_g} \omega_{pe}. \tag{4.26}$$

We can now easily write down the kinetic equation for the longitudinal plasmons in the large wavenumber region:

$$\frac{dW^{\mathrm{l}}_k}{dt} = J^{\mathrm{l}}_k - (\nu_e + |\gamma^{\mathrm{l}}|) W^{\mathrm{l}}_k + \alpha^{\mathrm{l}} W^{\mathrm{l}}_k \frac{\partial W^{\mathrm{l}}_k}{\partial k}. \tag{4.27}$$

We shall again consider stationary solutions. In the case when we can neglect the non-linear transfer through scattering by ions as well as collisional damping we get a constant spectrum:

$$W^{\mathrm{l}}_k = \frac{J^{\mathrm{l}}_k}{|\gamma^{\mathrm{l}}|} = \frac{m_e \omega_{pe}^2}{6\pi^2}. \tag{4.28}$$

In the region of very large k, but with $k \ll \frac{1}{3} d_e$, this spectrum is close to the spectrum of thermal pulsations:

$$W^{\mathrm{l}}_k = \frac{k^2}{2\pi^2} T_e \approx \frac{m_e \omega_{pe}^2}{18\pi^2}. \tag{4.29}$$

However, in contrast to the thermal spectrum, which decreases as k^2 with decreasing k, the spectrum (4.28) stays constant (see Fig. 4).

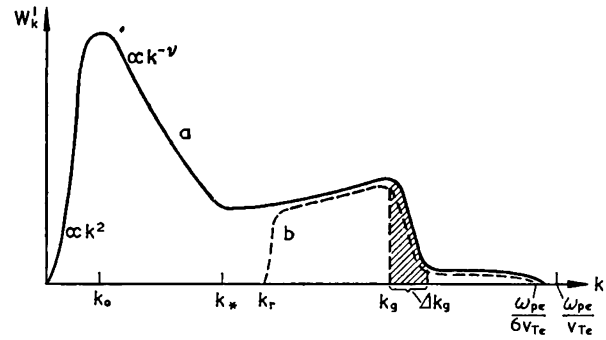

FIG. 4. Spectrum of Langmuir turbulence: (a) the case of weak radiation damping or collisional damping; (b) the case of strong radiation damping in the region of differential transfer.

If the plasma wave energy density is large, it is possible that the last term in (4.27) also turns out to be important and may dominate over the damping terms. The transfer of plasmons into the region $k > k_g$ will then cancel their gradual depletion due to the non-linear scattering by ions. In that case we have

$$W_k^l = \sqrt{\left(\frac{2}{\alpha^l}\int_k^{1/3d_e} J_{k'}\,dk'\right)} \approx \sqrt{\frac{32\pi}{15}\left(\frac{3m_i}{m_e}\right)^{3/4}\left(\frac{Q^l}{n_e T_e \omega_{pe}}\right)^{1/4}\frac{m_e\omega_{pe}^2}{2\pi^2}\, n_e^{1/2}k_g^{3/2}d_e^3}. \quad (4.30)$$

Equation (4.30) again leads to a flat spectrum which extends to the wavenumber $k \approx k_g + \Delta k_g$ where it sharply increases to the spectrum (4.8). For large k the flat spectrum (4.30) extends in an isothermal plasma ($T_e \cong T_i$) only up to $v_p \approx 6v_{Te}$, determined by (3.1); when k becomes even larger, the spectrum starts to decrease: $W_k \propto k^{-5/2}$ (Liperovskiĭ and Tsytovich, 1970). The plateau (4.30) extends up to $k \approx (3d_e)^{-1}$ in a non-isothermal plasma with $T_e \gg T_i$.

The magnitude of the jump in the spectrum at $k \approx k_g$ is equal to

$$\frac{W_{k>k_g}^l}{W_{k<k_g}^l} \approx 0{\cdot}15\left(\frac{k_g^3}{n_e}\right)^{1/2}\left(\frac{n_e T_e \omega_{pe}}{Q^l}\right)^{1/4}. \quad (4.31)$$

We shall consider the opposite case for scattering by electrons, when $k \approx k_0 \ll k_*$, in § 8.

If $k_g < (6d_e)^{-1}$ the spectrum starts with a section $W_k \propto k^{-5/2}$ due to scattering by electrons, and only when $k \approx 1/6d_e$ does it become flat. We give in Fig. 4 the qualitative behaviour of the longitudinal plasmon spectrum in both wavenumber regions.

So far we have assumed that the energy transfer along the spectrum is mainly determined by the non-linear scattering by thermal ions. In a strongly non-isothermal plasma the plasmon energy transfer is determined by their decay involving ion-sound waves. All relations remain as before, but the efficiency of the non-linear interaction is larger by a factor four and the level of the standard spectrum is lower therefore by a factor two. All other relations change in more or less the same way. The most important change in the spectrum for the case when $T_e \gg T_i$ occurs for $k < 1/6d_e$ when the section with $W_k \propto k^{-5/2}$ disappears and the flat spectrum extends up to the maximum k.

There are a number of reasons why we have given such a detailed analysis of the longitudinal plasmon spectrum. First of all, this spectrum is the easiest one to study. Secondly, longitudinal plasmon turbulence is just the one of most importance for plasmon astrophysics. And, thirdly, we tried to show by this example how one can study the spectra of plasma turbulence in other cases.

We now shall say a few words about non-stationary longitudinal plasmon turbulence.

It is well known that one of the solutions of a non-stationary equation such as (4.5) which is similar to a hydrodynamic equation, consists of simple Riemann waves which can turn into shock waves when propagating. It is clear that we can use that analogy to study the non-stationary equation (4.5). We shall give here only a qualitative analysis.

Let the turbulence source Q^l be switched on at a time $t = 0$ and then at once reach its full power and further remain constant. Along the wavenumber axis there appears then at once a "spectral density jump" which moves along the k-axis with a "velocity"

$$\frac{dk}{dt} = \alpha^l W_k^l \approx \sqrt{(2\alpha^l Q^l)} \approx \sqrt{\left(\frac{\pi m_e}{54 m_i}\right)\left(\frac{Q^l}{n_e T_e \omega_{pe}}\right)^{1/2} \frac{\omega_{pe}^2}{v_{Te}}}. \tag{4.32}$$

Behind the front of this jump the spectrum W_k^l remains constant—provided Q^l does not change with time for $t > 0$—and will correspond to the spectrum (4.8); this was used in (4.32). The width of the jump front will be of the order of the change Δk in a single scattering process (this is the analogue of the mean free path), that is, $\Delta k \approx k_*$. When we get to the region of integral transfer the "spectral density jump" will spread out (Fig. 5).

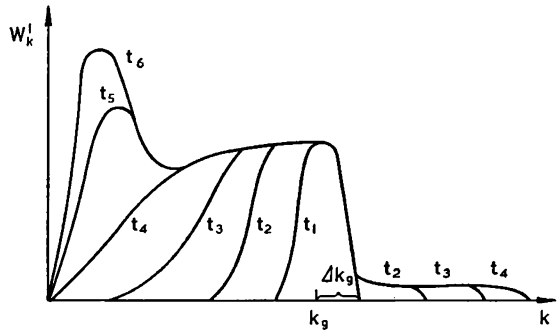

FIG. 5. Qualitative illustration of the dynamics of establishing the turbulence spectrum.

If the increase in the power of the turbulence sources Q^l with time after it has been switched on is not instantaneous but more gradual, the jump is not formed at once at $k \approx k_g$, but further along the wavenumber axis. If the increase in Q^l is even slower it is possible that the jump does not form at all and that the whole longitudinal plasmon turbulence spectrum increases together with Q^l, provided, of course, that the role of the parameters leading to the damping of the turbulence do not turn out to be a determining factor.

The spectral regions with $k < k_*$ and $k > k_g$ are formed after the spectrum in the region $k_* < k < k_g$ has been set up. A characteristic time for the formation of the spectrum is of the order of the time for spectral transfer and one can easily estimate it, using (4.32).

Under actual astrophysical conditions it is apparently always possible to establish a more or less stationary longitudinal plasmon turbulence spectrum, at any rate in the most important wavenumber ranges. We shall therefore not make a more complete study of the non-stationary aspects of the longitudinal plasmon spectrum, although it is not too difficult to do so.

We shall consider other properties of longitudinal plasmon turbulence in § 8.

Ion-sound turbulence and anomalous plasma resistivity

The generation and propagation of ion-sound waves is possible only in a non-isothermal plasma with $T_e \gg T_i$. Normally astrophysicists have been used to deal with an isothermal plasma, but one must bear in mind that in a plasma turbulence region the larger part of the dissipated energy usually goes to the electrons and that if the collision frequency is small the energy exchange between electrons and ions is slow. Moreover, we have already noted that the ion-sound turbulence itself creates temperature inequalities.

There can be many mechanisms to excite ion-sound turbulence. Most often one encounters the case where it is excited by strong electrical currents or strong magnetic field gradients. We shall consider just this case in more detail below.

Ion-sound turbulence is also excited by other forms of plasma waves. For instance, if energy is transferred along the longitudinal plasmon spectrum due to decay processes in Langmuir turbulence in a strongly non-isothermal plasma, ion-sound waves are also generated at the same time, that is, ion-sound turbulence arises besides Langmuir turbulence. The energy density which in this way is transferred to ion-sound waves is of the order of magnitude of

$$\frac{W^s}{n_e T_e} \approx \sqrt{\frac{m_i}{m_e}} \left(\frac{Q^l}{n_e T_e \omega_{pe}} \right)^{1/2}. \tag{4.33}$$

Under cosmic conditions this quantity is usually not very large.

The following mechanism is under astrophysical conditions the most important one for exciting ion-sound turbulence. Inhomogeneous magnetic fields create in a cosmic plasma electrical currents. If the density of these currents is sufficiently large a plasma instability can occur. The cause of this phenomenon lies in the fact that the current is connected with a drift of the electrons with respect to the ions. If the drift velocity is larger than the ion-sound wave velocity, Cherenkov excitation of ion-sound turbulence is possible. The presence of a current in the plasma leads also to the appearance of an electric field which can easily be determined if one knows the conductivity of the plasma and its dependence on the amplitude of the field strength. It is convenient to consider ion-sound turbulence in what follows, assuming the magnitude of the electric field to be known.

Let the plasma be in an electric field of field strength E. The electrons are then accelerated by a force eE. At the same time they are retarded by collisions with other particles. The frictional force is equal to $m_e v \nu_{eff}$, where $\nu_{eff} = \nu_e$ as long as the electron velocity is less than the thermal velocity, and $\nu_{eff} = \nu_e (v_{Te}/v)^3$ for larger electron velocities.

If the force eE is larger than the frictional force, which is true for sufficiently fast electrons, such electrons become "run-away" electrons and they must be retarded by other mechanisms—for instance, by falling on the walls of the apparatus in a laboratory plasma. When

the electric field is increased, a larger number of electrons become "run-aways" and for fields larger than the so-called Dreicer field E_D also electrons with velocities close to the thermal velocity run away. For E_D we have

$$E_D = \frac{m_e v_{Te} v_e}{e} = \frac{m_e \omega_{pe}^4}{e n_e v_{Te}^2} = \frac{4\pi e \omega_{pe}^2}{v_{Te}^2} = \frac{4\pi e}{d_e^2} . \tag{4.34}$$

The occurrence in a plasma of such an electric field leads to a number of important consequences which, however, occur also for fields appreciably less than E_D—to be precise, at $E_D(m_e/m_i)^{1/2}$, provided $T_e \gg T_i$.

We shall now consider how the plasma electrical conductivity σ behaves. As long as the electric field strength is small, one can evaluate the current strength in the plasma, using the normal Ohm law:

$$j = \sigma E = \frac{\omega_{pe}^3}{4\pi v_e} E. \tag{4.35}$$

After the electric field becomes larger than the Dreicer field, the resistivity of the plasma to a current should, it seems, decrease steeply and the current should increase steeply. However, in actual fact, for $E > E_D$ the resistivity not only does not decrease, but on the contrary it increases strongly: we have the so-called anomalous resistivity. This effect can be interpreted to correspond to an increase in the effective collision frequency of the accelerated electrons.

The explanation of this effect is as follows. An electric field $E > E_D$ causes the electrons and ions to move in different directions: drift. The drift velocity u, which we define in a moment, is a function of the electric field strength: when E increases, so does u. When the drift velocity turns out to be larger than the ion-sound velocity v_s Cherenkov excitation of ion-sound waves becomes possible. The relative motion of ions and electrons produces in the electron distribution function a section with positive derivative $\partial f_p/\partial p$. This means that ion-sound waves are excited.

As the growth rate and the damping rate differ only in the sign of the derivative $\partial f_p/\partial p$ the growth rate of ion-sound waves is proportional to their damping rate, with the opposite sign. More precisely:

$$\frac{\gamma^s}{\omega^s} = \sqrt{\left(\frac{\pi m_e}{8 m_i}\right)} \frac{u \cos\vartheta - v_s}{v_s}, \tag{4.36}$$

where ϑ is the angle between the wavevector of the excited wave and the electric field vector. The maximum growth rate occurs for waves propagating along the field. The ion-sound waves which are generated are enclosed within a Mach cone with angle $\vartheta_M = \arccos(v_s/u)$ and outside this cone the waves are only damped. Anisotropic turbulence occurs with wavenumbers up to $k \approx 1/d_e$ where the growth rate is at its largest.

When the level of the oscillations exceeds a certain limit either non-linear interactions may come into play or the effect of the oscillations upon the electrons may modify their distribution in such a way that the growth rate (4.36) decreases. We shall begin our discussion with the first case and we shall follow a recent analysis (Tsytovich, 1971b) which is based upon experimental results (Hamberger, Jancarik and Sharp, 1969; Sharp and Hamberger, 1970; Hamberger and Jancarik, 1971).

Different mechanisms for non-linear interactions, such as scattering by ions or decay processes, lead to the same form of the spectrum, $W_h \propto 1/k$, but with somewhat different numerical coefficients which all depend on the angular distribution of the ion-sound waves (Kadomtsev and Petviashvili, 1963; Sagdeev, 1967). If the spectrum is established through decay processes, we have

$$W_k^s \approx \frac{1}{4}\sqrt{\left(\frac{\pi m_e}{8m_i}\right)}\frac{n_e T_e}{k}\,|\ln(k\,d_e)|. \tag{4.37}$$

In the case of scattering by ions there appears a large factor $9T_e/T_i$. An estimate of the total energy density of ion-sound turbulence using the spectral function (4.37) gives

$$\frac{W^s}{n_e T_e} \approx \frac{1}{4}\sqrt{\left(\frac{\pi m_e}{8m_i}\right)}\,|\ln(k_0 d_e)|^2 \approx 0{\cdot}1 \quad \text{to} \quad 0{\cdot}2. \tag{4.38}$$

Here k_0 is a characteristic wavenumber which need not be determined exactly.

We can now turn to the electrical conductivity of a plasma in a strong electric field. After ion-sound turbulence has been established the accelerated electrons are braked by "collisions" with the ion-sound waves, and not by collisions with other particles. One can define the effective frequency of "collisions" between electrons and ion-sound waves, ν_{eff}, as the quantity which is proportional to the drift velocity:

$$\nu_{\text{eff}} = \frac{\omega_{pe}}{100}\frac{u}{v_{Te}}. \tag{4.39}$$

The numerical coefficient is determined by the spectrum (4.37). Sagdeev (1967) and Tsytovich (1971b) obtained theoretically a similar formula starting from somewhat different premises. In Sagdeev's formula there was an extra factor T_e/T_i and in Tsytovich's formula a factor $(\Delta\omega/\omega)^{1/2}$ where $\Delta\omega$ was the correlation width. They have been confirmed experimentally (Hamberger and Jancarik, 1971).

We proceed as follows to determine the drift velocity u. It is clear that we can express it in terms of the electric current density: $u = j/n_e e$. Moreover, we can express the current density in the regime considered through (4.35) if we replace ν_e by the effective frequency of "collisions" between electrons and ion-sound waves. As a result we get an elementary equation to determine the drift velocity, and it leads to

$$u = 10 v_{Te}\sqrt{\left(\frac{E}{N_D E_D}\right)}. \tag{4.40}$$

From this we get the following expression for the anomalous resistivity:

$$\sigma = \frac{10\omega_{pe}}{4\pi}\sqrt{\left(N_D\frac{E_D}{E}\right)}, \tag{4.41}$$

and to a corresponding change in Ohm's law:

$$j = \omega_{pe}\sqrt{(N_D E_D E)}. \tag{4.42}$$

This regime of ion-sound turbulence excitation occurs when the electron drift velocity is larger than v_s, but less than v_{Te}. We find thus from (4.40) the limits for the values of the electric field strengths which produce an intensive ion-sound turbulence:

$$\frac{m_e}{m_i}\frac{N_D}{100}E_D = E'_D < E < E''_D = \frac{N_D}{100}E_D. \tag{4.43}$$

If $E'_D > E_D$ there occurs generation of ion-sound turbulence in the electric field strength range $E_D(m_e/m_i)^{1/2} < E < E'_D$, albeit at a lower level, so that the non-linear effects appear weakly and the energy density of the ion-sound waves is, so to speak, "adjusted" to the condition $u = v_s$. The current density is then practically independent of the electric field strength (Rudakov and Korablev, 1966; Kovrizhnykh, 1967),

$$j = n_e e v_s. \tag{4.44}$$

The "cut-off" in the anomalous resistivity, if from the start $T_e \gg T_i$, occurs at electric field strengths less than E_D when (4.44) and (4.35) become comparable, that is, when

$$E = \frac{v_s}{v_{Te}}E_D = \sqrt{\left(\frac{m_e}{m_i}\right)}E_D. \tag{4.45}$$

Besides, if from the start $T_e \approx T_i$, the drift velocity due to the "running-away" of all particles becomes larger than v_{Te} only when $E > E_D$. However, a direct excitation of Langmuir turbulence does not occur, as one might have thought. Theoretical analysis and experimental data show that now the so-called Buneman instability (Buneman, 1959) occurs with a very large growth rate:

$$\gamma_B \approx \frac{\sqrt{3}}{2}\left(\frac{m_e}{2m_i}\right)^{1/3}\omega_{pe}, \tag{4.46}$$

when turbulent pulsations are excited with frequencies of the same order as the growth rate (4.46). The energy of these pulsations changes fast into heat, increasing v_{Te} and the system changes into a state of ion-sound turbulence with $T_e \gg T_i$ and $u \ll v_{Te}$. The non-

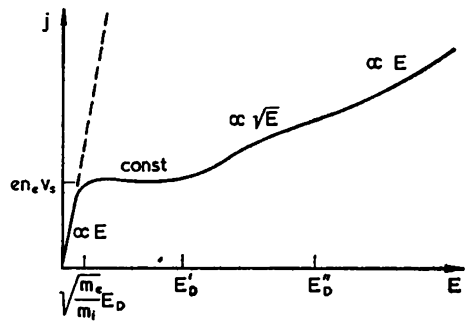

FIG. 6. Qualitative dependence of the current density on the magnitude of an applied electric field for the case when $T_e \gg T_i$. The dashed line shows the case of collisional resistivity.

linear stage of the Buneman regime has been studied poorly; experimental data show that here Ohm's law holds:

$$j = \frac{1}{2}\left(\frac{m_i}{m_e}\right)^{1/3} \omega_{pe} E. \tag{4.47}$$

We give in Figs. 6 and 7 graphs illustrating the way the current density and the conductivity change as functions of the electric field strength.

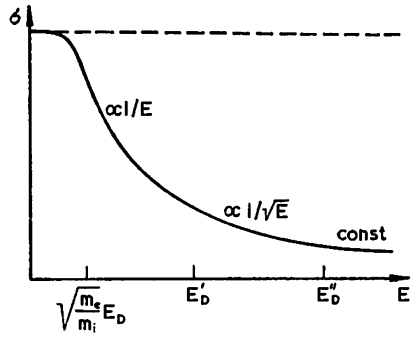

FIG. 7. Qualitative picture of the way the electrical conductivity of the plasma depends on the magnitude of the electric field for $T_e \gg T_i$.

MAGNETOHYDRODYNAMIC PLASMA TURBULENCE

The two examples of plasma turbulence considered above are important for astrophysical applications. One may expect that other kinds of turbulence may become of interest in the future.

We shall give here a set of equations describing the mutual transfer of energy between Alfvén and magneto-sound modes. It is in that case convenient to deal with W_ω^a and W_ω^m as the frequencies change little in the transfer while the wavenumbers change appreciably because of the differences in dispersion relations. Moreover, we assume that the turbulence sources are mainly excited by Alfvén waves and that basically the magneto-sound waves are damped—this assumption is often justified as $\gamma^a \ll \gamma^m$. The set of equations for the energy spectral densities, averaged over the angles, for the case where the energy transfer is due to induced scattering by ions has the form (Livshits and Tsytovich, 1970):

$$\left.\begin{aligned} \frac{dW_\omega^a}{dt} &= Q_\omega^a + \alpha^a \omega^2 W_\omega^a \left[3\frac{\partial}{\partial\omega}(\omega W_\omega^a) + 2\frac{\partial}{\partial\omega}(\omega W_\omega^m) + W_\omega^m\right], \\ \frac{dW_\omega^m}{dt} &= -\gamma^m W_\omega^m + \alpha^a \omega^2 W_\omega^m \left[\frac{\partial}{\partial\omega}(\omega W_\omega^m) + 2\frac{\partial}{\partial\omega}(\omega W_\omega^a) - W_\omega^a\right], \end{aligned}\right\} \tag{4.48}$$

where we have the non-linear transfer parameter

$$\alpha^a = \frac{\pi}{3}\,\frac{1}{n_e m_i v_A^2 (1 + T_e/T_i)^2}\,. \tag{4.49}$$

One checks easily that the stationary set (4.48), generally speaking, does not have a solution for which W_ω^a and W_ω^m remain positive for all frequencies. Of course, a solution with a

negative spectral density for the energy has no physical meaning. This means that there is in the system a continuous transfer of energy from one mode to the other so that, for instance, in a given frequency range at different times the energy density is different. So to speak, "pulsations" of the spectrum occur with a characteristic "frequency" $1/\tau$ and a characteristic time-averaged energy spectral density $\overline{W}_\omega^{\mathrm{a}} \approx \overline{W}_\omega^{\mathrm{m}}$ with

$$\overline{W}_\omega^{\mathrm{a}} \approx \frac{1}{\omega} \sqrt{\left(\int \frac{Q_\omega^{\mathrm{a}}}{\alpha^{\mathrm{a}}} \frac{d\omega}{\omega} \right)} \approx \frac{1}{\omega} \sqrt{\frac{\bar{Q}_\omega^{\mathrm{a}}}{\alpha^{\mathrm{a}}}}, \tag{4.50}$$

$$\frac{1}{\tau} \approx \omega^2 \alpha^{\mathrm{a}} \overline{W}_\omega^{\mathrm{a}} \approx \sqrt{(\alpha^{\mathrm{a}} \bar{Q}_\omega^{\mathrm{a}})} \tag{4.51}$$

We shall encounter such a behaviour of a two-mode turbulence in the next chapter when we shall consider the turbulence of longitudinal and transverse plasmons in the solar corona.

This concludes our discussion of plasma turbulence spectra. We shall give some other examples in later chapters.

Excitation of plasma turbulence and plasma instabilities

We saw that all parameters of plasma turbulence can be expressed essentially in terms of a single quantity Q, the power of the turbulence sources. The fact that we need here only one parameter is a great merit of the theory, but there remains the problem of how to determine that parameter.

There appear different mechanisms for the excitation of plasma turbulence in a plasma. Most of them are connected with plasma instabilities to a smaller or greater degree. It is well known that there are many such instabilities and it is most expedient to consider them separately for each concrete case. This is what we shall do. However, we shall give here a general classification of plasma instabilities which one may meet in astrophysical problems.

We are not interested in plasma instabilities in general, but in those instabilities which lead to the excitation of some plasma waves in well-defined frequency regions. This is possible when the induced emission of waves begins to dominate over their absorption, that is, when in the appropriate wavenumber range the growth rate $\gamma(k)$, first of all, will be essentially positive and, secondly, be larger than all damping rates which appear there.

It is clear that $1/\gamma(k)$ determines a characteristic time for the excitation of turbulence, and the power of the turbulence generating energy is equal to

$$Q_k = \gamma(k) W_k \tag{4.52}$$

for those wavenumber values which correspond to the range of positive $\gamma(k)$. One can determine the quantity W_k by constructing the balance equations which take into account the transfer of energy away from this range of wavenumbers. We show in the later chapters how this can be done in actual cases.

Depending on why the growth rate is positive, that is, why $(\boldsymbol{k} \cdot [\partial f_p / \partial \boldsymbol{p}]) > 0$, we can distinguish four of the most important kinds of plasma instabilities.

(a) Beam instability, when there is a "hump" on the particle distribution function which is connected with the fact that some of the particles have a velocity component along a preferred direction. A typical example of such an instability is a beam of particles which

have a velocity v_0 which is appreciably larger than the thermal velocity of the particles in the main plasma (Fig. 8; Bohm and Gross, 1949; Akhiezer and Faĭnberg, 1951). In that case $\gamma \propto \partial f_p/\partial p_\parallel$, that is, the growth rate is proportional to the derivative of the distribution function along the beam direction. If $v_0 \gg v_{Te}$, the waves excited are usually Langmuir waves.

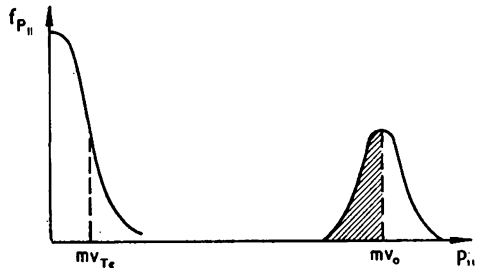

FIG. 8. Electron distribution function in a plasma with a beam. The unstable part is shaded.

(b) Electron–ion drift instability which occurs when all electrons in the plasma have a velocity along a preferred direction which is shifted with respect to the ions, that is, if there is a relatively high current in the plasma. Predominantly ion-sound waves are now excited, as we noted when we considered ion-sound turbulence. From a physical point of view, electron–ion drift instability is analogous to beam instability.

(c) Loss-cone instability which occurs when from the general particle distribution function particles "pour out" in a certain direction with velocities which lie within the limits of the loss cone (Rosenbluth and Post, 1965; Trakhtengerts, 1968). It is well known that particles moving in a magnetic field with convergent magnetic field lines with small values of the momentum component $p_\perp$, at right angles to the magnetic field, can leave freely along the field lines, while particles with large $p_\perp$ are reflected from the convergence of the magnetic field lines and go back. The instability is created here by the fact that there are fewer particles with small $p_\perp$ in the distribution function. The growth rate is proportional to $\partial f_p/\partial p_\perp$; such an instability occurs predominantly in strong magnetic fields with $\omega_{He} \gg \omega_{pe}$ as only under those conditions magnetic bottles are formed. Predominantly magnetic longitudinal waves with frequencies $\omega_{pe}|\cos\vartheta|$ are excited in a cone instability. The growth rate is.

$$\frac{\gamma^1}{\omega_{pe}} \approx \frac{n_*}{n_e}\left(\frac{\omega_{pe}}{kv_{T_\perp}}\right)^3 \frac{1}{\Theta}, \tag{4.53}$$

where n_* is the density of particles which belong to the anisotropic distribution and Θ is the angle of the "pouring-out cone". Equation (4.53) is valid provided $\Theta > \omega_{pe}/kv_{T_\parallel}$. Here k is, as usual, the wavenumber of the excited waves and $T_\parallel$ and $T_\perp$ are the longitudinal and transverse temperatures.

(d) Anisotropy instability which occurs when the velocity distribution function has different dispersions in different directions, for instance, when the longitudinal and transverse temperatures are different (Shafranov and Sagdeev, 1961). The growth rate is here proportional to $\partial f_p/\partial\theta$, where θ is the angle between the particle velocity and the direction of the magnetic field. When the temperature is anisotropic, we have for the growth rate

$\gamma \propto (T_\perp - T_\parallel)/T_\perp$. An anisotropy in the electron temperature leads to the excitation of whistlers, and an anisotropy in the ion temperature to the excitation of Alfvén waves.

All these forms of plasma instabilities are, essentially, maser effects, that is, they are due to an inversion in energy level populations (for the beam instability in $p_\parallel$, for the loss-cone instability in $p_\perp$, for the anisotropy instability in θ, and so on).

The excitation of plasma waves in this kind of instabilities leads to a fast flow of energy of the exciting particles into plasma waves. This causes also a fast "equalization" of any kind of inversion in the level populations (levelling of the "hump" of the beam, filling of the "pouring out cone") until the cause of the instability disappears.

This process is called quasi-linear relaxation and it plays an important role in the whole of plasma physics. We shall consider the effect of quasi-linear relaxation in later chapters. After the relaxation the excitation of plasma turbulence also disappears: in order to maintain it during a considerable period one must have a constant regeneration of the plasma instability.

Under cosmic conditions there are many possibilities for this. For instance, beams can be stabilized if the plasma waves are quickly removed from resonance with the beam (see § 8).

A sufficiently strong electric field maintains electron drift for a long time and a strong magnetic field prolongs the "pouring out" of particles from the loss cone, notwithstanding its filling up by the quasi-linear relaxation. Actual discussions of the causes of instabilities and the role played by quasi-linear relaxation under cosmic conditions will be given in later chapters while here we give a few general considerations.

We must bear in mind that the inhomogeneities and non-stationary character which are always inherent in cosmic objects must inevitably lead to instabilities. It is true that the inhomogeneities in the system give too large a characteristic time for instabilities such as the flute instabilities or drift wave instabilities, which are well known in plasma physics, to be generated directly by the source of the turbulence. For instance, a characteristic growth rate of a flute instability, $\gamma \approx (v_s/R)[\omega_{pi}^2/(\omega_{pi}^2+\omega_{Hi}^2)]^{1/2}$, is many orders of magnitude smaller than the collision frequency. However, such an inhomogeneity or non-stationarity can produce a small, but all the same important, anisotropy in the distribution function.

We shall consider what degree of anisotropy in the distribution function can lead to amplification. The damping rate is proportional to $\omega f_p^{(0)}$, where ω is the frequency of the absorbed waves and $f_p^{(0)}$ the isotropic (main) part of the distribution function. The growth rate will be proportional to the anisotropic part of the distribution function $f_p^{(1)}$ and to a characteristic frequency connected with the cause of the anisotropy. For instance, if the anisotropy is caused by the effect of a magnetic field the growth rate is proportional to $\omega_{He} f_p^{(1)}$. Thus, if waves can be emitted at frequencies $\omega < \omega_{He} f_p^{(1)}/f_p^{(0)}$, that is, if the damping rate is less than the growth rate, there will be a constant source to excite plasma waves. In particular, when the temperatures are anisotropic in a magnetic field, plasma turbulence can be excited and maintained at frequencies

$$\frac{\omega}{\omega_{He}} < 1 - \frac{T_\parallel}{T_\perp}. \tag{4.54}$$

If we take into account that the frequency of Alfvén waves is much less than ω_{He} and that these waves are weakly absorbed, it follows from this that even a weak anisotropy in the temperatures leads to an almost universal instability with respect to Alfvén waves (Kennel and Petschek, 1966). Such an anisotropy is inevitable when matter flows from an astrophysical object (for instance, it is constantly maintained in the solar wind).

There are also mechanisms for exciting plasma turbulence where the effects of the quasilinear relaxation do practically not affect the turbulence generation.

One of the mechanisms of this kind which are important for plasma astrophysics is the decay process when sufficiently strong electromagnetic radiation is present (Krivorutskiĭ and Tsytovich, 1970).

Indeed, a high-frequency electromagnetic wave with a frequency $\omega \gg \omega_{pe}$ passing through a plasma can decay in it into a plasma wave and another electromagnetic wave with frequency $\omega - \omega_{pe}$. The growth rate for this process in the case when the plasma wave energy density is small is determined by equation (3.37). Besides the effect described by (3.37) there is also the possibility for the spontaneous generation of longitudinal plasmons by this mechanism. It occurs, in particular, also in the case when the high-frequency electromagnetic radiation is isotropic.

It follows from the conservation laws that the minimum frequency of the electromagnetic waves which are able to excite plasma waves with a given wavenumber k is given by

$$\omega_{\min} = \frac{1}{2} kc = \frac{\omega_{pe} c}{2v_p}. \tag{4.55}$$

It is clear that the radiative power of the plasma waves is determined by integrating the decay probability of the electromagnetic waves over the whole of their spectrum from $\omega_{\min}$ to the highest frequencies. Omitting the details of the calculations we give the final result for the Langmuir wave radiative power:

$$Q_k^l = \frac{\pi}{8} \frac{c^2 \omega_{pe}^4 k^3}{n_e m_e} \int_{\omega_{\min}}^{\infty} \frac{(W_\omega^t)^2 \, d\omega}{\omega^6} \left[1 + \frac{2\omega_{\min}^4}{\omega^4} - \frac{2\omega_{\min}^2}{\omega^2} \right]. \tag{4.56}$$

This equation shows that the excitation of longitudinal plasmons proceeds predominantly at the highest allowable wavenumbers so that there also we may assume that Q_k^l is reasonably large only in a narrow range of wavenumbers, as we assumed in the section dealing with longitudinal plasmon turbulence.

We can use the following simplifications for astrophysical applications. Let ω_* be the frequency at which the spectral density of the electromagnetic wave energy density W_ω^t reaches its maximum. One may assume that Langmuir turbulence is excited at wavenumbers of the order $k_g \approx \omega_*/c$ (if $\omega_*/c < 1/d_e$). The total power of the turbulence energy is:

$$Q^l(t \to l + t') \approx \frac{\omega_{pe}^4}{\omega_*^3} \frac{(W^t)^2}{n_e m_e c^2}, \tag{4.57}$$

where W^t is the total electromagnetic wave energy density. Other kinds of plasma waves can be excited in a similar way, but for them the quantity Q is very small. For instance, for

ion-sound waves $Q^s \approx (m_e/m_i)Q^l$ and for whistlers $Q^w \approx (\omega_{pe}/\omega_{He})^4 Q^l$. We must substitute the quantity (4.57) in all formulae of longitudinal plasmon turbulence in order to obtain the properties of that turbulence when it is excited by isotropic high-frequency radiation. It is clear from (4.57) that the quasilinear effect of diminishing Q^l due to the action of the excited waves on the electromagnetic radiation which decreases the power Q^l could consist merely in increasing the characteristic frequency ω_*. Such a process exists, indeed, and we shall consider it in the next section. However, it is very slow compared to many other non-linear processes which occur due to the excitation of plasma oscillations.

There is here an additional absorption mechanism for longitudinal plasmons, namely, the inverse of their fusion with high-frequency electromagnetic waves. The damping rate for this process is

$$\gamma^l(l+t \to t') \approx -\frac{\omega_{pe}^4}{\omega_*^3} \frac{W^t}{n_e m_e c^2}. \tag{4.58}$$

The mechanism to excite longitudinal plasmons by isotropic radiation, considered here is a spontaneous process. Nevertheless it can lead to a very high level of turbulence. If the electromagnetic radiation is anisotropic it acts like a beam and causes an instability resembling a beam instability which excites Langmuir waves. In other words, when there is anisotropic radiation we have instead of the damping rate (4.58) a growth rate (Liperovskiĭ, Kovrizhnykh and Tsytovich, 1967):

$$\gamma^l(t' \to t+l) \approx \frac{\omega_{pe}^4}{\omega_*^3} \frac{W^l}{n_e m_e c^2} \frac{1}{(\Delta\vartheta)^2}, \tag{4.59}$$

where $(\Delta\vartheta)^2$ is the angular dispersion of the wavevectors of the electromagnetic radiation (see also Litvak and Trakhtengerts, 1971). Longitudinal plasmons with wavenumbers close to ω_{pe}/c are here predominantly excited. We shall consider in § 8 methods to evaluate longitudinal plasmon turbulence due to excitation by a beam (either of particles or of electromagnetic waves).

Of course, this brief survey has not covered all possible mechanisms to generate plasma turbulence under cosmic conditions. In later chapters we shall encounter other mechanisms. It is likely that when plasma astrophysics is further developed many new mechanisms to excite plasma turbulence will be discovered.

§ 5. Diffusion of charged particles and of electromagnetic radiation in the field of plasma turbulence

It is of great importance from the point of view of plasma astrophysics to know the interactions of relativistic particles and high-frequency electromagnetic radiation with plasma turbulence. Most of the present book is devoted to these problems. Here we shall consider only the nature of the equations describing such interactions. Although the interactions are different for particles and for radiation the derivation of the equations and their form have much in common.

FAST PARTICLE ACCELERATION IN A PLASMA

If by some means an intense plasma turbulence is excited in the plasma isotropically distributed fast particles which predominantly absorb plasma waves can be very appreciably accelerated (Tsytovich, 1963, 1966a). In other words, plasma turbulence can transfer energy from the turbulence sources to fast particles, increasing their energy even further.

Essentially, any absorption mechanism leads to an increase of particle energy. However, this increase in energy can be distributed over all particles—when one can speak of turbulent heating—or be given only to a small set of fast particles—when we are dealing with acceleration in the "pure form".

One understands easily that the absorption by electrons of ion-sound through Landau damping, for instance, leads to turbulent heating as here practically all electrons take part in the absorption. The change in temperature of the electron gas is described by the obvious relation

$$\frac{\partial T_e}{\partial t} = \frac{Q^s}{n_e}. \tag{5.1}$$

At the same time, Landau damping-type absorption for Langmuir waves can lead to an acceleration of particles. Indeed, a longitudinal plasmon with a phase velocity v_p can be absorbed in a non-bremsstrahlung mechanism by a particle only if its velocity is larger than v_p so that only a small number of particles which already had a high energy is accelerated. When their velocities increase, such particles can absorb a larger number of plasmons and thus further increase the rate of acceleration when their energy increases.

If the phase velocity of the longitudinal plasmons is small, of the order of v_{Te}, they can be absorbed by a large number of electrons and turbulent heating turns out to be more important than the acceleration of separate particles. However, we saw in the preceding section that non-linear transfer can "lead" longitudinal plasmons "away" from small to large phase velocities thus making the conditions for turbulent heating worse and easing the conditions for fast particle acceleration.

It is true, that along the "path" from small to large v_p part of the energy is lost as in each non-linear scattering process the wave frequency is decreased while the energy excess goes to the scattering particles. One usually calls this stochastic plasma heating. In the case of Langmuir plasmons with frequencies within a narrow range this effect is small but for other plasma waves with a stronger wavenumber dependence of the frequency it may turn out that stochastic heating is already more important than the heating through the direct absorption of plasma waves.

The transfer of longitudinal plasmons along the spectrum gradually gets them into a range of phase velocities which are larger than the velocity of light. Such plasmons are no longer absorbed by any particles in a Cherenkov process and acceleration once again stops. Both stochastic heating and heating due to the usual collisional damping remain.

Although turbulent heating, and especially the collisionless heating due to Landau damping is also important in many plasma astrophysics problems the process of the stochastic acceleration of fast particles is of most interest. The Langmuir waves are the most efficient for this. Ion sound, whistlers, and other low-frequency waves can play an important role under certain circumstances, for instance, in the injection of particles into the acceleration

mechanism by longitudinal plasmons. Ion-sound waves may be effective for accelerating heavy, multiply charged ions. Whistlers increase, due to their particular polarization properties, mainly the transverse component of the particle energy with respect to the magnetic field, helping them to be contained in magnetic traps.

We shall analyse in later chapters these actual conditions for the acceleration of particles by different modes. In particular, we shall pay special attention in Chapter 3 to the acceleration of particles through the absorption by them of electromagnetic waves. This acceleration effect differs, in essence, from the case of acceleration through plasma turbulence, which we have just discussed, only in the energy absorption mechanism. In the present section we give merely a description of the method through which particle acceleration can be evaluated.

Let us write down an equation for the change in time of the number of particles in a given phase volume element. We shall only take into account the induced emission and absorption processes for plasma waves as in a turbulent plasma they are far more effective than the spontaneous processes.

One understands easily that the change of the number of particles in a given phase volume element, that is, of the quantity f_p, is determined by the balance between the number of waves absorbed and emitted by these particles:

$$\frac{\partial f_p}{\partial t} = \int N_k[u_{k,p}(f_{k+p}-f_p)-u_{k,p-k}(f_p-f_{p-k})]\frac{d^3k}{(2\pi)^3}$$
$$= \int N_k\left(k\cdot\frac{\partial}{\partial p}\left[u_k\left(k\cdot\frac{\partial f_p}{\partial p}\right)\right]\right)\frac{d^3k}{(2\pi)^3} = \sum_{i,j}\frac{\partial}{\partial p_i}\left(D_{ij}\frac{\partial f_p}{\partial p_j}\right). \tag{5.2}$$

We have taken into account here all "transitions" between particle energy levels, both up and down. Moreover, we have taken into consideration that the change in the particle momentum, which is equal to the wave momentum, is small compared to the magnitude of the particle momentum itself.

One can easily track the analogy between (5.2), describing the diffusion of particles in momentum space, and the usual diffusion equation describing particle diffusion in coordinate space. In both cases the diffusion is connected with the statistical, fluctuation nature of the collisions of the diffusing particles with particles in the medium in the case of coordinate space diffusion and with plasma waves in the case of stochastic acceleration. In the last term in (5.2) we have introduced the particle acceleration diffusion tensor:

$$D_{ij} = \int k_i k_j u_k N_k \frac{d^3k}{(2\pi)^3}. \tag{5.3}$$

As the emission probability u_k depends on the momentum of the emitting particle, the diffusion coefficients correspondingly also depend on the momenta of the accelerating particles.

We shall not consider the diffusion coefficients D_{ij} in their general form. There are many difficulties, especially if one takes into account the anisotropy of the turbulence and the influence of the magnetic field. We shall give merely the relations which are most important for plasma astrophysics.

We assume that both the particle distribution function and the plasma turbulence are isotropic. There remains then only one term on the right-hand side of (5.2) and it is convenient to change from f_p to $f_p = (p^2/2\pi^2) f_p$. As a result (5.2) becomes

$$\frac{\partial f_p}{\partial t} = \frac{\partial}{\partial p}\left(p^2 D_{\parallel} \frac{\partial}{\partial p}\left(\frac{f_p}{p^2}\right)\right), \tag{5.4}$$

where the "longitudinal" diffusion coefficient,

$$D_{\parallel} = \int \frac{(\boldsymbol{k}\cdot\boldsymbol{p})^2}{p^2} u_k N_k \frac{d^3\boldsymbol{k}}{(2\pi)^3}, \tag{5.5}$$

describes the change in the particle momentum (velocity, energy) when absorbing or emitting plasma waves. We can obtain similarly $D_{\perp}$ which describes the process of making the particle distribution isotropic and which differs from (5.5) in that instead of the scalar product $(\boldsymbol{k}\cdot\boldsymbol{p})^2$ now we have the vector product, $([\boldsymbol{k}\wedge\boldsymbol{p}]\cdot[\boldsymbol{k}\wedge\boldsymbol{p}])$. Substituting here, for instance, expression (2.24) for the coefficient of Cherenkov emission of Langmuir waves and integrating, using the fact that only plasmons with $\omega/k < v$ can produce acceleration, we have

$$D_{\parallel}^{l} = \frac{2\pi^2 e^2 \omega_{pe}^2}{v^3} \int_{\omega_{pe}/v}^{1/d_e} W_k^{l} \frac{dk}{k^3} = \frac{\pi}{2} \frac{m_e \omega_{pe}}{n_e} \int_{\omega_{pe}/v}^{1/d_e} \left(\frac{\omega_{pe}}{kv}\right)^3 W_k^{l}\, dk. \tag{5.6}$$

The velocity dependence of the diffusion coefficient can here be either strong or weak; this is connected with the nature of the longitudinal plasmon spectrum. If the spectrum $W_k^{l} = (2Q^{l}/\alpha^{l})^{1/2}$ extends up to phase velocities close to the velocity of light, an estimate gives us from (5.6):

$$D_{\parallel}^{l} = \frac{\pi}{4} \frac{m_e \omega_{pe}^2}{n_e v} W_k^{l} = 3\sqrt{\left(\frac{3\pi m_i}{2m_e}\right)} \left(\frac{Q^{l}}{n_e T_e \omega_{pe}}\right)^{1/2} \frac{v_{Te}}{v} m_e T_e \omega_{pe}, \tag{5.7}$$

that is, $D_{\parallel}^{l} \propto 1/v$. If we are dealing with the acceleration of relativistic particles, we can put $v = c$ and then $D_{\parallel}^{l} =$ constant.

If we know how the diffusion coefficient depends on the velocity or the energy of the particles we can—as is well known in diffusion theory—without solving the equation itself find the characteristic change with time of the parameters. We shall apply this method to relativistic particles for which the energy and momentum are connected through the relation $\varepsilon = pc$ and for which we have just found that $D_{\parallel}^{l} =$ constant. It is well known that the dispersion (in this case the square of the particle energy) increases according to

$$\overline{p^2} = \frac{\overline{\varepsilon^2}}{c^2} = 2D_{\parallel}^{l} t. \tag{5.8}$$

From this we can obtain the following relations for the average value of the rate at which particles with a given energy are accelerated:

$$\frac{d\varepsilon}{dt} = \frac{c^2 D_{\parallel}^{l}}{\varepsilon} = \frac{\pi}{4} \frac{\omega_{pe}^2}{n_e} \frac{m_e c}{\varepsilon} W_k^{l} = 3\sqrt{\left(\frac{3\pi m_i}{2m_e}\right)} \left(\frac{Q^{l}}{n_e T_e \omega_{pe}}\right)^{1/2} \frac{v_{Te}}{c} \frac{m_e c^2}{\varepsilon} T_e \omega_{pe}, \tag{5.9}$$

and also for the characteristic growth rate for the particle energy:

$$\gamma^{\varepsilon} = \frac{c^2 D_{\parallel}^{l}}{\varepsilon^2} \approx \omega_{pe}\left(\frac{m_e c^2}{\varepsilon}\right)^2 \frac{W^l}{n_e m_e c^2} \ln \frac{c}{v_{Te}}. \tag{5.10}$$

Here W^l is the total energy density of longitudinal plasmons with phase velocities below the velocity of light.

Similarly we have for the rate of acceleration for non-relativistic particles, when we substitute $p^2 = 2m\varepsilon$:

$$\frac{d\varepsilon}{dt} = \frac{D_{\parallel}^{l}}{m_e} = 3\sqrt{\left(\frac{3\pi m_i}{2m_e}\right)}\left(\frac{Q^l}{n_e T_e \omega_{pe}}\right)^{1/2} \frac{v_{Te}}{v} T_e \omega_{pe}, \tag{5.11}$$

and for the acceleration growth rate:

$$\gamma^{v} \approx \frac{\omega_{pe}^2 W_k^l}{n_e m_e v^3} \approx 3\sqrt{\left(\frac{3\pi m_i}{2m_e}\right)}\left(\frac{Q^l}{n_e T_e \omega_{pe}}\right)^{1/2}\left(\frac{v_{Te}}{v}\right)^3 \omega_{pe}. \tag{5.12}$$

The rate of acceleration of particles by longitudinal plasmons with a flat spectrum decreases thus with increasing energy as $\varepsilon^{-1/2}$ for non-relativistic velocities and as ε^{-1} for relativistic energies. In rare, but possible, cases when the major part of the longitudinal plasmons have phase velocities appreciably smaller than the velocity of light the acceleration rate may be proportional to $\varepsilon^{-3/2}$ for particles with large velocities.

We can also find an increase in the acceleration rate with increasing energy if the scale k_s for which the transition from non-linear scattering to four-plasmon interaction takes place in Langmuir turbulence corresponds to a range of phase velocities less than the light velocity. In that case W_k^l behaves as $k^{-\nu}$ $(2.84 < \nu < 4)$ so that increasing the particle velocity and energy makes it possible to absorb more and more plasma waves. The rate of acceleration increases thus as $d\varepsilon/dt \propto \varepsilon^{(\nu-1)/2}$, that is, in the most favourable case as $\varepsilon^{3/2}$ (Pikel'ner and Tsytovich, 1969b). Apparently, in real astrophysical conditions the flat spectrum extends all the way to phase velocities comparable with the velocity of light. We give in Fig. 9 the qualitative behaviour of the acceleration rate as function of the particle energy.

The acceleration of particles by low-frequency plasma waves can, as a rule, not produce very large (in particular, relativistic) energies. On the other hand, these mechanisms are

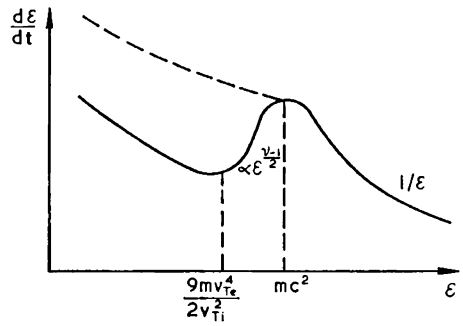

FIG. 9. The behaviour of the acceleration rate as function of particle energy for not too high energies. The full-drawn curve corresponds to the case $T_e/T_i < 2m_e c^2/9m_i v_{Te}^2$, and the dashed curve to the case when the opposite inequality holds.

very efficient as injection mechanisms which produce particle energies up to the threshold where the acceleration by Langmuir waves can start, that is, up to velocities which are larger than the minimum phase velocity of longitudinal plasmons. Indeed, the phase velocities of low-frequency waves in a plasma are small and these waves can thus accelerate particles with low energies. Moreover, they can also more easily accelerate heavy ions.

When acceleration occurs through low-frequency plasma waves a magnetic field can also exert a large influence. Indeed, the acceleration is connected with Cherenkov absorption and this is possible only when the wavelength is much smaller than the Larmor radius. Kaplan, Tsytovich and Chikachev (1971) have considered the complete problem of evaluating the diffusion coefficients for the acceleration by plasma waves taking the effect of a magnetic field into account. We shall here give merely some diffusion coefficients for the acceleration by low-frequency waves and the influence of the magnetic field is taken into account simply by restricting the range of the wavenumbers of the plasma waves which are absorbed in the acceleration process.

Particle acceleration by whistlers is determined by the diffusion coefficient

$$D_{\parallel}^{\mathrm{w}} = \frac{\pi}{6} \frac{m_{\mathrm{e}}\omega_{H\mathrm{e}}^2 c^2}{n_{\mathrm{e}}\omega_{\mathrm{pe}}^2 v} \int_{\omega_H/v}^{\omega_{\mathrm{pe}}/c} W_k^{\mathrm{w}} k \, dk \leqslant \frac{\pi}{6} \frac{m_{\mathrm{e}}\omega_{H\mathrm{e}}^2}{n_{\mathrm{e}}\omega_{\mathrm{pe}}} \frac{c}{v} W^{\mathrm{w}}. \tag{5.13}$$

Here ω_H is the gyro-frequency of a particle in the magnetic field which is equal to eH/mc for a non-relativistic particle and to eHc/ε for a relativistic particle. In inequality (5.13) we assumed that with decreasing k, W_k^{w} does not increase faster than k^{-2}. Electrons are accelerated by whistlers only when their velocities are larger than $v_{\mathrm{A}}(m_{\mathrm{i}}/m_{\mathrm{e}})^{1/2}$, as only then the lower limit of the integral in (5.13) is less than the upper limit.

Ion-sound waves accelerate particles with a velocity which is already larger than v_{s}. The acceleration diffusion coefficient is

$$D_{\parallel}^{\mathrm{s}} = \frac{\pi}{2} \frac{m_{\mathrm{e}}}{m_{\mathrm{i}}} \frac{m_{\mathrm{e}}\omega_{\mathrm{pe}}}{n_{\mathrm{e}}} \left(\frac{v_{T\mathrm{e}}}{v}\right)^3 W^{\mathrm{s}}. \tag{5.14}$$

It is clear that all ion-sound waves take part in the acceleration. The small factor $m_{\mathrm{e}}/m_{\mathrm{i}}$ diminishes this diffusion coefficient, but if we bear in mind that the ion-sound turbulence density may be large, even comparable with the thermal energy of the gas, the acceleration by ion-sound waves turns out to be important. This mechanism can accelerate particles in a cold plasma starting from low energies.

We finally give the expression for the diffusion coefficient for acceleration by Alfvén waves for $v \gg v_{\mathrm{A}}$ (Tsytovich, 1966a; Tverskoĭ, 1967, 1968):

$$D_{\parallel}^{\mathrm{a}} = \frac{\pi^2}{3} \frac{e^2 v_{\mathrm{A}}^3}{(1+v_{\mathrm{A}}^2/c^2)c^2 v} \int_{\omega_H v_{\mathrm{A}}/v}^{\omega_{H\mathrm{i}}} W_\omega^{\mathrm{a}} \frac{d\omega}{\omega} \leqslant \frac{\pi}{12} \frac{m_{\mathrm{i}}\omega_{H\mathrm{i}}^2}{n_{\mathrm{e}}\omega_H} W^{\mathrm{a}}. \tag{5.15}$$

We have taken from under the integral sign in (5.15) the frequency at the lower limit. The larger the particle mass, the smaller ω_H and the larger the efficiency of the acceleration by Alfvén waves (Melrose, 1968).

One can easily determine using the expressions for the diffusion coefficients for acceleration by low-frequency waves, the rate at which energy is accumulated, and the acceleration rate in a similar way as was done earlier for the case of acceleration by Langmuir waves.

The problem of particle acceleration is one of the central problems of plasma astrophysics and we shall several times come back to it in later chapters. We shall here restrict ourselves to only the following remarks.

We first of all note that the acceleration through absorption of plasma waves in a Cherenkov mechanism is by no means the only mechanism through which particles can accumulate energy. Absorption through the cyclotron mechanism increases the velocity component at right angles to the magnetic field and thus makes the containment of particles in a trap much easier. We mentioned this effect in connection with whistlers, but cyclotron absorption also occurs for other waves in a plasma. For instance, high-frequency electromagnetic waves are also absorbed through the cyclotron mechanism and give a similar result. Synchrotron absorption of high-frequency waves also leads to an appreciable acceleration. Finally, scattering effects, such as the Compton effect, are also able to accelerate particles.

Each acceleration mechanism is most effective in its "own" range of velocities and energies. We give in Figs. 10 to 12 qualitative curves which illustrate the role of different acceleration mechanisms under different conditions.

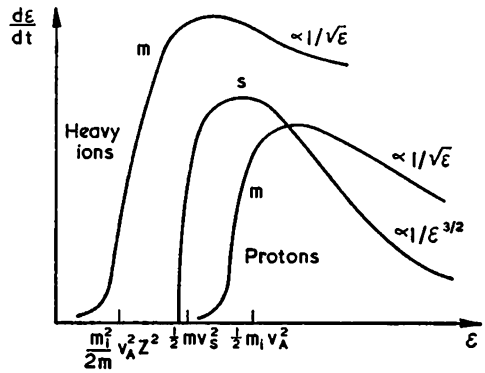

FIG. 10. Acceleration of non-relativistic ions by ion-sound waves (indicated by s) and by fast magneto-sound waves (indicated by m).

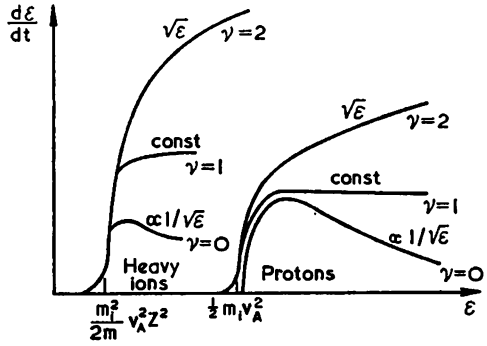

FIG. 11. Acceleration of non-relativistic ions by Alfvén waves with a spectrum $W_\omega^a = \text{const.}/\omega^\nu$ for different values of the index ν.

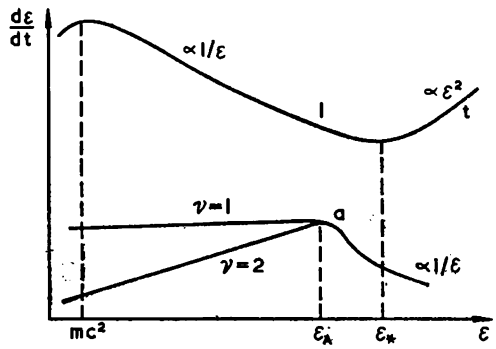

FIG. 12. Acceleration of relativistic particles by Langmuir waves (index l), by Alfvén waves (index a), and by transverse waves (index t). Here ε_A is the energy for which the Larmor radius is equal to the largest wavelength of Alfvén waves and ε_* the energy at which the acceleration through absorption of electromagnetic radiation begins to dominate (see Chapter 3).

We turn now to the problem of conditions for isotropy. Equations (5.4) and (5.5), and also all the subsequent discussion, depend on an assumed isotropy of the accelerated particles. How important is this?

We must first of all note that the components of the diffusion tensor which describe the change in the directions of the particle momentum are, as a rule, much larger than the component $D_\parallel$ which characterizes the change in the absolute magnitude of the momentum. There is thus associated with each acceleration a much faster rate of making the directions of the particle velocities isotropic. If the approach to isotropy is small, there will also practically not be any acceleration.

At the same time, anisotropic distributions in the directions of the particle velocities are, as a rule, unstable and they excite strong plasma turbulence and this turbulence, in turn, will affect the anisotropic distributions (quasi-linear relaxation) and will make them quickly isotropic. An anisotropic distribution exists only for a short time under cosmic conditions while the acceleration process needs a long time. The assumption that there is isotropy, which was made to consider acceleration processes, therefore corresponds rather well to actual cosmic conditions in those plasma regions where the acceleration occurs at all.

Of course, when we consider some plasma turbulence model we must include in it also acceleration problems, taking all factors into account. It is, however, important to note that the calculation of acceleration rates does not require the introduction of new parameters: they are determined by the same power Q of the turbulence sources as are the plasma turbulence spectra. It is of interest that the energy of the fast particles can in final reckoning become larger than the turbulence energy as it is gradually stored up, dissipates slowly, and does not change back into plasma turbulence. This can explain the large energy density of fast particles in astrophysical objects.

Observations show that the energy distribution function of relativistic particles under cosmic conditions usually follows a power law, that is, $f_p \propto \varepsilon^{-\gamma}$. The acceleration through plasma turbulence can lead to the formation of such distributions, provided the acceleration and energy losses of the particles are coordinated in the sense that the same mechanism

is responsible both for the acceleration and for the energy absorption. If the acceleration and deceleration mechanisms are "uncoordinated" the distribution function either becomes Maxwellian or has a modified form such as $f_p \propto \exp(-\text{const.}\, \varepsilon^a)$. For instance, in the case when the particles are accelerated by Langmuir turbulence but lose their energy through collisions a Maxwell distribution is established with an effective temperature

$$T_{\text{eff}} \approx m_e c^2 N_D \sqrt{\frac{m_i}{m_e}} \left(\frac{Q^1}{n_e T_e \omega_{pe}} \right)^{\frac{1}{2}}. \tag{5.16}$$

However, collisions are rare for fast particles, and they lose most of their energy through radiation.

We shall give in later chapters a more detailed analysis of the acceleration of particles by plasma turbulence applicable to actual astrophysical objects.

Scattering of electromagnetic waves in a turbulent plasma

A second effect which we shall discuss here is physically different from the particle acceleration process, but is also described by diffusion type equations.

We consider the transfer of high-frequency ($\omega \gg \omega_{pe}$) electromagnetic radiation in a turbulent plasma. We shall to begin with restrict ourselves for the sake of simplicity to the case when the wavenumber of the electromagnetic wave is also much larger than the wavenumbers of the plasma turbulence ($\omega \gg c\omega_{pe}/v_p$). Afterwards we shall show what happens, if we lift this restriction.

It is clear that if the intensity of the plasma turbulence is large, that is, if there are many plasma waves per unit volume, there will be a large probability for the fusion or decay of electromagnetic waves with the participation of the appropriate plasma wave. We discussed the physics of this process in § 3. We shall now apply these considerations to the problem under discussion here.

Consider an electromagnetic quantum $\boldsymbol{k}$ which undergoes fusion or decay involving some plasma wave $\boldsymbol{k}_1$ while another electromagnetic quantum $\boldsymbol{k}'$ results, with a frequency and wavevector obtained from the conservation laws (3.29):

$$\omega = \omega' \pm \omega_1, \quad \boldsymbol{k} = \boldsymbol{k}' \pm \boldsymbol{k}_1. \tag{5.17}$$

As the plasma wave phase velocity is often much smaller than the velocity of light, the relative change in the wavevectors of the electromagnetic waves is considerably larger (by a factor c/v_p) than the change in their frequencies. The main effect is thus a change in direction of the wavevectors while their magnitude changes little. In other words, high-frequency electromagnetic waves undergo in a turbulent plasma scattering which affects the direction of propagation while their frequency is relatively little changed.

If $\boldsymbol{k}_1$ is comparable in magnitude with $\boldsymbol{k}$, the wavevector of the electromagnetic wave will even in a single scattering process turn over an angle of order unity; this is integral scattering. The change in the frequency is, as before, small: $\Delta\omega \approx \omega_1 \ll \omega$. If, however, $k_1 \ll k$, the scattering is also differential scattering as far as the angular variable is concerned.

The wave emission probability in a fusion process was determined by equation (3.28). We now introduce the probability u_{k,k',k_1} for fusion per unit phase volumes of the waves $\boldsymbol{k}$, $\boldsymbol{k}'$, and $\boldsymbol{k}_1$, in such a way that

$$u_k = \int u_{k,k',k_1} N^t_{k'} N_{k_1} \frac{d^3k'\, d^3k_1}{(2\pi)^6} . \tag{5.18}$$

One can easily use this probability to write down balance equation involving all fusions and decays of electromagnetic and plasma waves, similar to what we did for equation (5.2). Using the conservation laws (5.17) we have (Kaplan and Tsytovich, 1969d, 1970a, c; Sakhokiya and Tsytovich, 1968; Colgate, Lee and Rosenbluth, 1970):

$$\begin{aligned}\frac{dN^t_k}{dt} &= \int N_{k_1}[(N^t_{k-k_1} - N^t_k) u_{k,k-k_1,k_1} + (N^t_{k+k_1} - N^t_k) u_{k,k+k_1,k_1}] \frac{d^3k_1}{(2\pi)^3} \\ &= \int N_{k_1}\left(\boldsymbol{k}_1 \cdot \frac{\partial}{\partial \boldsymbol{k}} \left\{ u_{k,k,k_1}\left(\boldsymbol{k}_1 \cdot \frac{\partial N^t_k}{\partial \boldsymbol{k}}\right)\right\}\right) \frac{d^3k_1}{(2\pi)^3} = \sum_{i,j} \frac{\partial}{\partial k_i}\left(\tilde{D}_{ij} \frac{\partial N^t_k}{\partial k_j}\right),\end{aligned} \tag{5.19}$$

where $\tilde{D}_{ij}$ is the scattering diffusion tensor involving fusions and decays:

$$\tilde{D}_{ij} = \int k_{1i} k_{1j} u_{k,k,k_1} N_{k_1} \frac{d^3k_1}{(2\pi)^3} . \tag{5.20}$$

All components of the tensor $\tilde{D}_{ij}$ can be expressed in terms of two parameters in the case of isotropic plasma turbulence. One of those describes the change in the frequency of the electromagnetic waves:

$$\tilde{D}_{\parallel}(k) = \int \frac{(\boldsymbol{k}\cdot\boldsymbol{k}_1)^2}{k^2} u_{k,k,k_1} N_{k_1} \frac{d^3k_1}{(2\pi)^3} . \tag{5.21}$$

We shall call this quantity the longitudinal component of the tensor. The other parameter—the transverse component of the tensor—determines the change in the direction of the wavevector:

$$\tilde{D}_{\perp}(k) = \int \frac{([\boldsymbol{k}\wedge\boldsymbol{k}_1]\cdot[\boldsymbol{k}\wedge\boldsymbol{k}_1])}{2k^2} u_{k,k,k_1} N_{k_1} \frac{d^3k_1}{(2\pi)^3} . \tag{5.22}$$

One can easily evaluate these quantities if one knows the fusion and decay probabilities (see Appendix). We shall give the final expressions for the transverse components of the tensor for scattering by different modes of isotropic plasma turbulence.

Scattering by longitudinal plasmon turbulence:

$$\tilde{D}^l_{\perp}(k) = \frac{\pi}{16} \frac{\omega^2_{pe}}{ck^2} \int_0^k \frac{W^l_{k_1} k_1^3\, dk_1}{n_e m_e c^2} ; \tag{5.23}$$

scattering by ion-sound turbulence:

$$\tilde{D}^s_{\perp}(k) = \frac{\pi}{16} \frac{\omega^4_{pe}}{c^3k^2} \int_0^k \frac{W^s_{k_1} k_1\, dk_1}{n_e T_e} ; \tag{5.24}$$

scattering by magneto-sound turbulence:

$$\tilde{D}_{\perp}^{\mathrm{m}}(k) = \frac{\pi}{16}\,\frac{\omega_{\mathrm{pe}}^4}{c^3k^2}\int_0^k \frac{W_{k_1}^{\mathrm{m}} k_1\,dk_1}{n_e m_i v_{\mathrm{A}}^2}. \tag{5.25}$$

The coefficient for diffusive scattering by Alfvén waves is very small; it differs from (5.25) by an extra small factor $(k_1c/\omega_{\mathrm{pi}})^2$ in the integrand. Such scattering occurs also through collisional sound waves (Makhan'kov, Sakhokiya and Tsytovich, 1969).

We shall in what follows often use instead of $\tilde{D}_{\perp}$ the effective elastic scattering coefficient:

$$\sigma_{\perp} = \frac{2\tilde{D}_{\perp}}{k^2}. \tag{5.26}$$

Let us consider in somewhat more detail the case of scattering by waves of Langmuir and ion-sound turbulence. If the wavenumber k of the electromagnetic wave is less than the wavenumber of the excited longitudinal plasmons k_g, the elastic scattering coefficient will be constant, that is, independent of the frequency of the electromagnetic waves, if the plasma turbulence spectrum is flat:

$$\sigma_{\perp}^{\mathrm{l}} = \frac{2\tilde{D}_{\perp}^{\mathrm{l}}}{k^2} = \frac{\pi}{32}\,\frac{\omega_{\mathrm{pe}}^2 W_k^{\mathrm{l}}}{n_e m_e c^3} = \frac{\pi}{32}\,\frac{\omega_{\mathrm{pe}}^2}{n_e m_e c^3}\sqrt{\frac{2Q^{\mathrm{l}}}{\alpha^{\mathrm{l}}}}. \tag{5.27}$$

For large frequencies ($\omega > ck_g$) the elastic scattering coefficient decreases with increasing frequency as $\sigma_{\perp} \propto \omega^{-4}$.

The coefficient for elastic scattering by ion-sound turbulence waves always decreases with increasing frequency. When $\omega \leqslant c/d_e \approx c\omega_{\mathrm{pe}}/v_{Te}$ we have (using (4.37)):

$$\sigma_{\perp}^{\mathrm{s}} = \frac{\pi}{64}\sqrt{\left(\frac{\pi m_e}{8m_i}\right)}\,\frac{\omega_{\mathrm{pe}}^4}{\omega^3}\,|\ln\,(kd_e)| \approx 3\times 10^{-3}\,\frac{\omega_{\mathrm{pe}}^4}{\omega^3}, \tag{5.28}$$

and for larger frequencies, $\omega > \omega_{\mathrm{pe}}\,c/v_{Te}$ we have again $\sigma_{\perp}^{\mathrm{s}} \propto \omega^{-4}$.

The longitudinal diffusion coefficient can be calculated similarly. We shall use here instead of $\tilde{D}_{\parallel}$ the effective inelastic scattering coefficient:

$$\sigma_{\parallel} = \frac{\tilde{D}_{\parallel}}{k^2}. \tag{5.29}$$

For scattering by isotropic Langmuir turbulence waves we have

$$\sigma_{\parallel}^{\mathrm{l}} = \frac{\pi}{16}\,\frac{\omega_{\mathrm{pe}}^4}{c^3k^4}\int_0^k \frac{W_{k_1}^{\mathrm{l}} k_1\,dk_1}{n_e m_e c^2} = 2\left(\frac{\omega_{\mathrm{pe}}}{\omega}\right)^2\sigma_{\perp}^{\mathrm{l}} \tag{5.30}$$

for frequencies less than $\omega \approx ck_g$. In the case when the wavenumber of the electromagnetic wave is larger than the maximum wavenumber of the longitudinal plasmon turbulence we find the relation $\sigma_{\parallel}^{\mathrm{l}}/\sigma_{\perp}^{\mathrm{l}} = (v_p/c)^2$ where v_p is the minimum longitudinal plasmon phase velocity. The inelastic scattering coefficient decreases here thus always with increasing frequency and as a rule is much smaller than the elastic scattering coefficient.

In the case of interactions with ion-sound waves the ratio of the elastic and the inelastic scattering coefficients is constant:

$$\frac{\sigma_{\parallel}^{s}}{\sigma_{\perp}^{s}} = \left(\frac{v_s}{c}\right)^2 \ll 1 . \tag{5.31}$$

The diffusion equation (5.19) is essentially an equation for the transfer of electromagnetic radiation. We can put it in the form which is the usual one for astrophysics. In order not to complicate the formulae we assume that the elastic and inelastic scattering coefficients are independent of the angles (which is always correct for scattering of waves in isotropic turbulence). Moreover, we assume that we have differential scattering as far as the angles are concerned.In that case we have

$$\frac{dI_\omega}{dt} = \frac{\partial I_\omega}{\partial t} + (v_g \cdot \nabla I_\omega) = \sigma_\perp (\nabla^2)_\perp I_\omega + \frac{1}{\omega^2}\frac{\partial}{\partial \omega}\left(\omega^4 \sigma_\parallel \frac{\partial I_\omega}{\partial \omega}\right), \tag{5.32}$$

where $(\nabla^2)_\perp$ is the angular Laplace operator:

$$(\nabla^2)_\perp = \frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial}{\partial\vartheta}\right) + \frac{1}{\sin^2\vartheta}\frac{\sigma^2}{\partial\varphi^2} . \tag{5.33}$$

For electromagnetic waves with $k = \omega/c$ of the order of the wavenumbers of the plasma turbulence we have integral scattering as far as the angles are concerned. Instead of the angular Laplace operator one has here the usual integral expression from transfer theory (Kaplan and Tsytovich (1970a, c) have considered this case in detail).

We shall not study here the solution of (5.32) but one can immediately draw a few general conclusions.

Firstly, if narrow-beam radiation passes through a turbulent plasma, its angular dispersion $\overline{(\Delta\vartheta)^2}$ will broaden as the path R increases:

$$\overline{(\Delta\vartheta)^2} = 2\sigma_\perp t = 2\sigma_\perp \frac{R}{c} . \tag{5.34}$$

This relation can be used to study plasma turbulence under cosmic conditions (Tsytovich, 1965; Gordon, 1968). For instance, if one observes a radio-source with small angular size ϑ_0 while one knows that the radiation has traversed a path R in a medium where longitudinal plasmon turbulence is possible, its spectral energy density must satisfy the inequality

$$W_k^l \leqslant \frac{m_e^2 c^4}{e^2 R}\vartheta_0^2 . \tag{5.35}$$

Similarly one can obtain limits on the turbulent energy density for other modes.

Secondly, if a very narrow spectral radio-line emission is observed, one can use arguments about inelastic scattering to estimate the plasma wave energy density (Colgate, 1967; Kaplan and Tsytovich, 1970c).

It is here, however, important that the elastic scattering coefficient is much larger than the inelastic scattering coefficient so that the increase in the angular dispersion is much more considerable. Only in the case when the angular size of the source is not too small and the

spectral line very narrow can one use more effectively inelastic scattering data. In the most favourable case when a narrow line is observed from a small angular diameter source one could use for the analysis of the turbulence properties the following obvious relation, provided the effect of the turbulence is appreciable:

$$\overline{\left(\frac{\Delta\omega}{\omega}\right)^2} = \frac{\sigma_\parallel}{\sigma_\perp}\,\overline{(\Delta\vartheta)^2}. \tag{5.36}$$

AMPLIFICATION OF HIGH-FREQUENCY ELECTROM AGNETIC RADIATION DURING ITS PROPAGATION THROUGH A TURBULENT PLASMA

We took into account only decay interactions in the transfer equation (5.32). In the general case we must add terms taking into account spontaneous and induced emission and scattering mechanisms. There are many such processes in a medium with plasma turbulence and it is difficult to write down the transfer equation in its general form. In later chapters we shall consider separate cases which are important for plasma astrophysics. We shall, however, note some properties of such an equation here in the general context of plasma turbulence theory.

First of all it is important that electromagnetic radiation can be amplified when passing through a turbulent medium, that is, one must add to equation (5.32) terms of the form $\gamma^t I_\omega$ with a positive value of the growth rate γ^t. There are many mechanisms for amplifying radiation in a turbulent plasma; in later chapters we shall consider several of them.

In the present section we restrict ourselves to describing a single mechanism of this kind which can be explained qualitatively as follows (Tsytovich, 1971c). Normally the damping of electromagnetic waves is connected with the fact that electrons affected by the wave lose their energy through colliding with other particles. In a turbulent plasma the electrons "collide" mostly with plasma waves. However, in that case they can not only give off energy, but also acquire it. For instance, it turns out that if the wavevectors of electromagnetic and ion-sound waves are in the same direction and if the ion-sound wave energy density is larger than some minimum value (see below) energy is transferred from the ion-sound to the electromagnetic wave, and the electron is affected by both waves. If the wavevectors are in opposite directions energy is transferred from the electromagnetic to the ion-sound wave. It is important that the direction in which the energy is transferred depends on the anisotropy of the plasma turbulence. There is no amplification when the turbulence is strictly isotropic.

The growth or decay rate for the case of interactions between electromagnetic and ion-sound waves can be written as follows:

$$\frac{\gamma^t}{\omega_{pe}} = \sqrt{\left(\frac{\pi m_e}{8m_i}\right)}\left(\frac{\omega_{pe}}{\omega}\right)^3 \int \frac{(\boldsymbol{k}\cdot\boldsymbol{k}_1)}{n_e m_e \omega_{pe}^2} W^s_{k_1}\, d^3k_1. \tag{5.37}$$

We note that ion-sound turbulence is, as a rule, not fully isotropic; for instance, the wavevectors of ion-sound waves excited by an electric field are predominantly directed along it. If the electromagnetic radiation also propagates in that direction, its amplification

coefficient is

$$\frac{\gamma^t}{\omega_{pe}} \approx \sqrt{\left(\frac{\pi m_e}{8m_i}\right)}\left(\frac{\omega_{pe}}{\omega}\right)^2 \frac{v_{Te}}{c}\,\frac{W^s}{n_e T_e}. \tag{5.38}$$

As amplification is possible only when its growth rate is larger than the always present collisional damping rate, $(\omega_{pe}/\omega)^2\,\nu_\vartheta$, the ion-sound wave energy density must necessarily satisfy the inequality

$$\frac{W^s}{n_e T_e} \gg \sqrt{\left(\frac{8m_i}{\pi m_e}\right)}\frac{c}{v_{Te}}\,\frac{1}{N_D}. \tag{5.39}$$

To obtain an appreciable amplification of the electromagnetic waves it is also necessary that the optical depth relating to the growth rate must be much larger than unity, that is,

$$\tau_\gamma = \frac{\gamma^t R}{v_g} = \frac{\gamma^t R}{c} \gg 1. \tag{5.40}$$

It is possible that condition (5.40) is not satisfied for various amplification mechanisms in astrophysical sources with a small size R. Nevertheless, if there is an intensive plasma turbulence in the sources, the amplification of electromagnetic radiation is possible, as was shown by Kaplan and Tsytovich (1970a, c), even though the inequality which is the reverse of (5.40) is true. In a turbulent plasma elastic scattering of electromagnetic waves through the decay interactions, which were considered in the preceding section, leads to an appreciable lengthening of the effective path traversed by the wave, if $\overline{(\Delta\vartheta)^2}$ becomes larger than unity. In other words, an electromagnetic wave is "trapped" in the medium, passes a longer time in it, and, hence, the conditions for its amplification become more favourable.

A considerable "trapping" of the electromagnetic waves occurs when the optical depth with respect to elastic scattering is much larger than unity, that is,

$$\tau_\sigma = \frac{\sigma_\perp R}{c} \gg 1. \tag{5.41}$$

For the amplification of electromagnetic waves it is now necessary that the following condition holds:

$$\sqrt{(\tau_\sigma \tau_\gamma)} = \sqrt{(\sigma_\perp \gamma^t)}\,\frac{R}{c} \gg 1, \tag{5.42}$$

which can be satisfied even when $\tau_\gamma < 1$, provided τ_σ is sufficiently large. We shall use in later chapters this kind of intensification of the amplification mechanism for electromagnetic waves when we consider actual astrophysical objects.

Some non linear transfer equations for epectromagnetic radiation

Recently many people have studied non-linear radiative transfer equations. There are many reasons why the equations become non-linear and there is as yet no general non-linear transfer theory. Non-linearity can, for instance, be connected with the fact that the

level populations depend on the field of the radiation itself or with the change in dielectric constant in that field, and so on.

There is, however, yet another important cause for the appearance of non-linearity in the transfer equation, namely, the influence of induced processes. In particular, Weymann (1965) and Kompaneets (1957) obtained such an equation for the case of induced Compton scattering by the thermal electrons in the plasma. We note that this equations has been used for a number of astrophysical studies (Zel'dovich and Syunyaev, 1969; Syunyaev, 1971).

Indeed, when induced processes are taken into account the growth rate is, in the differential approximation, proportional to the derivative of the spectral function and the corresponding term in the transfer equation is thus proportional to the product of the intensity and its derivative, or to the derivative of the square of the intensity. In particular, in the already mentioned case the transfer equation which takes into account the induced Compton scattering contains a term

$$\gamma^{t}(t'+1\to t)I_{\omega}=\frac{2\pi^{2}}{3}\frac{\omega_{pe}^{4}}{n_{e}m_{e}c^{3}}\omega\frac{\partial}{\partial\omega}\left(\frac{I_{\omega}}{\omega}\right)^{2}. \tag{5.43}$$

In the framework of a non-linear plasma theory when all plasma particles participate in the interaction, the scattering by particles is unimportant compared to decay processes (Tsytovich, 1971a, 1972b). At the same time, just the decay processes lead also to equation (5.43).

Indeed, constructing the expression for $\gamma^{t}I_{\omega}$ with the growth rate for the process $t'\to t+1$ determined by equation (3.37), we get exactly equation (5.43). At the same time we meet with an important restriction which is connected with the fact that this expression is valid only when the number of transverse waves per unit phase volume is much larger than the number of longitudinal waves per unit phase volume, that is, provided $N_{k}^{t}\gg N_{k_1}^{l}$. Only in that case is the intensity of decay processes proportional to $N_{k}^{t}N_{k-k_1}^{t}$. If, however, $N_{k}^{t}\ll N_{k_1}^{l}$, the intensity of decay processes is proportional to $N_{k}^{t}N_{k_1}^{l}$ and the transfer equation now becomes linear in the intensity of the electromagnetic radiation, that is, one is reduced to the case of the scattering of radiation by the plasma turbulence waves.

The condition $N_{k}^{t}\gg N_{k_1}^{l}$ as applied to a cosmic plasma turns out to be rather stringent. In fact, powerful electromagnetic radiation for which it is necessary to take the non-linearity into account itself makes the plasma turbulent thanks to the same decay processes. If the non-linearity appreciably affects the intensity of the electromagnetic radiation, this means at the same time that an appreciable part of its energy is transferred to plasma waves (see (4.56)) and as the energy of a single plasma wave is much smaller than the energy of an electromagnetic wave, it follows from this that $N_{k}^{t}\ll N_{k_1}^{l}$. The case is possible, however, where the plasma waves are strongly absorbed so that the level of their energy remains low. At any rate, the problem of the non-linear transfer of electromagnetic waves in a plasma can, apparently, not be divorced from a study of the excitation of plasma turbulence and of the interaction of the radiation with the plasma turbulence, in particular, the study of scattering and of frequency increase. This problem must thus be solved in the framework of plasma astrophysics problems, taking into account a whole range of other non-linear processes, besides the process (5.43).

§ 6. Laboratory studies of plasma turbulence and possible laboratory models for cosmic plasma phenomena

The astrophysics of plasma turbulence studies objects of a particular kind and conditions which can usually not be reproduced in a laboratory experiment. On the other hand, it cannot but be based on the manifold experimental studies of plasma turbulence in laboratories. On the one hand, there are cosmic objects, such as the plasma in the circumterrestrial space or the solar plasma, for which the conditions are in many respects similar to those encountered in laboratory experiments, while, on the other hand, one finds in laboratory experiments often many qualitative peculiarities which one meets with under cosmic conditions on a larger scale. It is therefore natural to base a theoretical analysis of processes which are characteristic for a cosmic plasma and which are difficult to realize in a laboratory on general considerations about the nature of the plasma turbulence studied in laboratory experiments.

The peculiar role of cosmic rays in the astrophysical plasma is well known. We must thus first of all consider the problem of whether one can observe in a laboratory experiment effects which are similar to the creation of cosmic rays. Another important problem, mentioned in the introduction, is the strong increase in dissipative processes when turbulence is present—a strong increase in entropy; we shall here also draw attention to important experimental work. Finally, a third problem concerns the observations of an anomalous epithermal emission by a turbulent plasma which is of special interest for astrophysicists. The observation of anomalously strong scattering of electromagnetic waves by a turbulent plasma is also connected with this problem.

Before we discuss those experiments which are of most importance from the astrophysical point of view and which indicate both the large efficiency of the above-mentioned processes of acceleration, anomalous emission, and anomalous dissipation as well as the general prevalence of the turbulent state of a plasma, we must make two observations.

Firstly, there are a large number of experiments where turbulent processes have been observed at very low frequencies, in particular, below the collision frequency or at frequencies of the order of the drift frequency (Kadomtsev, 1965; Kadomtsev and Pogutse, 1969, 1970). No effects involving particle acceleration or anomalous emission have then been observed. As far as the anomalous dissipation is concerned, it always accompanies turbulence and is observed in most experiments. In particular, an anomalous plasma diffusion is connected with drift turbulence which prevents its magnetic confinement in experimental devices. We shall not be concerned here with those experiments as neither dynamic nor drift turbulence are topics of our present discussion and these kinds of turbulence do not lead to an anomalous acceleration or emission which are of particular interest in astrophysics. Drift turbulence has, by the way, so far not found a satisfactory theoretical description. The effects of the anomalous acceleration and emission have been well observed and studied just in high-frequency plasma turbulence for which satisfactory methods to describe them have been worked out as well as clear elementary physical processes which form the basis for them and, as a rule, the experiments are explained by the existing theory.

Secondly, we must emphasize that almost all ways to produce a laboratory plasma are

connected with making it at the same time turbulent. Most plasma sources—except perhaps the so-called Q-machines which are based upon thermal emission—produce apparently a turbulent plasma. More important, however, is the following: even relatively weak external influences make the plasma turbulent and produce various kinds of oscillations. This is what we have in mind when we say that a plasma is extraordinarily unstable. As a rule, the hotter the plasma, the easier it is to make it turbulent. For a temperature of 1 eV and a density $n_e \approx 10^{10}$ cm^{-3} even a field of $E \approx 0{\cdot}01$ V/cm leads to an intense ion-sound oscillation excitation. Even weak electron beams, a slight angular anisotropy in the plasma particle distribution, or the action of a not very intense high-frequency electric field, and so on, all will produce strong turbulence. The experimental data on plasma turbulence accumulated up to the present are manifold and the examples given below are thus fragmentary and only serve to illustrate the general trends.

Some experimental data on particle acceleration in a turbulent plasma

We now turn to a brief discussion of experiments on particle acceleration in a plasma. We must note that in almost all experiments in which one observes the excitation of plasma turbulent pulsations, particle acceleration has also been observed. Moreover, the observation of fast particles has now become an indicator which can be used to diagnose the development of plasma turbulence. Recently it has been ascertained that the assumption that in most laboratory set-ups the plasma is a two-temperature one, that is, that it contains a hot component (accelerated particles) and a cold one, is a good working hypothesis which enables one to understand the observational results. In many respects this corresponds to the astrophysical observations where one has a cold plasma and the cosmic ray plasma. The conditions of a laboratory experiment differ, however, appreciably from the cosmic conditions, on the one hand, in the time scales—the turbulence is excited in a relatively short time while the plasma is still contained in the device—and, on the other hand, in the spatial dimensions— the accelerated particles can be contained by the magnetic field only as long as their Larmor radius does not become comparable with the size of the device. Therefore in laboratory experiments the accelerated particles generally speaking do not have relativistic energies. All the same, in most experiments in which the excitation of plasma turbulence has been observed there were also accelerated particles. Such effects have, for instance, been observed when electron beams have interacted with a plasma. The beam-plasma interaction problem itself is now a large part of plasma physics which has been the subject of books and special annual international conferences. A large amount of experimental data on the appearance of accelerated particles has already been accumulated. Amongst those we must note first of all the work by U.S.S.R. groups led by Faĭnberg, Kharchenko, Zavoĭskiĭ, Golovin and many others, and foreign groups, for instance, those led by Smullin, Alekseev, Hopman and others. Experimental results and detailed reviews can be found in the proceedings of recent international conferences on thermonuclear fusion and ionized gas phenomena (see References at the end of this book).

In different experimental devices acceleration effects due to various plasma oscillations such as Langmuir waves, whistlers, or ion-sound waves, have been observed. Both accelerated electrons and accelerated ions have been observed. More or less arbitrarily we have

chosen from these experiments a number of the most illuminating ones, realizing that such an illustration necessarily is somewhat one-sided.

Experiments by Kharchenko, Faĭnberg, Kornilov, Lutsenko, and Pedenko (1962) showed the appearance of energetic electrons in plasma-beam interactions. For an initial beam energy of 5 keV the energy of the fast electrons was 80 to 100 keV and their number about 5 per cent of the total number of electrons in the plasma (see also Berezin, Faĭnberg, Bolotin and Berezina (1965), Kornilov, Kovpik, Faĭnberg and Kharchenko (1965), Kornilov, Kovpik, Faĭnberg, Bolotin and Kharchenko (1965), and a review by Faĭnberg, 1968). In experiments by Smullin (1968) a beam with a relative density $n_*/n_e \approx 10^{-3}$ to 10^{-2} ($n_* \approx 10^{10}\,\mathrm{cm}^{-3}$, $n_e \approx 10^{13}\,\mathrm{cm}^{-3}$) excited strong Langmuir oscillations at frequencies ω_{pe} which were accompanied by a powerful emission by the plasma at frequencies of the order of ω_{pe} (it was impossible to distinguish between ω_{pe} and $2\omega_{pe}$ because of the inhomogeneity of the plasma which is peculiar to laboratory experiments). The energy of the accelerated electrons which fell on the side-walls of the gas-discharge chamber was determined by the X-ray emission and turned out to be about 100 keV for beam particle energies of 7 to 10 keV. The density of the accelerated particles was about 0·1 per cent of the density of the cold plasma; this was measured exactly by a low-power detecting beam. The Langmuir oscillations were the main mechanism for the particle acceleration. It is characteristic that the total energy of the accelerated particles is large compared to the energy of the cold plasma. In the experiments by Plakhov, Ryutov and Shapkin (1969) it was shown in detail that when plasma-beam interactions occur, which are excited in magnetic traps, the electron acceler-

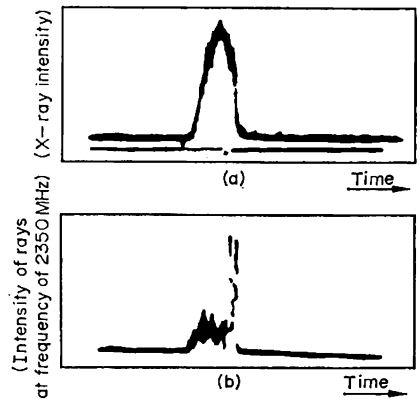

FIG. 13. Correlation of the acceleration of fast particles and the excitation of plasma oscillations.(a) Oscillogram of the X-ray emission generated by the fast particles. (b) Oscillogram of the plasma waves at a frequency of $\omega = 1{\cdot}5\times10^{10}\ \mathrm{s}^{-1}$ on the same time scale.

ation occurs basically outside the region where the beam and the plasma interact (the size of the beam was limited). The Langmuir oscillations and the beam particles diffuse out of the region of the beam interaction and their interaction then leads to a particle acceleration by the Langmuir oscillations. In Karkhov's experiments (1969) it was shown that the acceleration is particularly effective when the electron gyrofrequency is a multiple of the plasma frequency; initially intensive Langmuir oscillations are excited which even outside the mag-

netic trap are converted into whistlers—or, more precisely, into their high-frequency part where they join up with the longitudinal oscillations with frequency $\omega_{He}|\cos\vartheta|$. The condition that ω_{He} be a multiple of the plasma frequency corresponds apparently to the condition that the non-linear conversion of Langmuir waves into whistlers be efficient. In the magnetic trap region there are basically whistlers which efficiently accelerate electrons. It has been established that the oscillation spectra are excited just by a beam and not by the accelerated electrons and they are correlated with the X-ray emission (Fig. 13) which is generated by the accelerated particles. The energy of the accelerated electrons exceeded by a factor 100 the energy of the beam particles. Nezlin (1967) observed an acceleration of ions which was correlated with the appearance of oscillations at the ion cyclotron frequency. Zavoĭskiĭ (1967) observed a particularly efficient ion acceleration when the condition for resonance between the ions and the ion-sound oscillations was satisfied. In the majority of experiments in which ion-sound turbulence occurs—for instance in a plasma in an external electric field (Zavoĭskiĭ and Rudakov, 1967) or in experiments with collisionless shock waves (Alikhanov, Alinovskiĭ, Dolgov-Savel'ev, Eselevich, Kurtmullaev, Malinovskiĭ, Nesterikhin, Pil'skiĭ, Sagdeev and Semenov, 1969)—accelerated ions have been observed. In the experiments by Veksler, Gekker, Gol'ts, Delone, Kononov, Kudrevatova, Luk'yanchikov, Rabinovich, Savchenko, Sarksyan, Sergeĭchev, Silin, Tsopp, Levin and Muratov (1965) on the interaction of high-frequency strong waves with a plasma, accelerated ions with a maximum energy of around 100 eV and a subsequent falling-off extending to 10 keV have been observed.

Of course, acceleration effects might be connected not with plasma turbulence, but, for instance, with the appearance of some kind of potential wells or humps by which particles are accelerated. It is important that there are experimental means to solve the problem of the nature of the observed acceleration. Firstly, the development of turbulence means a rather strong exchange of energy between different modes and hence a rather wide spectrum of excited oscillations. Secondly, particles accelerated stochastically by the random fields of the turbulent pulsations must have a rather wide energy spectrum. In most experiments both these conditions turn out to be satisfied. However, the most unambiguous proof for the acceleration by turbulent pulsations is given by experiments in which the turbulence through some means is excited periodically (for instance, due to some relaxation processes in changing the plasma parameters). It turns out that the appearance of accelerated particles is strictly correlated under those conditions with the periods of the generation of turbulence in the plasma and in a number of cases with the appearance of only one well-defined kind of pulsations. This confirms directly that the acceleration of the particles is connected with the action of the turbulent plasma oscillations on the particles.

Under some circumstances whistlers can also accelerate particles very efficiently; this is exclusively connected with the fact that a whistler which propagates mainly along the magnetic field increases the particle energy at right angles to the field and this assists, for instance, the containment of accelerated particles in the case of magnetic traps. However, as a rule, the acceleration is the more efficient, the higher the frequency of the pulsations—for the same turbulence energy. The acceleration has been observed in magnetic traps and through Langmuir oscillations. Accelerated ions appear when strong high-frequency fields act upon the plasma and when laser radiation interacts with the plasma.

Another distinguishing feature of all experimental results is that a relatively small fraction of the total number of particles is accelerated. This points, on the one hand, to the existence of powerful dissipative processes which accompany the increase of the entropy in the turbulent regime but, on the other hand, does not mean at all that the total energy of the accelerated particles cannot be comparable with the energy of the cold component.

The rather prolonged existence of the turbulent regime can lead to the fact that the total energy of the accelerated particles will be comparable with the energy of the thermal particles. Moreover, it has been suggested that the diamagnetic effect which has been measured in turbulent heating experiments and which is proportional to the total number of particles may be produced by just the hot component. This problem was discussed extensively in 1965–7. It has now become clear that in those experiments in which one could relatively clearly distinguish the hot and the cold components the two have comparable energies. For instance, in Karkhov's experiments (1969) the energy of the accelerated particles was about 30 per cent of the energy of the main mass of thermal particles. One can compare this result with the well-known astrophysical fact that the energy of the cosmic rays is approximately equal to other forms of energy, such as the energy of the cold cosmic plasma and the magnetic field energy.

All experiments described here undoubtedly give evidence for the fact that the appearance of accelerated particles is explained by the interaction of particles with turbulent plasma oscillations, for the fact that there is a deep-lying resemblance between laboratory and cosmic phenomena, and for the fact, finally, that the generation of fast particles is a general property of plasma turbulence.

Anomalous plasma resistivity

Let us now turn to experiments showing the anomalous amplification of dissipative processes when plasma turbulence is developed. The simplest way to excite plasma turbulence is the action of relatively strong electric fields. The turbulent regime occurs when the field E is larger than the Dreicer field E_D, as was already noted in § 4. In such a field the "run-away" electrons are decelerated by the plasma turbulence waves.

This effect has been called the plasma anomalous resistivity. It has been studied experimentally by many authors (Suprunenko, 1964; Fanchenko, Demidov, Elagin, and Ryutov, 1964; Burchenko, Vasilenko, Volkov, Potapenko and Tolok, 1969; Hamberger, Jancarik and Sharp, 1969; Paul, 1969; Daughney, Holmes, and Paul, 1970; Daughney, Holmes, Paul and Summers, 1970; Jancarik and Hamberger, 1970; Hamberger and Jancarik, 1971). The anomalous resistivity effect has been observed in toroidal devices with magnetic plasma containment, such as, for instance, the Tokamak (Artsimovich, 1970), and also under conditions when the so-called neutral magnetic field lines appear, near which the magnetic field strength changes sign. These observations are of particular interest for an interpretation of the chromospheric solar flares which usually appear near neutral lines. It is plausible to connect the observed anomalous dissipation of magnetic fields which occurs when chromospheric flares occur with the anomalous resistivity.

We have given in Fig. 14 the results of an experimental study of the way the average plasma electric resistivity depends on the electric field; these results were obtained recently

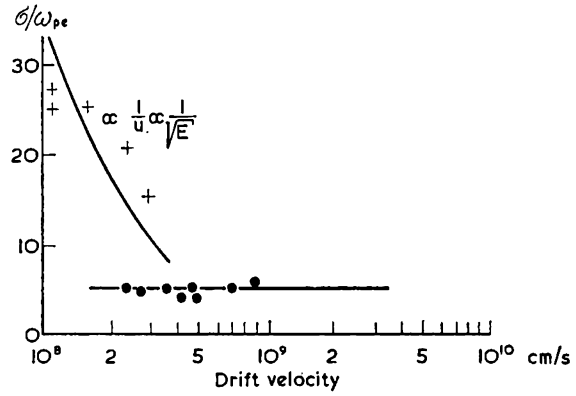

FIG. 14. The behaviour of the turbulent plasma electrical conductivity as a function of the magnitude of the relative drift velocity u. The region $\sigma \propto 1/u$ corresponds to ion-sound turbulence and the region $\sigma \approx$ constant to the Buneman instability.

by Hamberger and Jancarik (1971). The applied electric field lay between the limits 100 to 1500 V/cm in a hydrogen plasma with $n_e \approx 10^{11}$ to 10^{13} cm^{-3}. This is just the range of parameters in which the transition occurs from ion-sound turbulence when $u < v_{Te}$ to the so-called Buneman turbulence when $u > v_{Te}$. In accordance with what we said in § 4 the anomalous conductivity changes from $\sigma \propto 1/u$ to $\sigma \approx$ constant (see (4.39) to (4.47)) as is

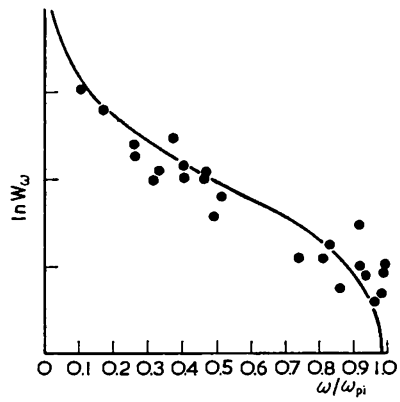

FIG. 15. Ion-sound oscillation spectrum in a plasma with an applied electric field and with the following parameters: $n_e = 10^{13}$ cm^{-3} and $u = 10^9$ cm/s. Full-drawn curve: $W_\omega \propto (1/\omega) \ln(\omega_{pi}/\omega)$.

observed experimentally. The transition from one regime to another is accompanied by a sudden change in the turbulence spectrum. In the ion-sound regime the spectrum is proportional to $1/k$ (Fig. 15) and corresponds to the theory (Kadomtsev, 1965; also Sagdeev, 1967; Tsytovich, 1971 b), while for $u > v_{Te}$ it is concentrated around frequencies which are approximately equal to $\frac{1}{2}\omega_{pe}(m_e/m_i)^{1/3}$ (see Fig. 16 and equation (4.46)). The turbulent pulsations propagate mainly along the applied electric field.

Strong ion-sound oscillations were observed in the experiments of Zavoĭskiĭ and Rudakov (1967) when their level was not less than $10^{-2}\, n_e T_e$. The effective collision frequencies

correspond to $10^9 s^{-1}$ which is higher by many orders of magnitude than the electron-ion collision frequency and this shows clearly how efficient the dissipative processes are in the turbulent regime. Under conditions when the applied electric field is not able to maintain the inequality $u > v_{Te}$ because of the anomalous resistivity which appears in that regime,

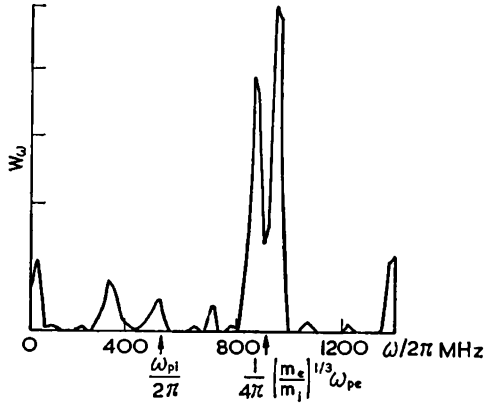

FIG. 16. Plasma oscillation spectrum in the Buneman instability regime for $E = 60\,E_D$ and $u = 3\times10^8$ cm/s. The maximum of the spectrum occurs at a frequency $\nu=(m_e/m_i)^{1/3}\,\omega_{pe}/4\pi > \omega_{pi}/2\pi$.

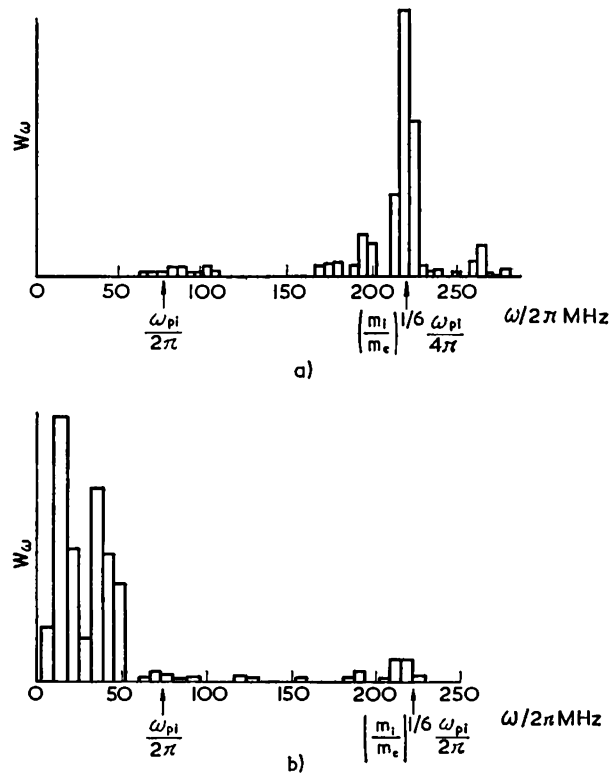

FIG. 17. Change with time in the oscillations spectrum as a result of the development of plasma instabilities in a strong electric field. (a) Spectrum in the initial stage (Buneman instability); (b) ion-sound spectrum in the next stage (after 70 ns).

the plasma changes automatically to the ion-sound regime because of the heating of the electrons This was demonstrated experimentally by Hamberger, Jancarik and Sharp (1969; see Fig. 17). The characteristic time for this transition is of the order of $(m_i/m_e)^{1/3}\omega_{pe}^{-1}$, that is, very small; this is particularly important for astrophysical applications. We are concerned here with the fact that if $E > E_D$, but $T_e = T_i$, the ion-sound oscillations can not be excited directly. There are therefore initially "run-away" electrons after which all electrons in the plasma are heated up when $u > v_{Te}$ and the conditions arise which are necessary for the development of ion-sound turbulence. Hence follows the important conclusion that under astrophysical conditions the anomalous resistivity must usually be connected with ion-sound turbulence. The anomalous resistivity effects occur under all conditions when there occurs in the plasma a directed motion of the electrons with respect to the ions, whatever forces cause it. Although together with ion-sound oscillations other oscillations are also excited, the contribution of the latter to the anomalos resistivity effect turns out to be small. The origin of magnetic perturbations in the solar wind may, for instance, occur in this way.

Apart from an anomalously large friction of the electrons with respect to the ions, one has also observed an anomalously fast approach to isotropic electron and ion distributions. Such effects arise when there is no directed motion of electrons relative to the ions. Effects of a fast approach to isotropy lead to an anomalously large loss of particles from magnetic traps.

In a plasma with hot ions and cold electrons the cause for the approach to isotropy for $\omega_{He} \gg \omega_{pe}$ is the excitation of magnetic Langmuir oscillations with frequencies $\omega_{pe}|\cos\vartheta|$; if $\omega_{He} \ll \omega_{pe}$ the approach to isotropy of the electrons is mainly connected with the excitation of whistlers, and of the ions with the excitation of collisionless Alfvén and ion-sound waves. These effects are important for an understanding of phenomena occurring in the Earth's magnetosphere and of the turbulence of the solar wind in which an anisotropy in the particle distribution has been observed. Processes facilitating the appearance of such an anisotropy are the motion of plasma particles in the direction of decreasing magnetic field, plasma heating and so on.

The fast turbulent plasma heating is another kind of anomalous dissipation when plasma turbulence is developed. Many papers have been devoted to experimental studies of this effect and it constitutes now a large section of plasma physics. In the U.S.S.R. these problems have been studied by groups under the leadership of Zavoĭskiĭ, Faĭnberg, Krachenko and others and elsewhere by Alekseev, Kolb, Hamberger, Wharton and others. As an example we give here the results obtained by Zavoĭskiĭ, Nedoseev, Rudakov, Rusakov, Skoryupin and Franchenko (1969).

In the experiments a plasma of density 10^{12} to 10^{13} cm^{-3} was heated by a solenoidal field $E = 100$ to 250 V/cm. After a time of order 3×10^{-7} s a value $n_eT_e \approx 3\times10^{16}$ eV cm^{-3} was reached. The heating was accompanied by a powerful emission and the appearance of a large anomalous resistivity. The rate of heating was so fast that it is completely impossible to explain it by binary collisions. The possibility of such a fast heating has an important value for astrophysics, for instance, for chromospheric flare problems or for the analysis of plasma heating in the vicinity of such objects as pulsars. Finally, when plasma turbulence develops an anomalously large diffusion and plasma thermal conductivity have been

observed. While the first of these is connected solely with the presence of low-frequency drift oscillations or, at any rate, oscillations in which the ions of the plasma take part, the second may be connected also with high-frequency oscillations in which only the electrons of the plasma participate. This is caused by the fact that the thermal conductivity is usually electronic. A change in the plasma parameters, such as its temperature or density, due to anomalous heating, diffusion, or thermal conductivity can influence the characteristics of the dynamic turbulence which must be taken into account when interpreting astrophysical observations.

Anomalous collisionless dissipative processes play an important role in the formation of collisionless shock waves (Adlam and Allen, 1958; Sagdeev, 1958, 1962). It is well known that for the appearance of shock waves one must have an increase of entropy at its front or a well-defined dissipation mechanism. If the width of the front is less than the mean free path of the particles such a mechanism may be turbulent dissipation where a non-linear wave in the plasma itself becomes a source for the excitation of turbulent oscillations. For instance, at the front of a wave propagating at right angles to a magnetic field, electrons with a directed motion which produce a current can have a velocity larger than their average thermal velocity. Buneman turbulence then occurs which changes fast into ion-sound turbulence. The dissipation at the front of such a wave will be connected with the anomalous plasma resistivity. Another example is the case which occurs at the front of a wave, when the magnetic field is compressed: an anisotropic particle distribution is created which leads to the excitation of whistlers and collisionless magnetohydrodynamic oscillations. If the amplitude of the non-linear wave is not very large the anomalous dissipation does not occur. The structure of collisionless shock waves has been studied experimentally by many investigations, in particular, in the USSR by groups under Sagdeev, Nesterikhin, Zavoĭskiĭ and others (see Sagdeev, 1966; Alikhanov, Alinovskiĭ, Dolgov-Savel'ev, Eselevich, Kurtmullaev,

FIG. 18. Profile of the front of a collisionless shock wave in a hydrogen plasma ($n_e = 5 \times 10^{13}$ cm^{-3}).

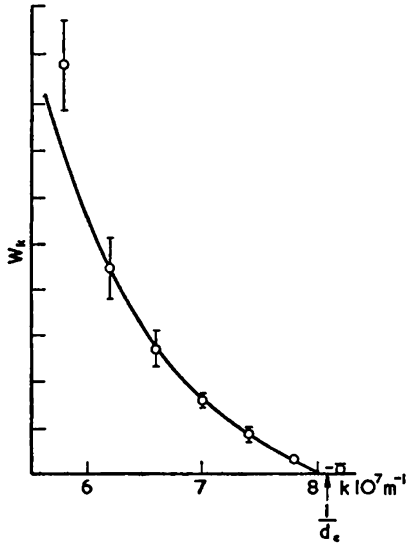

FIG. 19. Ion-sound wave spectrum at the front of a shock wave, measured by the laser light scattering method. Full-drawn curve: $W_k \propto k^{-1} \ln (kd_e)$.

Malinovskiĭ, Nesterikhin, Pil'skiĭ, Sagdeev, and Semenov, 1969; and Zavoĭskiĭ, Nedoseev, Rudakov, Rusakov, Skoryupin, and Fanchenko (1969), and elsewhere by Daughney, Holmes, Paul and Summers (1970), De Silva, Düchs, Goldenbaum, Griem, Hintz, Kolb, Kunze and Vitkovitsky (1969), Woods (1970) and others.

The width of the shock wave front propagating either at right angles or an another angle to the external magnetic field is for Mach numbers $M < 2$ to 4 determined by the anomalous resistivity and for $M > 4$ by the anomalous ion viscosity. In Fig. 18 we have given an example of the structure of a shock wave for $M > 4$ obtained in the experiments of Alikhanov, Alinovskiĭ, Dolgov-Savel'ev, Eselevich, Kurtmullaev, Malinovskiĭ, Nesterikhin, Pil'skiĭ, Sagdeev and Semenov (1969). In Paul's experiments (Daughney, Holmes, Paul and Summers, 1970) a detailed study was made of the spectrum of the turbulent oscillations excited at the front of a shock wave propagating at right angles to the magnetic field, using the scattering of laser light (Fig. 19). This spectrum corresponds completely to the ion-sound turbulence case. From the shift in the frequency of the laser light one can conclude that ion-sound waves propagate mainly in the direction of the electric field which occurs at the front of the wave.

Plasma turbulence emission

Let us now dwell upon the observations of anomalous emission when plasma turbulence is present. These effects appear very strikingly when beams interact with a plasma in experiments on turbulent plasma heating by an external electric field. In the experiments of Faĭnberg and coworkers (Berezin, Faĭnberg, Bolotin and Berezina, 1965; Kornilov, Kovpik, Faĭnberg, and Kharchenko, 1965; Kornilov, Kovpik, Faĭnberg, Bolotin and Kharchenko 1965) a powerful radiation was observed at frequencies close to the measured frequencies of the excited plasma oscillations.

This result indicates that the conversion of turbulent oscillations occurs without any appreciable change in frequency. Demidov, Elagin, and Fanchenko (1966) observed emission at a frequency $2\omega_{pe}$ when longitudinal oscillations had been excited in the plasma. This can be explained by the fusion of two longitudinal plasmons (see § 3).

One can use the intensity of the radiation to determine the turbulence energy; the results of such a determination agree with other independent measurements. However, in a number of cases continuous emission has been observed up to frequencies $6\omega_{pe}$ which can not be explained by successive fusion processes. It is, apparently, connected with the conversion by fast particles as only under those conditions can an appreciable change of frequency occur in a conversion process. We have already mentioned that the energy of the accelerated particles under cosmic conditions is appreciably higher and this conversion effect plays a considerably larger role in that case, and is one of the important plasma emission mechanisms. A particularly large change in frequency occurs in the conversion involving relativistic particles.

Sharp and Hamberger (1970) have also observed the radiation by a plasma in the ion-sound turbulence regime. When $u < v_{Te}$ the emission is mainly at the plasma frequency. When one changes to the regime $u > v_{Te}$, the emission increases steeply and stretches up to $10\omega_{pe}$ which possibly is connected with conversion involving accelerated particles.

Apart from the anomalous emission, anomalous scattering also occurs in many experiments in which plasma turbulence was excited. Demidov and Fanchenko (1964, 1965),

Hamberger, Jancarik and Sharp (1969), and Wharton, Prono and Sandel (1969) have observed this effect in turbulent heating experiments.

If the electron wave transmitted in the plasma is sufficiently monochromatic, as a result of its being scattered there occur combination frequencies which differ from the frequency of the original wave by a frequency of the turbulent plasma oscillations. These effects were clearly observed in experiments by Ascoli-Bartoli, Martelluci and Martone (1965) about the interaction of laser light with a plasma.

These effects are important for the detection of plasma turbulence under astrophysical conditions and could also serve for plasma diagnostics, in particular, to determine its density through measuring the combination frequencies.

Of course, in the brief survey which we have given we could not hope to do justice to the wide-spread experimental studies of plasma turbulence to which so many researchers are contributing nowadays. We wished merely to indicate the great similarity of the phenomena in a "terrestrial" plasma and many of the phenomena observed under cosmic conditions.

Laboratory models for cosmic plasma phenomena

If one wants to make a model for some plasma turbulence phenomena under cosmic conditions it is desirable to observe the well-defined principles of similarity. Podgornyĭ and Sagdeev (1970) and Schindler (1968) have analysed the possibilities for constructing such models and in particular they considered the case of the shock wave formed at the solar side of the Earth's magnetosphere.

The problem of the terrestrial magnetosphere lies outside the framework of the present book and we shall therefore only very briefly consider the problem of models for its interaction with the solar wind, referring the reader to the above-mentioned review by Podgornyĭ and Sagdeev.

It turns out that a reproduction in a laboratory of the phenomena in the near-terrestrial space, which is "exact" from the point of view of similarity theory, requires the generation of plasma currents with such a temperature and the obtaining of such magnetic field strengths that their attainment would practically mean a solution of the problem of controlled thermonuclear fusion. In other words, an exact model of cosmic plasma processes has so far been impossible but a restricted model enables one to reproduce in a laboratory some cosmic phenomena. By a restricted model we mean the following.

Dimensionless parameters of order unity under cosmic conditions must be of the same order of magnitude in the laboratory. Dimensionless parameters much less than unity in cosmic conditions must also be much less than unity in the laboratory, but if under cosmic conditions this parameter is less than unity by several orders of magnitude one can restrict oneself under laboratory conditions to a smaller difference. For instance, the ratio of the size of the magnetosphere to the mean free path in the solar wind is of the order of 10^{-3} while one takes this ratio in a laboratory experiment to be of order 10^{-1}.

Several experimental studies of a solar wind model have been made; we briefly mention an experiment by Podgornyĭ and Managadze (1968). In those experiments the formation of a collisionless shock wave was shown and the turbulent pulsation spectrum was found: $W_\omega \propto \omega^{-3}$ which apparently indicates that perhaps the dissipation mechanism is connected

with the excitation of whistlers due to the anisotropy in the particle distribution. We note that the oscillatory structure of the collisionless shock wave was experimentally studied under the laboratory conditions mentioned earlier (Fig. 20). Its width was of the order of a few times c/ω_{pi} as in the experiment performed by Podgornyĭ and collaborators. These

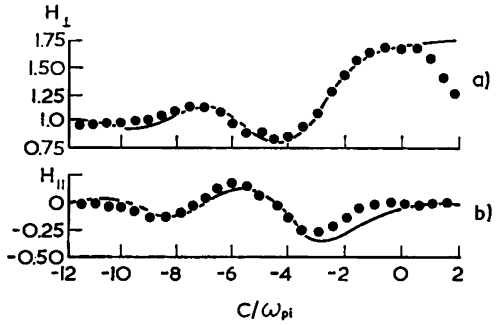

FIG. 20. Oscillatory structure of the magnetic field in a shock wave with Mach number $M = 2(n_e = 2\times10^{14}\ \mathrm{cm}^{-3})$. The distance is reckoned from the front of the shock wave in units c/ω_{pi}; H is in units of the field strength in front of the wave front: (a) field component perpendicular to the wave velocity; (b) field component along the wave velocity.

results correspond to the earlier developed theoretical explanation (for instance, Sagdeev, 1962, 1966; Karpman, 1964).

In another series of experiments by Podgornyĭ and collaborators (see Podgornyĭ and Sagdeev, 1970) the "tail" of the magnetosphere was simulated with the formation of a neutral magnetic field sheet; in this sheet the magnetic field was smaller by a factor 20 to 30 than on average in the whole of the "tail".

A study of the plasma phenomena near neutral lines is also important for simulating processes occurring in chromospheric flares. Syrovatskiĭ, Frank, and Khodzhaev (1970) have performed such experiments in which the magnetic field was pinched into a neutral

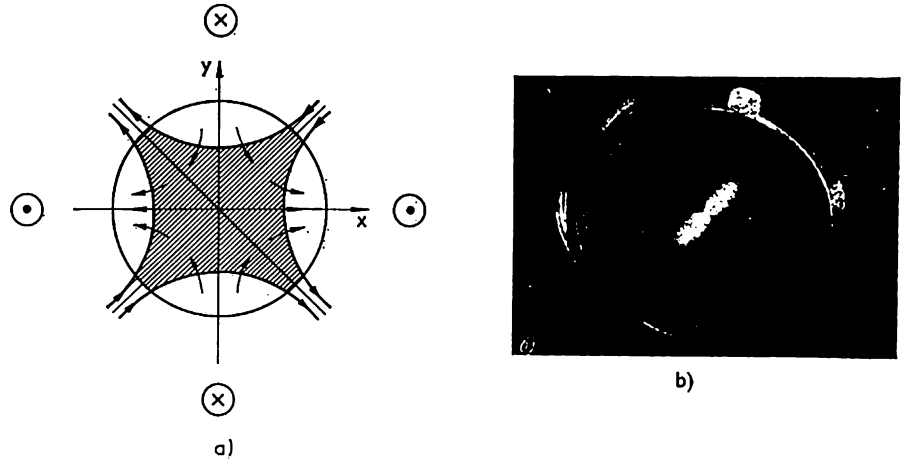

FIG. 21. Formation of a turbulent current sheet near magnetic field neutral lines. (a) Plan of the disposition of the conductors and of the direction of the magnetic field line deformation, leading to the formation of a current sheet; (b) observed radiation of a turbulent current sheet.

region (Fig. 21). The results of the experiment indicate that the initial stage of the development of the process corresponds to the picture of a magnetohydrodynamic cumulation in a laminar regime which was theoretically worked out by Syrovatskiĭ (1966a, b). Later on the effets of the anomalous resistivity appear, that is, plasma turbulence develops, thanks to which an increased dissipation of the magnetic fields occurs. The role of the anomalous resistivity in the dynamics of chromospheric flares was discussed in detail by Friedman and Hamberger (1968, 1969).

An interesting group of problems is connected with the possibility of making a model of those processes in which the emission from a turbulent plasma plays an important role. For this it is necessary that the optical depth for the emission of fast particles is larger than unity and, preferably, that the particles are relativistic. This requires a rather large fast particle density and an even larger cold plasma density. At the moment experiments have essentially been performed in the high-density plasma region, up to 10^{20} to 10^{22} cm^{-3} and at the same time powerful beams of relativistic electrons were created. As a result the possibility appears now to create "mechanically" a two-component plasma and to study in it radiative processes leading to powerful acceleration of particles and their emission. These problems are of great interest for the problem of the construction of models for plasma processes in such objects as quasars or pulsars.

CHAPTER II

Sporadic Radio-emission of the Sun

(Turbulent Processes in a Non-Relativistic Plasma)

THE most striking appearance of plasma turbulence is the sporadic radio-emission of the Sun, observed by Hey as early as 1942. One may consider the first discussions of the nature of the nature of this emission (Shklovskiĭ, 1946; Ginzburg, 1946; Martin, 1947) to be the start of plasma astrophysics. Many hypotheses and theories explaining various peculiarities of this phenomenon have been proposed and we cannot consider all of them. There are as yet no well-established views. The authors of the present book will present their own point of view. Our aim is, however, not to find a "uniquely correct" explanation of the phenomena of the sporadic radio-emission of the Sun—this is probably at present anyhow impossible to do—but rather to find a solution of some plasma astrophysics problems. The statement of these problems is based upon an analysis of the observational data and their solution aids the interpretation of those complicated phenomena which occur in the solar atmosphere and which have as a consequence the sporadic solar radio-emission and other occurrences of solar activity. We shall start with a brief summary of the observational data.

§ 7. Observational data on the sporadic solar radio-emission

As we are restricted by the size of the present book we cannot even give a brief summary of the result of the observations of all solar activity phenomena. We certainly cannot enter into a detailed discussion of the results of the large number of actual observations. The reader can find all necessary details in many books and review articles (Shklovskiĭ, 1965a; Zheleznyakov, 1970a; de Jager, 1959; Pikel'ner, 1965, 1966; Sweet, 1969; Kaplan, 1966; Zirin, 1966). We give only a general survey of the characteristic solar activity phenomena which are necessarily connected with plasma turbulence. Generally speaking, it is as yet not very clear where and when on the Sun one can find plasma turbulence. It is, for example, possible that the formation of the solar corona and its heating are also connected with plasma turbulence. So far this connection has not been traced and the problem of the heating of the corona is up to now one of the most important problems in solar physics. We shall therefore restrict ourselves to those solar phenomena where this connection is certain.

ELECTRON DENSITY AND TEMPERATURE DISTRIBUTION IN THE OUTER SOLAR ATMOSPHERE

It is necessary to know the electron density and temperature distribution in the solar atmosphere in order to set the "scene" on which the turbulent processes "perform".

Unfortunately, these data which can be obtained only from observations and not on the basis of theoretical calculations are not very reliable. Particularly bad is our knowledge of these quantities above the active solar regions—just there where plasma turbulence may develop.

We give in Table 1 some data which are assumed to be more reliable. Once we know n_e and T_e we can evaluate also the quantities ω_{pe}, v_{Te}, N_D, and ν_e which are important in plasma astrophysics. It is unfortunately difficult to observe the magnetic field in the solar

TABLE 1. HEIGHT-DEPENDENCE OF THE ELECTRON DENSITY AND TEMPERATURE AND OF OTHER PARAMETERS

Height above photosphere level (cm)	n_e (cm^{-3}) above active regions	ω_{pe} (s^{-1})	T_e (°K)	v_{Te} (cm/s)	d_e (cm)	N_D	ν_e (s^{-1})
0	1×10^{14}	6×10^{11}	$4{\cdot}4\times10^{3}$	$2{\cdot}5\times10^{7}$	4×10^{-5}	6	10^{11}
4×10^{8}	1×10^{12}	6×10^{10}	1×10^{4}	4×10^{7}	6×10^{-4}	200	3×10^{8}
6×10^{8}	1×10^{11}	2×10^{10}	2×10^{4}	5×10^{7}	3×10^{-3}	3×10^{3}	1×10^{7}
1×10^{9}	3×10^{9}	3×10^{9}	1×10^{5}	2×10^{8}	0·06	7×10^{5}	4×10^{3}
4×10^{9}	1×10^{9}	2×10^{9}	1×10^{6}	4×10^{8}	0·2	1×10^{7}	200
1×10^{10}	3×10^{8}	1×10^{9}	2×10^{6}	5×10^{8}	0·5	3×10^{7}	30
4×10^{10}	3×10^{7}	3×10^{8}	2×10^{6}	5×10^{8}	1·5	1×10^{8}	3
7×10^{10}	3×10^{6}	1×10^{8}	2×10^{6}	5×10^{8}	5	4×10^{8}	0·4
$1{\cdot}4\times10^{11}$	3×10^{5}	3×10^{7}	1×10^{6}	4×10^{8}	10	3×10^{8}	0·1

corona. So far one cannot really trust the estimates based on some interpretation or other of the sporadic solar radio-emission.

The solar corona is anisotropic and the electron density in it depends on the state of activity of the underlying region. Apparently, plasma turbulence is developed there where the electron density is largest. We have therefore given in Table 1 the maximum estimates of n_e and the corresponding ω_{pe}.

The electron temperature in the solar corona is more or less uniform. However, in lower layers, in the chromosphere, there are at the same height regions with vastly different temperatures, ranging from 6000°K in cold regions to 30,000°K in hot regions. We have given an average value in Table 1. It is possible that the ion temperature in the corona is less than the electron temperature, but there is no reliable information about this.

A consideration of Table 1 enables us to reach a number of conclusions which are important for what follows. Firstly, one can conclude from the data in the table that the equivalent height l_n, that is, the distance over which the density changes by a factor e, increases when we move away from the surface, changing from $1{\cdot}4\times10^{7}$ cm to 10^{10} cm.

Secondly, one can reach some conclusions about the escape of electromagnetic waves from the solar corona, using these data. It is clear that radiation with a given value of the frequency ω—or the corresponding wavenumber k—can leave the corona without great loss if the optical thickness corresponding to the equivalent height l_n is of order or less than unity. The emission will be weakened by, say, two orders of magnitude, if we have for the optical thickness:

$$\tau \approx \left(\frac{\omega_{pe}}{\omega}\right)^2 \frac{\nu_e l_n}{v_g} \approx \frac{\nu_e l_n \omega_{pe}^2}{c^2 k(\omega_{pe}^2 + c^2k^2)^{1/2}} \approx 5. \tag{7.1}$$

Equation (7.1) determines the minimum wavenumber of electromagnetic waves which can leave the corona. We find from the data in Table 1 that radiation with $k \gtrsim 100\omega_{pe}/c$ can leave the region at a height of 10^9 cm. Radiation generated at a height of 10^{10} cm (decimeter band) escapes at even lower wavenumbers, $k \gtrsim 0{\cdot}5\omega_{pe}/c$. The corona radiation (meter band) generated, say, at a height of 7×10^{10} cm has as a lower wavenumber limit $k \gtrsim 0{\cdot}2\omega_{pe}/c$. We must emphasize that these conditions impose a considerable limitation on the conditions for the escape of radiation from plasma turbulence regions. The limiting values of k may even turn out to be higher, if one takes into account refraction, inclined propagation, and so on.

Chromospheric flares

The origin of most of the sporadic solar radio-emission lies in the so-called chromospheric flares. Up to now there is no clear picture of what occurs. The following is observed: at heights of the order of 2×10^8 to 5×10^9 cm (usually below 2×10^9 cm) bright "knots" of size $\sim10^8$ cm are formed in the chromosphere, and afterwards ribbons, cones and prominences grow. Visible radiation extends over an area of the order of 10^{19} cm^2. The radiation cocurs in the hydrogen H_α lines, the calcium Ca II lines and, rarely, in the continuous spectrum. The brightest flares exceed in brightness the continuous spectrum of the unperturbed photosphere by a factor 25. Notwithstanding the considerable increase in brightness the plasma temperature in the region of a chromospheric flare does not increase greatly, to $1{\cdot}5\times10^4$ °K, and the electron density also increases to 10^{13} cm^{-3}. Many researchers feel that the visible radiation of a chromospheric flare is a secondary phenomenon.

In the flare region various motions are observed. The region itself shifts slowly—the radial velocity of the radiating gas is of the order of 10^6 cm/s, but the tangential velocities can be larger, up to 2×10^7 cm/s. It is very important that many flares are accompanied by gas ejections. The strongest ejections are accompanied by a sudden additional increase in brightness ("burst" phase). The ejection velocities may be small, $\sim10^7$ cm/s, in which case the material afterwards comes back to the flare region. Other bursts have velocities up to $1{\cdot}5\times10^8$ cm/s and their effect on other corona features is appreciable at distances of 6×10^{10} cm.

The volume of the gas mass which is involved in the motion and the radiation is of order 10^{27} cm^3. For a density $n_e \approx 10^{13}$ cm^{-3} this gives a mass of 10^{16} g. The kinetic energy of the gas motion is of the order of 10^{30} erg, and approximately the same energy is radiated during the time the flare exists, that is, during 10^3 s.

The complex trajectories of the gas motion show clearly that it is controlled by the magnetic field. However, it has so far not been possible to observe the magnetic fields directly at those chromosphere levels where the flares occur. Magnetic fields are observed in the photosphere and in the lower layers of the chromosphere. As a rule the magnetic field underneath flares has a complicated structure. The first "knots" to flare up are often observed near neutral magnetic field lines.

Apparently, chromospheric "knots" are connected with regions of a maximum gradient of the magnetic field (gradients up to 10^{-6} Oe/cm have been observed), that is, regions with the greatest electric current density. Closure of magnetic field lines has been observed: possibly one finds here an anomalous electrical conductivity leading to a violation of the "freezing-in principle". After a flare the magnetic field structure in an active region of the solar surface sometimes simplifies somewhat and the gradients are smoothed out. If the magnetic field in the flare region is of the order of 10^2 Oe, the total magnetic field energy in the flare region will be of the order of 10^{30} erg, that is, comparable with the energy of the gas motion and its radiation.

An important peculiarity of chromospheric flares is the appearance of a large number of fast particles. These particles are observed through their X-ray emission and through their influence on the terrestrial atmosphere (Sun–Earth coupling) and have, finally, been observed by direct measurements with rockets. A strong chromospheric flare accelerates 10^{36} to 10^{37} electrons to an energy in the range from 10 to 100 keV, that is, from 10^{-8} to 10^{-7} erg, and approximately 10^{31} to 10^{33} electrons to energies from 0·1 to 3 MeV (10^{-7} to 5×10^{-6} erg).

The electron energy distribution function has, apparently, the form

$$f_\varepsilon = \text{constant}\times e^{-\varepsilon/\varepsilon_0}, \tag{7.2}$$

where $\varepsilon_0 \approx 200$ keV $= 3\times10^{-7}$ erg.

The electrons are accelerated very fast during a time of the order of 1 s so that during the lifetime of one flare the particle acceleration process can be repeated several times. Protons are also accelerated in flares. They acquire a larger energy (from 5×10^{-7} to 3×10^{-2} erg) because of their larger mass, but it takes a longer time interval, of the order of 10^3 s.

The total energy liberated in a chromospheric flare, predominantly in the form of fast particles, in a time 10^3 s is thus of the order of 10^{30} erg. This energy can be stored in the region of a future flare in the form of a magnetic field.

Fast-drifting radio-bursts (type III, V, and U bursts)

The original classification of solar radio-emission bursts was undertaken when the mechanisms of their excitation were not yet clear. In the exposition of the experimental data we shall therefore not adhere to the historical order, but group the properties of the bursts according to their physical features.

Type III radio-bursts are the most striking manifestation of sporadic solar radio-emission. The region of their emission shifts in the solar corona with a velocity from 0·2c to 0·8c which shows up in an observed decrease in the frequency of the emission (frequency drift). The average displacement velocity is $v_0 \approx 0{\cdot}33c = 10^{10}$ cm/s. We note that no case has

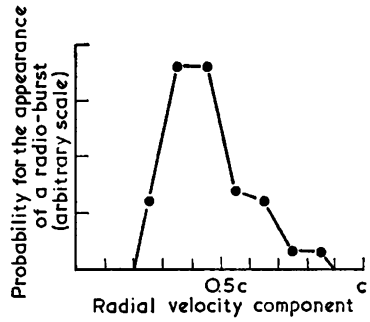

FIG. 22. Probability for the occurrence of a type III radio-burst as function of the dirft velocity.

been observed with velocities less than $0{\cdot}2c = 6\times10^9$ cm/s (Wild, Sheridan and Neylan, 1959; see Fig. 22).

At each "instantaneous level" (distance from the photosphere) radiation is generated at frequencies close to the Langmuir and twice the Langmuir (second harmonic) frequency corresponding to the electron density at that height. Usually the emission starts to escape from the corona at a level corresponding to frequencies between $\omega_{pe} \approx 6\times10^8$ s^{-1} and $\omega_{pe} \approx 2\times10^9$ s^{-1} (that is, at heights between 4×10^9 cm and 10^{10} cm), but sometimes the height at which the emission starts lies lower, around 2×10^9 cm (Fig. 23). Rocket observations have made it possible to track this type of emission up to distances of the order of 3×10^{11} cm from the Sun. The second harmonic usually has a frequency somewhat less than twice the frequency of the first harmonic: $\omega^t/\omega^p \approx 1{\cdot}8$ to $2{\cdot}0$ (Fig. 24). Both harmonics

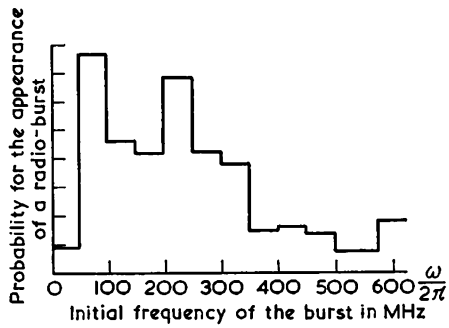

FIG. 23. Probability for the occurrence of radio-bursts as function of the initial frequency.

are found in most type III radio-bursts and the intensity of the emission at both harmonics is about the same. The solid angle of the emission at the second harmonic is, probably, somewhat larger than at the first harmonic. The band width of the instantaneous emission (that is, at any given time) is rather large and is comparable with the corresponding frequency of the emission. At any rate, $\Delta\omega/\omega \geqslant 0{\cdot}2$ (Fig. 25).

The duration of each event is 3 to 10 s. The duration of the emission at high frequencies, that is, at lower levels in the corona, may be short, down to 0·2 s (at a height of 4×10^9 cm), but at greater heights the duration of the emission increases to 5 to 10 s or even longer (at a distance of $\approx 4\times10^{10}$ cm from the Sun's surface). At each frequency one notices

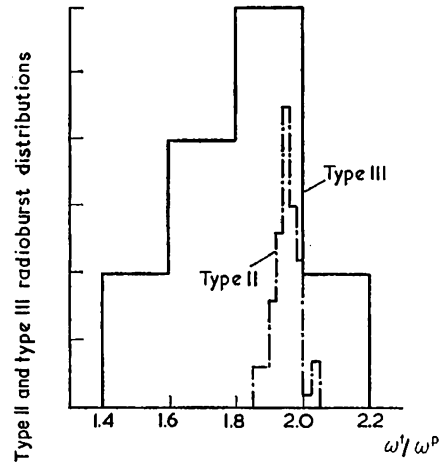

FIG. 24. Distribution of type II and type III radio-bursts as function of the ratio of the second to the first harmonic frequency.

first a fast rise in intensity and than a slow exponential fall. Sometimes one notices gaps in the emission at some frequency but after a fraction of a second the emission occurs again at a somewhat lower frequency (see Fig. 25).

The polarization of the emission of type III bursts is usually not more than 50 per cent, usually elliptical, but sometimes linear or circular. It is possible that it is connected not

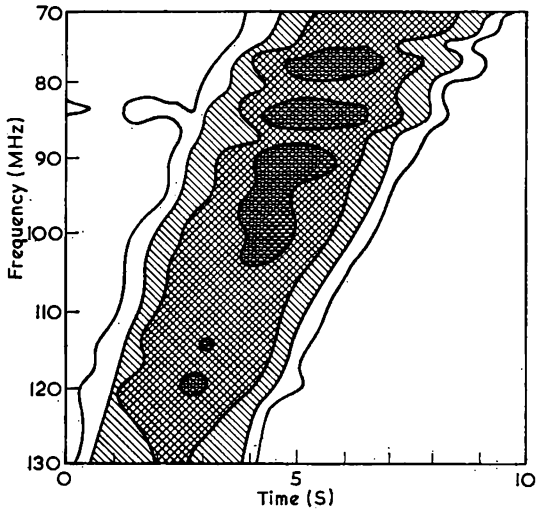

FIG. 25. Dynamic spectrum of type III radio-bursts. The increase in emission intensity is indicated by an increase in the degree of shading.

with the source of the emission, but with the conditions for the propagation of the electromagnetic waves in the corona which generate them.

The instantaneous region of emission consists of a small nucleus of size of the order of 10^{10} cm at lower heights, and of size up to 2 to 3×10^{10} cm at greater heights, and a halo

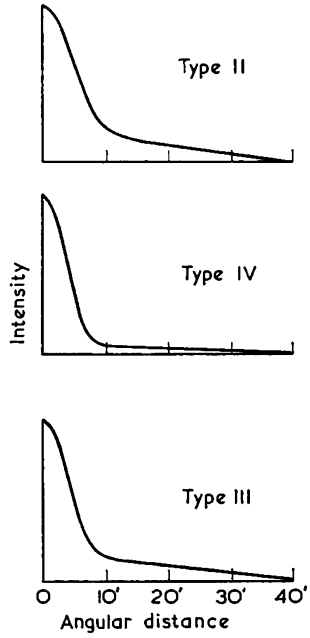

FIG. 26. Intensity distribution of type II, IV, and III radio-bursts as function of the angular distance to the centre of the source.

which has a size 4 to 5 times that of the nucleus (Fig. 26). At lower heights the greater part of the radiation intensity is concentrated in the nucleus and at greater heights the radiation is "smeared out" over the halo.

The radiation intensity of different bursts varies within wide limits. Fluxes of the order of 3×10^{-18} W m^{-2} Hz^{-1} have been recorded which corresponds to a spectral intensity

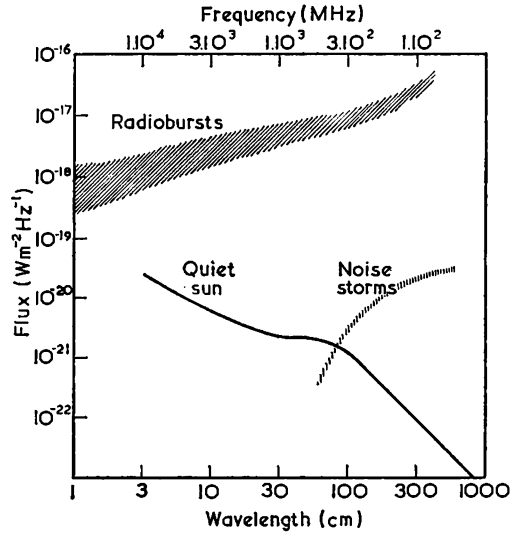

FIG. 27. Diagram giving the frequency- or wavelength-dependence of the intensity of the sporadic solar radio-emission.

$I_\omega \approx 10^{-10}$ erg/cm^2 for a source solid angle $\Omega \approx 5\times10^{-7}$. This intensity corresponds to a brightness temperature of order 10^{11} °K (Figs. 27 and 28). This value of the intensity refers to emission at the higher layers of the solar corona ($\sim 5\times10^{10}$ cm). The radiation intensity of the lower layers is several times smaller; many bursts have yet smaller intensities.

After type III bursts one observes in interplanetary space electrons with an energy of 3 to 6×10^{-8} erg, which corresponds to a velocity $v_0 \approx 10^{10}$ cm/s. A very rough estimate

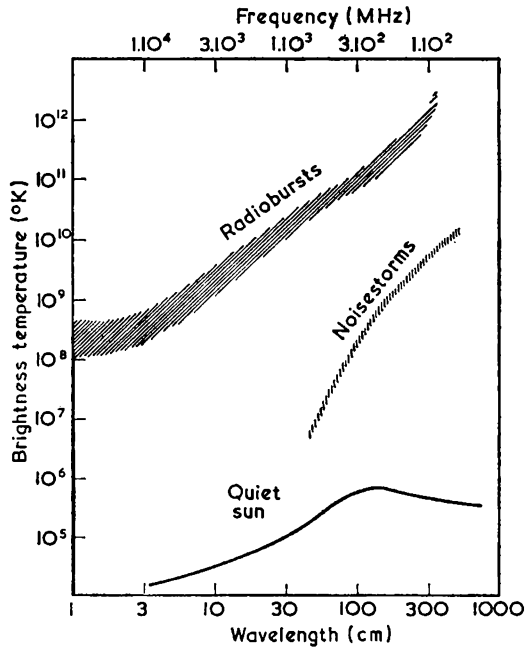

FIG. 28. The same as in Fig. 27, but with the intensity expressed in terms of the brightness temperature.

gives for the number of those electrons an order of magnitude of 10^{32} to 10^{35}. Their escape into the cosmic space indicates that the magnetic field in the upper layers of the corona is not closed. However, sometimes this field is apparently stronger and turns the particle flux the other way, producing *U*-type radio-bursts. Here the frequency of the emission at first decreases as in the normal type III bursts, and afterwards its change ceases, after which it starts to increase: the emission region shifts downwards, back to the solar surface. The frequency at the "turning" point of *U* type bursts is of the order of $\omega_{pe} \approx 1$ to 3×10^8 s^{-1}, that is, corresponding to heights of 2 to 4×10^{10} cm.

Often one observes type V bursts after type III bursts. Their frequency is always less than 10^8 s^{-1}. The radiation in them occurs over a wider band and is more strongly (elliptically or linearly) polarized than for type III bursts. There is here also a brighter nucleus of size $\sim10^{10}$ cm and an extended halo. The emission region also shifts but with a smaller velocity which is approximately 3×10^8 cm/s. The intensity of the radiation is of the same order as for type III bursts. Type III radio-bursts occur always after the start of a chromospheric flare but they can anticipate its maximum phase (Figs. 29 and 30).

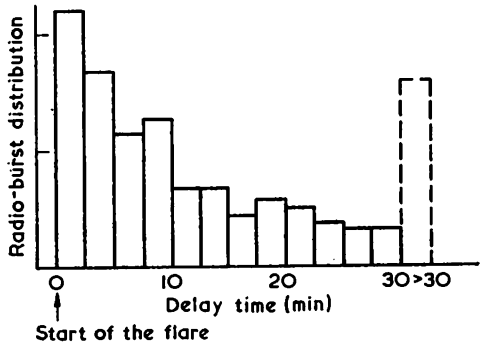

FIG. 29. Distribution of the number of type III radio-bursts relative to the start of chomospheric flares.

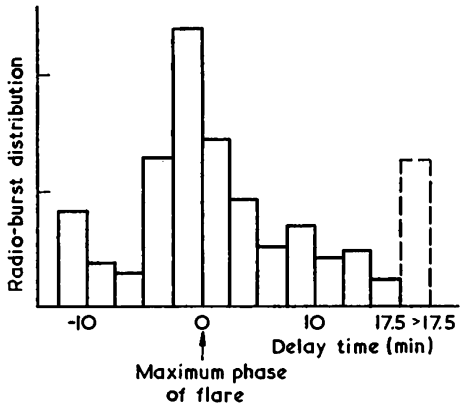

FIG. 30. The same as in Fig. 29 but relative to the maximum phase of chromospheric flares.

It follows from what we have said that the most probable explanation of these kinds of sporadic radio-emission consists of the following. A number of fast electrons, accelerated in a chromospheric flare during a very short time (~ 1 s) form a beam penetrating the solar corona with a velocity $v_0 \approx 10^{10}$ cm/s practically without energy loss; the spread of velocities in the beam is $\Delta v_0/v_0 \approx 0{\cdot}5$. (It is important that for velocities less than $0{\cdot}2c \approx 6\times 10^9$ cm/s the electron beam is unable to pass through the corona, to judge from the observational data.) Along its path through the corona the beam excites plsmas turbulence and we observe its emission in the form of radio-bursts.

We shall give in the next section an analysis of the passage through the solar corona of a beam of fast electrons, of the plasma turbulence excited by them, and of the corresponding electromagnetic radiation.

SLOWLY DRIFTING OR STATIONARY RADIO-BURSTS (TYPE II, IV, AND I BURSTS NOISE STORMS)

In type II bursts one also observes a shift of frequency with time, but a slower one. The velocity with which the turbulence region shifts lies within the limits of 3×10^7 to 10^8 cm/s (Figs. 31 and 32). In contrast to type III bursts the velocity can change appreciably

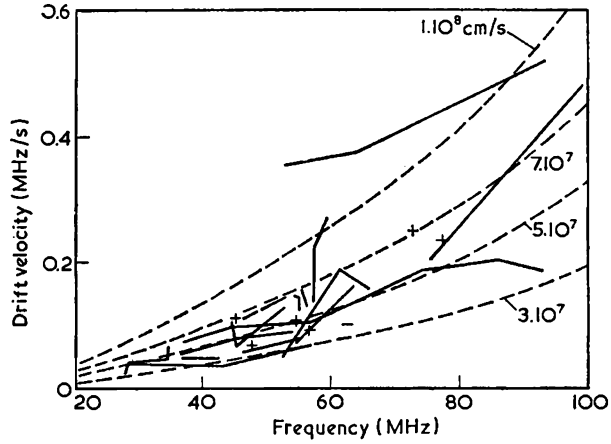

FIG. 31. Frequency dependence of the drift velocity of type II radio-bursts. The full-drawn lines correspond to the measured drift velocities at several frequencies for different bursts, the crosses to measurements at one frequency, and the dashed lines are the curves giving the change in drift velocity for a constant velocity of the motion of the radiation source (for instance, a shock wave).

when the turbulent region rises. The radiation starts at frequencies $\omega_{pe} \approx 6\times10^8$ to 10^9 s^{-1} (heights around 1 to 2×10^{10} cm) and it is observed up to the limits of observational possibilities. The duration of the bursts is of the order of 5 to 10 minutes, sometimes up to 20 minutes.

The second harmonic of the radiation appears very often. The ratio of the frequencies of the two harmonics is on average equal to 1·95, but values from 1·90 to 2·05 are possible

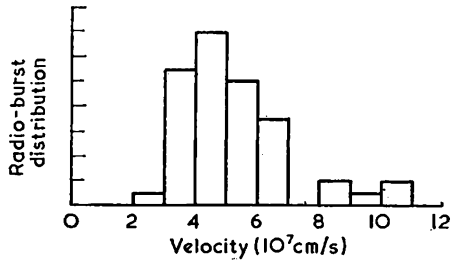

FIG. 32. Distribution of type II radio-bursts as function of the source motion velocity.

(see Fig. 24). The intensities of the two harmonics are equal on average, but often the second harmonic is stronger. At high frequencies ($\omega > 10^9$ s^{-1}) one observes predominantly the second harmonic.

There are many cases where the emission at both harmonics is cut off suddenly and, moreover, simultaneously. Sometimes after a certain time the emission starts again at lower frequencies but with the old frequency drift velocity.

A peculiar feature of type II bursts is the splitting of each of the harmonics into two bands. The frequency distance between the bands of the first harmonic is usually equal to 5 to 10×10^6 s^{-1} (Fig. 33). This splitting is repeated in the second harmonic, often even in details, but the distance is twice as large. Often it is attributed to a magnetic field. If one

equates the frequency of the split to ω_{He} one obtains for the magnetic field strength a value of the order of 1 Oe.

The width of the instantaneous frequency range is, apparently, of the order $\Delta\omega \approx 0{\cdot}2\,\omega$. One often encounters a peculiar "treelike" structure of type II bursts. One finds that wide-band, but short-lived (less than 1 s) bursts split off from a slowly drifting burst and drift both upwards and downwards. These small bursts resemble in their properties type III

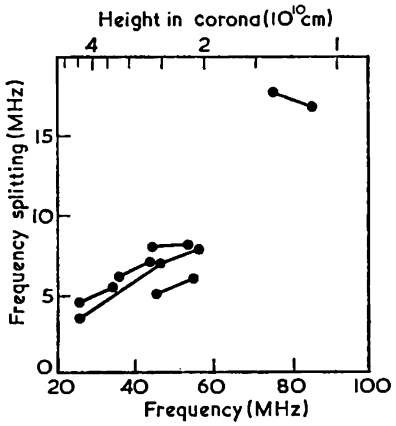

FIG. 33. Dependence of the splitting into bands of the first harmonic for type II bursts. Each of the points corresponds to a separate measurement. The lines connect points corresponding to the same burst.

bursts, but they differ from them in that they propagate both upwards and downwards from the type II burst region. They do not necessarily appear in pairs; cases are known when the type II burst itself fades away and the "tree-like" structure remains. Such a structure or at least its traces have been observed in 20 per cent of all type II bursts.

The observed region of a type II burst consists also of a bright nucleus and a diffuse halo. The size of the nuclear region is around $1{\cdot}3\times10^{10}$ cm and this value is more or less constant both for different bursts and for one burst at different heights. The size of the halo is 6 to 7 times larger.

The radio-flux from a type II burst also varies within wide limits, from the observational threshold to values above 10^{-17} W m^{-2} Hz^{-1}. The intensity of the emission can reach a value of $I_\omega \approx 10^{-9}$ erg/cm^2. As a rule the emission is unpolarized, even in the region of the above-mentioned splitting attributed to the influence of a magnetic field.

Type II bursts are encountered considerably less often than type III bursts, but they are correlated with them. In 60 per cent of the cases type II bursts either anticipate or accompany type III bursts. They are also very closely connected with chromospheric flares: 94 per cent of bursts arise at the maximum of the corresponding chromospheric flare. However, only a few chromospheric flares are able to produce type II bursts. As a rule, the intensity of type II burst radio-emission is larger than the intensity of type III bursts.

The relatively narrow-band low-frequency type II bursts are closely connected with the very wide-band type IV bursts. The latter often consist of three consecutive phases (Fig. 34).

The first phase (type IV A) is the emission from a small source (size less than 10^9 cm) at a height of less than 4×10^9 cm. The source does not move noticeably. It appears several minutes before a type II burst, but its emission can last from 10 minutes to 2 hours, and sometimes even up to 6 hours. Sometimes these bursts appear also without a corresponding type II burst. The emission is generated in a very wide range of frequencies, at least from 2×10^9 to 6×10^{10} s^{-1}. The emission displays circular polarization, the degree of which decreases with decreasing frequency. The emission is initially produced simultaneously at all frequencies and the variations in intensity, which are usually small, also comprise a

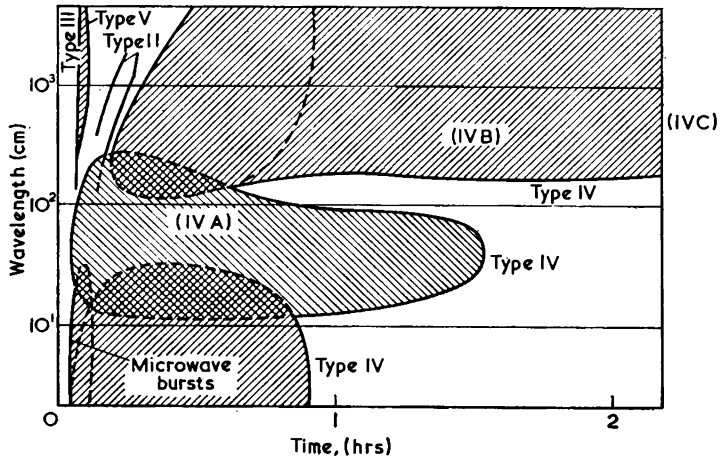

FIG. 34. Scheme of the dynamic spectra of different kinds of radio-bursts. The shaded regions show where the wavelengths and durations of radio-bursts occur.

wide frequency interval. The average intensity decreases with decreasing frequency. The brightness temperature changes within the limits of 10^6 to 10^9 °K.

The second phase (IV B) appears a few minutes after a type II burst and also lasts from 10 minutes to 2 hours, usually somewhat longer than the first phase of the same burst. Moreover, this phase is seldom unaccompanied by a type II burst. The size of the source is larger (of the order of 2 to 3×10^{10} cm) in this phase and it shifts in the corona from heights of 10^{10} to 10^{11} cm with a velocity of the order of or larger than 10^8 cm/s. The radiation is generated at lower frequencies than in the phase IV A (from 6×10^8 s^{-1} to the lowest registrated frequencies). A weak circular polarization of the emission has been noted. The emission varies little with time. Generally speaking, type IV bursts are relatively quieter than type II bursts, and the higher the frequency, the weaker the variations in intensity. The brightness temperature lies within the limits of 10^7 to 10^9 °K.

The third phase (IV C) also often (but not always) follows a type II burst but only after tens of minutes after its start. The duration of its emission is large, from several hours to several days. The size of the source is small ($\sim 10^{10}$ cm) and its motion not noticeable. In contrast to phases IV A and IV B the source here lies in the corona, apparently, at the level where its frequency corresponds to the Langmuir frequency.

The frequencies of the radiation in IV C are less than 2×10^9 s^{-1} and they possess a rather high degree of circular polarization. The direction of the emission is for these bursts well

defined. The flux can be larger than 10^{-19} W m^{-2} Hz^{-1} which corresponds to an intensity of 3×10^{-11} erg/cm^2 or a brightness temperature of 10^{10} °K.

The most common sporadic solar radio-emission phenomenon is the noise storm: in a period of maximum solar activity they comprise up to 13 per cent of the whole time. A noise storm is a strong emission, variable in time, in a very wide range of frequencies, from 2×10^9 s^{-1} down to the lowest observed frequencies, that is, less than 6×10^7 s^{-1}

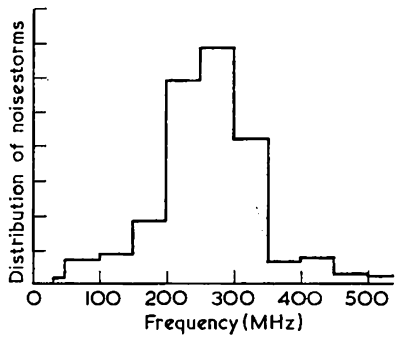

FIG. 35. Noise storm frequency distributions.

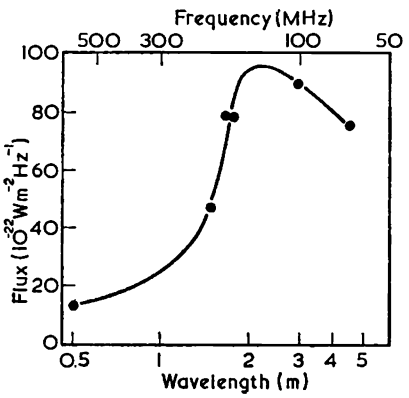

FIG. 36. Average emission sprectrum constructed from the observations of twenty-eight noise storms.

(Figs. 35 and 36). Superimposed upon this radiation are the short-lived (around 1 s) narrow-band type I bursts ($\Delta\omega \approx \omega/10$). The emission of the latter can be very different at frequencies spaced only over $\Delta\omega \approx 10^7$ s^{-1} (Fig. 37). Noticeable is its very strong circular polarization, reaching up to 100 per cent.

Noise storms, apparently, occur in sources with a small size of order of 10^{10} cm and the fast bursts superimposed upon these storms are generated in even smaller sources of size $\sim 3\times10^9$ cm.

One can sometimes notice a shift of the sources, but there are apparently no systematic velocities. The intensity of the radiation is of the same order as in other kinds of sporadic radio-emission; it can reach $I_\omega \approx 10^{-10}$ erg/cm^2 with brightness temperatures of up to 10^9 to 10^{10} °K.

The sources of the wide-band noise storms and of the narrow-band spectra of type I bursts are situated high in the solar corona (from 2×10^{10} cm upwards) and the plasma frequency at the level where these sources are situated is appreciably lower than the frequency of the radiation, although it is possible that the frequencies of some short-lived bursts may be close to ω_{pe}.

The position of the noise storm sources and of type I bursts is very closely connected with magnetic fields on the Sun. Usually the sources are situated above a large group of sunspots. There is no strong connection with chromospheric flares.

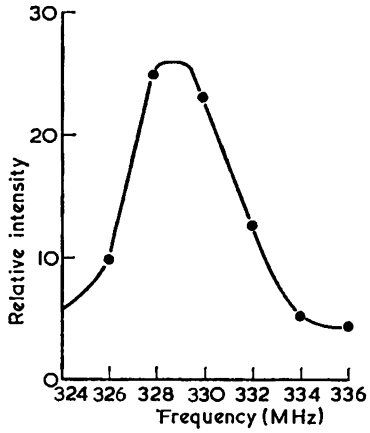

FIG. 37. Line profile of a narrow-band type I radio-burst.

In contrast to the fast drifting type III bursts where the source of the turbulent plasma—the fast electrons—is clearly revealed, an explanation of the slowly drifting or stationary bursts, such as type II bursts, is more complicated. Later we shall discuss in detail various hypotheses and models for these phenomena. Apparently, for them we see an excitation of plasma turbulence by macroscopic motions, such as shock waves, causing steep magnetic field gradients.

Other solar activity phenomena such as centimetre and decimetre bursts, or coronal condensations have also been observed. However, their connection with plasma turbulence, though possible, is so far not clear and we shall not consider these phenomena.

RADAR OBSERVATIONS OF THE SUN

Radar sounding of the Sun allows us, at least in principle, to obtain important data on the state of the upper layers of the solar corona. One is concerned here with the fact that one can only observe radio-signals, reflected from the Sun at frequencies less than $6\times10^{8}\,s^{-1}$: higher frequencies are not so much reflected, but absorbed by the corona, if one uses estimates for a non-turbulent plasma. It is true that one may expect also reflection at higher frequencies from a thin transitional layer between the corona and the chromosphere where n_e increases rapidly with depth and from regions with a strong magnetic field where the height of the reflection layer may be higher than the height of the layer with $\omega \approx \omega_{pe}$.

A reliable radar study of the Sun has so far been performed at a frequency $\omega_0 = 2{\cdot}4\times10^8 s^{-1}$ (James, 1966). Reflection has been demonstrated from corona layers at heights of 3·5 to $4{\cdot}9\times10^{10}$ cm. The reflected signals are systematically shifted to the short-wavelength side by $\Delta\omega = 2{\cdot}5\times10^5\ s^{-1}$ which may be attributed to the presence of an outward motion in the reflecting layer with a velocity $v = 1{\cdot}6\times10^6$ cm/s (solar wind). Moreover, a large, symmetric broadening of the reflected signal has been observed which is always larger than $\Delta\omega = 2\times10^6\ s^{-1}$ and sometimes reaches a value of $10^7\ s^{-1}$ (Fig. 38). If it is connected with

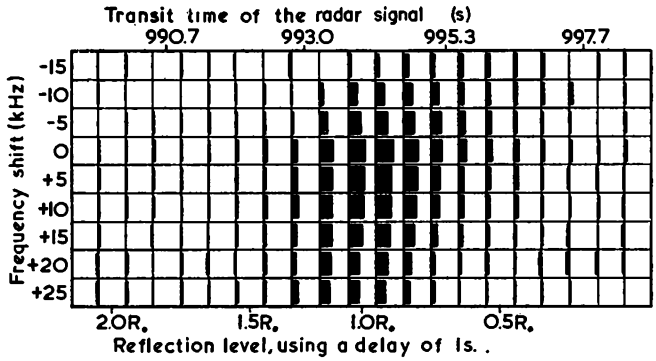

FIG. 38. Diagram showing the dependence of the intensity of a signal reflected by the corona on the time of transit and on the frequency shift. The delay of the reflected signal may be connected with the level of reflection in the corona and the shift with a Doppler effect or with a change in frequency in the scattering process. The size of the blackened regions corresponds to the intensity of the reflected signal.

a Doppler effect, one can from this conclude that there are in the corona at these heights random motions present with velocities up to $2{\cdot}4\times10^7$ cm/s or higher. It is, however, possible that the broadening of the reflected signal is caused by the influence of plasma turbulence on the reflection and scattering of radio-waves.

The scattering cross-section, that is, the reflection area, varies within wide limits. The geometrical cross-section at the reflecting level of the corona is equal to $2{\cdot}5\pi R_\odot^2$, but the observed reflection cross-sections of the radar signal turn out to be both much smaller and much larger than this value (up to $800\pi R_\odot^2$). It is possible that this effect is also connected with plasma turbulence.

We conclude herewith our brief survey of the observational data indicating the possible occurrence of plasma turbulence on the Sun. In the next two sections we shall give a solution of some plasma astrophysics problems on the basis of which one can try to interpret these observational data. The last section of this chapter will be devoted to details of this interpretation and to a discussion of various hypotheses.

§ 8. Excitation of longitudinal and transverse plasmon turbulence by a beam instability

The usually accepted explanation of type III bursts consists in saying that some electrons accelerated in the region of a chromospheric flare pass through the solar corona and excite in it Langmuir waves which in turn are transformed into electromagnetic radiation at

frequencies close to ω_{pe} and $2\omega_{pe}$. We shall consider in the present section the picture of the processes occurring in that case.

Quasi-linear beam relaxation

We assume that a beam of electrons, with an average velocity $v_0 \gg 3v_{Te}$ and a small spread in velocities by an amount $\Delta v_0 \ll v_0$ and in directions by an amount $\Delta\theta_0 \ll 1$, passes through the solar corona. We shall assume that the particle density in the beam n_* is very small compared with the electron density in the main plasma, n_e. As we explained in detail in § 4, a beam instability will then occur leading to the generation of longitudinal plasmons with wavenumbers around k_g. In an interval Δk_g, where

$$k_g \approx \frac{\omega_{pe}}{v_0}, \quad \frac{\Delta k_g}{k_g} \approx \frac{\Delta v_0}{v_0}, \tag{8.1}$$

the plasmon wavevectors are directed predominantly along the direction of motion of the beam and the growth rate is determined by equation (2.37). Outside this interval waves are excited at a large angle to the beam axis with a growth rate

$$\frac{\gamma^l}{\omega_{pe}} \approx \frac{n_*}{n_e}\frac{\omega_{pe}^2}{k^2 v_0^2}\frac{1}{(\Delta\theta_0)^2}, \tag{8.2}$$

which decreases with increasing wavenumber, and plasmons are excited up to $k \approx \omega_{pe}/3v_{Te}$ where Landau damping starts to become important. If we neglect non-linear processes the so-called quasi-linear relaxation will occur here (Vedenov, Velikhov and Sagdeev, 1961, 1962). The theory of the quasi-linear relaxation has by now been developed fairly completely and can be found in many books and review articles (see, for instance, Vedenov, 1967; Tsytovich, 1970).

If we restrict ourselves to the maximum growth rate (2.37) for waves moving along the beam, we can use a one-dimensional model.

Induced longitudinal plasmon emission is produced by particles which are in that part of the "hump" which corresponds to the lower velocities, as only then $(\boldsymbol{k}\cdot[\partial f_p/\partial \boldsymbol{p}]) > 0$ (shaded part in Fig. 8 on p. 70). These particles thus lose their energy rapidly. As a result the "inside slope of the hump" begins to move into the region of even lower velocities. The slope then becomes steeper (particles with a higher energy also lose it faster; see Ivanov and Rudakov, 1967). Particles corresponding to the outside slope of the hump do not radiate (here $(\boldsymbol{k}\cdot[\partial f_p/\partial \boldsymbol{p}]) < 0$) but they also practically do not absorb as the longitudinal plasmon energy density is here small—this "slope of the hump" so to speak stays put.

As a result the distribution function of the beam particles changes from being "bell-shaped" to being "table-shaped", increasing in width to one side and correspondingly losing its height as the total number of particles in the beam is conserved.

We note here that the distribution function of the beam particles can also broaden in the direction of higher velocities (Ryutov, 1970; Seidl and Šunka, 1969) if the beam passes through an inhomogeneous plasma in the direction of diminishing electron density. In that case the phase velocity of the plasma waves created by the beam increases in accord-

ance with the effect described by equation (4.2). Such plasmons "overtake" the beam and turn out to be in resonance with particles on the outside slope of the hump. These particles now absorb the plasmon energy and increase their energy while the energy of the slower particles decreases. This effect depends strongly on the magnitude of the electron density gradient and it probably does not accelerate the beam particles appreciably under cosmic conditions.

We now turn to the quasi-linear relaxation effect in a uniform plasma. The "slope" of the already table-shaped beam particle distribution function which is moving along the velocity axis finally reaches the electron distribution function of the main plasma. A so-called plateau is then formed on the total distribution function and there is then no longer a velocity range for which $(\boldsymbol{k}\cdot[\partial f_p/\partial \boldsymbol{p}]) > 0$ for plasma waves with wavevectors along the direction of motion of the beam. As a result the further generation of those plasma waves is cut off. The plateau formation and the cut-off in the plasma wave generation is called the quasi-linear relaxation effect. A characteristic time for the formation of a plateau is of the order of

$$t_{\text{plat}} \approx 40 \frac{n_e}{n_* \omega_{pe}}, \tag{8.3}$$

where the numerical factor which is approximately equal to 40 appears because the beam particle distribution function initially turns out to be table-shaped and only gradually broadens to form a plateau. With the appearance of the plateau the dissipation of the beam is, however, not terminated. It is true that plasmons with wavevectors along the beam axis are no longer generated, but the induced excitation of plasmons with a $\boldsymbol{k}$ making an angle with the beam axis is very well possible provided there is no external reason—such as, for instance, a strong magnetic field—which prevents the generation of such plasmons. As a result the density of all epithermal particles decreases in general and the beam dissipates almost completely. The time for such a dissipation is somewhat longer than (8.3) and we can estimate it to be of the order of

$$t_{\text{dis}} \approx 10^2 \frac{n_e}{n_* \omega_{pe}}. \tag{8.4}$$

We shall see below that on the Sun the characteristic time (8.4) is of the order of 1 second. This is a very important fact. The time t_{dis} is under astrophysical conditions always—and not only in the corona—small compared to the cosmic time scales. The existence of beams can thus only occur when such a quasi-linear relaxation is somehow suppressed. For instance, for an ion beam the characteristic smearing-out time is $m_i/m_e \approx 10^3$ times longer. The quasi-linear relaxation of a relativistic beam also requires a longer time, by a factor $\varepsilon_*/m_e c^2$ where ε_* is the average energy of its particles.

Finally, the quasi-linear relaxation can be stopped by non-linear processes. We shall consider this effect in more detail in the next subsection.

We shall give here, without a detailed study, the main equations which describe the quasi-linear relaxation. This effect is usually considered in the framework of a one-dimensional model. The plasma distribution function of the beam particles is then determined by the

diffusion equation (5.2) which in the one-dimensional case has the form

$$\frac{df_p}{dt} = \frac{\partial}{\partial p}\left(D^1 \frac{\partial f_p}{\partial p}\right). \tag{8.5}$$

The coefficient of the diffusion interaction of fast particles with a one-dimensional plasma turbulence is also different from the expressions found in § 5 when we assumed that the turbulence was isotropic. We have here no longer an integral over the wavenumbers which appeared because a particle with velocity v can absorb all plasma waves with phase velocities $v_p < v$. Now a particle with velocity v can absorb from the one-dimensional longitudinal plasmon spectrum only waves with a phase velocity equal to $v_p = v$. We have therefore for the coefficient for diffusion along the velocity axis:

$$D^1(v) = \frac{\pi m_e \omega_{pe}}{n_e} k W_k^1 = \frac{\pi m_e \omega_{pe}^2}{n_e v} W^1_{k=\omega_{pe}/v}. \tag{8.6}$$

The change in the velocity dispersion is for the case of one-dimensional diffusion:

$$\frac{d}{dt}(\Delta v)^2 = \frac{2D^1}{m_e^2} = \frac{2\pi\omega_{pe}^2}{n_e m_e v} W^1_{k=\omega_{pe}/v}. \tag{8.7}$$

Strictly speaking, equation (8.7) is valid only as long as $\overline{(\Delta v)^2} \ll v^2$. One can, however, use it to get an estimate also for $\overline{(\Delta v)^2} \lesssim v^2$. The second equation of the quasi-linear relaxation theory describes the change in the spectral density of the longitudinal plasmon energy:

$$\frac{dW_k^1}{dt} = \gamma^1(k) W_k^1 \approx \omega_{pe} \frac{n_*}{n_e}\left(\frac{v}{\Delta v}\right)^2 W^1_{k=\omega_{pe}/v}. \tag{8.8}$$

A quantitative analysis of the set (8.7) and (8.8) leads to the estimate (8.3) for the characteristic time of quasi-linear relaxation, and shows that at the moment when the plateau is formed the total energy density of the longitudinal plasmons is

$$W^1 = \int_{\omega_{pe}/v_0}^{\omega_{pe}/v_{Te}} W_k^1 \, dk \approx \tfrac{1}{6} n_* m_e v_0^2. \tag{8.9}$$

For a more exact analysis equation (8.7) is replaced by the kinetic equation for the beam particle distribution function and in equation (8.8) the magnitude of the growth rate in its explicit form is expressed in terms of the derivative $\partial f_p/\partial p$ of the beam particle distribution function. We refer to Vedenov (1967) for a detailed discussion of these problems.

Non-linear beam stabilization

There are several possible causes for beam stabilization but they are all based upon a fast escape of the plasma waves from resonance with the beam (Tsytovich and Shapiro, 1965). We shall elucidate the physics of this effect by considering the example of beam stabilization by plasma inhomogeneities (Ryutov, 1970) although this is, apparently, not applicable to the case of the solar corona.

Consider a beam moving along the electron density gradient in an inhomogeneous plasma. When the beam moves, the plasma wave phase velocity will change all the time while the particle velocity remains constant when there is no appreciable relaxation. If the growth rate γ^1 turns out to be appreciably less than the reciprocal of the time of escape from resonance, the increase in the plasma wave amplitude ceases and the beam no longer loses energy by exciting plasma waves. If we denote by L the characteristic scale of the inhomogeneity, we get as a criterion for the stabilization of a beam by plasma inhomogeneities:

$$L < \frac{v_g}{\gamma^1} \lesssim \frac{n_e}{n_*} \frac{3v_{Te}^2}{v_0 \omega_{pe}}. \tag{8.10}$$

This criterion is not satisfied in the solar corona conditions.

Of course, beams will be stabilized if the growth rate γ^1 is less than the collisional damping rate. However, one then gets only spontaneous excitation of longitudinal plasmons and their energy density turns out to be low.

The most important mechanism for the stabilization of beams is the transfer of energy along the longitudinal plasmon spectrum due to the non-linear scattering by the thermal ions in the plasma (Tsytovich and Shapiro, 1965). It acts two-fold.

Firstly, in the non-linear scattering process the wavevector direction changes rapidly and thereby the resonance of the plasmons with the particles is at once violated.

Secondly, the transfer along the wavenumber spectrum—always in the direction of decreasing k—also removes oscillations from their resonance with particles. The probability for induced emission which is proportional to the number of resonance plasmons is decreased and the beam is stabilized. One must, incidentally, stress that the stabilization can be partial: a spreading out of the beam takes place all the time but on a much longer time scale. If we neglect binary collisions total stabilization is also possible.

The turning of the wavevector in non-linear scattering gives a certain effect: through stochastic rotations one obtains a more or less isotropic distribution in the directions of $\boldsymbol{k}$. As a result it turns out that part of the waves will all the time be in resonance with the beam. The relative fraction of resonance waves is of the order of the relative spread of the beam velocities over the directions. We shall in what follows understand by γ^1 the magnitude of the effective growth rate, taking into account this decrease.

The non-linear transfer along the wavenumber spectrum not only removes plasmons from resonance with the beam but also, by virtue of its "one-sidedness" does not return them again into the resonance region. This effect is therefore more important. We can for an analysis use again equation (8.7) as the maximum contribution to the diffusion coefficient comes from waves excited along the beam. One must, however, add to equation (8.8) a term taking into account the non-linear scattering. For the case of a differential transfer in the scattering by thermal ions we have here equation (4.5) where Q_k^1 must be replaced by $\gamma^1 W_k^1$:

$$\frac{dW_k^1}{dt} = (\gamma^1 - \nu_e) W_k^1 - \beta^1 \left(\frac{W_k^1}{k}\right)^2 + \alpha^1 W_k^1 \frac{\partial W_k^1}{\partial k}. \tag{8.11}$$

We assume that there is indeed a partial stabilization of the beam, that is, we shall assume that $\overline{(\Delta v)^2}$ changes relatively slowly when compared with the characteristic time $1/\gamma^1$. We

can then use the quasi-stationary solution of equation (8.11) to determine W_k^1, assuming here that γ^1 is a slowly varying quantity and dropping dW_k^1/dt.

We substitute the value of W_k^1 obtained in this way into (8.7) and this enables us to determine the relatively slow spreading out of the partly stabilized beam.

We need only know W_k^1 in the resonance region of the spectrum when substituting it into (8.7). If the quantity k_r from (4.12) is less than $k_g \approx \omega_{pe}/v_0$, we can then in (8.11) drop the term in β^1 and we get for the quasi-stationary value of W_k^1.

$$W_k^1 = \frac{\gamma^1 - \nu^e}{\alpha^1}(k_g - k) = \frac{18}{\pi}\left(1+\frac{T_e}{T_i}\right)^2 \frac{n_e m_i v_{Te}^4}{\omega_{pe}^3}(\gamma^1 - \nu_e)(k_g - k). \tag{8.12}$$

We have assumed here that for $k > k_g$ there is no induced emission and that the energy density of the longitudinal plasmons there remains at a low level.

Substituting (8.12) into (8.11) and taking for $k_g - k$ the average value of this quantity in the resonance region, that is, putting $k_g - k \approx k_g/2$, we get

$$\frac{d}{dt}\left(\frac{\Delta v}{v_0}\right)^2 = 18\left(1+\frac{T_e}{T_i}\right)^2 \frac{m_i}{m_e}\left(\frac{v_{Te}}{v_0}\right)^4 \frac{\Delta k_g}{k_g}(\gamma^1 - \nu_e). \tag{8.13}$$

Putting $\Delta k_g \approx k_g(\Delta v/v)$ we can rewrite this expression as follows:

$$\frac{d}{dt}\left(\frac{\Delta v}{v_0}\right) = \left(\frac{v_*}{v_0}\right)^4 (\gamma^1 - \nu_e), \tag{8.14}$$

where we have introduced a critical beam stabilization velocity:

$$v_* = \left(9\frac{m_i}{m_e}\right)^{1/4}\left(1+\frac{T_e}{T_i}\right)^{1/2} v_{Te} = 11\left(1+\frac{T_e}{T_i}\right)^{1/2} v_{Te}. \tag{8.15}$$

In an isothermal plasma $v_* = 16\, v_{Te}$. When the ratio T_e/T_i increases the critical stabilization velocity increases but when $T_e > 3T_i$ the removal of longitudinal plasmons from the resonance region with the beam is not through the non-linear scattering by ions but through the decay process $l \to l' + s$ (stabilization by ion-sound waves). This case can be described by the same equations but with $1 + T_e/T_i$ replaced by unity so that here the critical stabilization velocity is reduced to $11 v_{Te}$.

The process of the fusion of longitudinal plasmons with the formation of transverse electromagnetic waves of frequency $2\omega_{pe}$ is an additional stabilization influence on the beam.

It follows from (8.14) that if the beam velocity is larger than v_* the beam relaxation time is larger than the time needed to excite plasma waves. As the ratio v_*/v_0 occurs in that formula to the fourth degree, even a relatively small excess of v_0 over v_* suffices for the non-linear stabilization effect to become appreciable.

The minimum value of $v_* = 11\, v_{Te}$ (assuming a non-isothermal plasma) is in the conditions of the solar corona equal to 6×10^9 cm/s $= 0{\cdot}2c$ which is, indeed, close to the observed minimum velocity of beams which excite type III bursts. Expression (8.15) for the critical velocity v_* is only an estimate and the exact value of the numerical coefficient may be some-

what different from the one given here. One can from this conclude that the role of non-linear beam stabilization may be considerable in the conditions of the solar corona. However, in order that the beam passes without considerable spreading through the observed distance which is by far longer than the quasi-linear relaxation length, it is necessary that $v_0 > v_*$. It is true that according to (8.14) the spreading-out steeply drops when v_0 increases, but in order that the path of the beam is, for instance, two orders of magnitude greater than the quasilinear length it is necessary that $v_0 \geqslant 3v_*$. We note that in a sufficiently cold plasma $v_0 \gg v_*$ may occur (we give below a more rigorous criterion) and a total, not just a partial, stabilization is possible.

The non-linear stabilization which we have just considered takes into account the influence of the wavenumbers of only the neighbouring part of the spectrum (that is, the case of differential transfer) on the removal of longitudinal plasmons from the resonance region. The form of the remaining part of the longitudinal plasmon spectrum was then not taken into account. However, we saw in § 4 that in actual conditions of Langmuir tubulence there is often a build-up of longitudinal plasmons in the region of very small wavenumbers (around the main scale-length wavenumber k_0). If an integral transfer of longitudinal plasmons is possible, waves of the main scale-length can thus very strongly "pump" longitudinal plasmons from the resonance range of wavenumbers. There occurs here also an absolute stabilization, when each plasmon generated in the region $k \approx k_g$ is rapidly removed to the region $k \approx k_0$ so that $W^l_{k_g}$ becomes vanishingly small.

Integral transfer occurs in the case of scattering by thermal ions when $k_g \lesssim k_*$ or $v_0 > 130 v_{Te}$. In the conditions of the solar corona this condition is not satisfied. We shall therefore consider non-linear scattering by electrons. It is well known that it leads to an appreciable change $\Delta k \lesssim (3d_e)^{-1}$ already in a single scattering process. For the transfer of energy corresponding to relatively small values of Δk the non-linear scattering by electrons is unimportant compared to the non-linear scattering by ions. For a transfer of longitudinal plasmons from one end of the spectrum to the other end in a single process, however, the role of the scattering by ions is, by contrast, very (exponentially) small and the scattering by electrons dominates. Moreover, if in the main scale-length region the longitudinal plasmon energy density becomes large, then a single transfer of resonance plasmons through scattering by electrons into the main scale-length region may exceed the transfer caused by ions from that part of the spectrum which is immediately adjoining the resonance region. In this case scattering by electrons leads to absolute stabilization (Rudakov, 1970, 1971). Let us consider the transfer of resonance plasmons due to scattering by electrons into the small wavenumber region $k_0 < k_g(m_e/m_i)(v_0/v_{Te})^3$. This inequality is practically always satisfied in the solar corona for type III bursts if k_0 corresponds to the main turbulence scale. The transfer growth rate is here independent of the detailed shape of the plasmon spectrum and determined only by their total energy density (Tsytovich, 1970):

$$\frac{\gamma^l(1+\mathrm{e} \rightleftarrows 1'+\mathrm{e}')}{\omega_{\mathrm{pe}}} = \frac{2\sqrt{(2\pi)}}{81}\left(\frac{m_e}{m_i}\right)^2\left(\frac{\omega_{\mathrm{pe}}}{k_g v_{Te}}\right)^3 \frac{W^l}{n_e T_e} \approx \left(\frac{m_e}{3m_i}\right)^2\left(\frac{v_0}{v_{Te}}\right)^3 \frac{W^l}{n_e T_e}, \quad (8.16)$$

where k_g is the wavenumber of the plasmons which are removed.

Comparing (8.2) and (8.16) we find the longitudinal plasmon energy density in the main-scale-length region of the spectrum which is necessary for beam stabilization through

integral non-linear scattering by electrons:

$$W^1 \approx n_* m_e v_0^2 \left(\frac{3m_i}{m_e}\right)^2 \left(\frac{v_{Te}}{v_0}\right)^5. \tag{8.17}$$

The determination of the stabilization criterion is reduced to explaining how the maximum values of W^1 can be reached. When the beam is not restricted in space the magnitude of W^1 can clearly not exceed the energy density of the particles in the beam itself, that is, $\frac{1}{2} n_* m_e v_0^2$. The criterion for stabilization by non-linear scattering by electrons therefore has here the form:

$$v_0 > \left(3\sqrt{2}\frac{m_i}{m_e}\right)^{2/5} v_{Te} \approx 40 v_{Te}. \tag{8.18}$$

The situation is changed, if we take into account that the beam is limited. We are here dealing with the following. Faĭnberg and Shapiro (1965), and also Tsytovich and Shapiro (1966), have shown that when a beam enters a bounded plasma an accumulation of longitudinal plasmon energy occurs at its leading front; then W^1 is larger than the beam particle energy density by a factor $v_p/v_g \approx \frac{1}{3}(v_p/v_{Te})^2$. This effect is general in nature and is due to a difference in the velocity of energy transfer (group velocity) of the beam particles and the plasmons. One can therefore expect that if a bounded beam moves in a plasma, behind its front there is formed a region with increased longitudinal plasmon energy density of the order of $\frac{1}{6} n_* m_e v_0^2 (v_p/v_{Te})^2$. The non-linear scattering by electrons could then be important when

$$v_0 > \left(3\sqrt{6}\frac{m_i}{m_e}\frac{v_{Te}}{v_p}\right)^{2/5} v_{Te} \lesssim 10 v_{Te}, \tag{8.19}$$

if we bear in mind, that according to Chapter 1 in the main scale-length region $v_p > \omega_{pe}/k_* = 130 v_{Te}$.

If we assume that in the main scale-length region the energy (8.17) is accumulated, we must also estimate the time during which this occurs. It is clear that this time is determined by the ratio of W^1, corresponding to (8.17), to Q^1, the power of the plasma turbulence generation. Assuming as before that the excited plasmons in the resonance range of wave-numbers are in the first stage of the excitation of the turbulence transferred through the non-linear scattering by ions we have

$$Q^1 \approx Q_k^1 \Delta k_g \approx \gamma^1 W_k^1 \Delta k_g \approx \frac{(\gamma^1 \Delta k_g)^2}{2\alpha^1}. \tag{8.20}$$

Hence

$$\frac{1}{t_{st}} = \gamma_{st} \approx \frac{Q^1}{W^1} \approx \frac{n_*}{2n_e}\left(1+\frac{T_e}{T_i}\right)^2 \frac{m_i}{m_e}\frac{v_0}{v_{Te}}\omega_{pe}. \tag{8.21}$$

The growth rate γ_{st} of the total stabilization is smaller than the growth rate γ^1 of the beam instability and becomes of the order of (8.4)—the time of quasi-linear dissipation. Thus, if we put $v_0 \approx 15 v_{Te}$, we get $t_{st} \approx 10^2 n_e/n_* \omega_{pe} \approx t_{dis}$ and in a non-isothermal plasma, when we must drop the factor $(1+T_e/T_i)^2$ in (8.21), an even smaller magnitude. This means that it is sufficient to prevent the spreading-out of the beam only for a time of the order of

the quasi-linear relaxation time. This can according to (8.14) be appreciable if v_0 is rather close to v_*.

From what we have said so far we can conclude that the beam stabilization process in the solar corona, if it occurs at all, must be rather complex in nature and must depend on the total longitudinal plasmon spectrum on which, by the way, the transverse plasmon spectrum is superimposed. The case is also possible when the beam is stabilized only with regard to the excitation of plasma waves moving at an appreciable angle to the beam axis.

Unfortunately, the conditions in the solar corona are such that simple estimates turn out to be insufficient for using a theory constructed for the limiting cases of a stabilized or a quasi-linearly relaxed beam since the appropriate parameter is of order unity. However, it is undoubtedly impossible to neglect non-linear transfer effects either by electrons or by ions, or the effects of the quasi-linear relaxation in the initial stage. It is clear that if we take the boundedness of the beam and the accumulation effect into account, and also the fact that all quasi-linear and non-linear processes are three-dimensional one can only obtain a solution of the complete problem by numerical methods. However, all physical processes are here well-defined and clear and one may hope that such a study will be made in the not too far distant future.

For the further discussion we shall assume that in the first stage after the beam has arrived in the given region of the corona plasma turbulence will be excited and resonance plasmons will be removed by non-linear scattering on ions. The duration of this stage is somewhat shorter than the characteristic time (8.3) of the quasi-linear relaxation and the power of the turbulence generation is determined by equation (8.20), that is,

$$Q^1 \approx 9\left(1+\frac{T_e}{T_i}\right)^2\left(\frac{n_*}{n_e}\right)^2\frac{n_e m_i v_{Te}^4}{v_0^2}\omega_{pe}. \tag{8.22}$$

Using the definition of v_* we have also

$$\frac{Q^1}{n_e T_e \omega_{pe}} \approx \left(\frac{n_*}{n_e}\frac{v_*^2}{v_{Te}v_0}\right)^2. \tag{8.23}$$

After a sufficient longitudinal plasmon energy density has been accumulated in the whole of the spectrum the non-linear transfer by electrons will limit the further increase in W^1 and leads thereby to a beam stabilization. The longitudinal plasmon energy density in the resonance region is now determined by the balance of the removal (8.16) and the transfer into this region of plasmons from the main scale-length region through spontaneous scattering for which we have the estimate:

$$J_k^1(1+e \rightleftharpoons 1'+e') = \frac{4\sqrt{(2\pi)}}{27}\left(\frac{m_e}{3m_i}\right)^2\left(\frac{\omega_{pe}}{kv_{Te}}\right)^3\frac{m_e\omega_{pe}^3}{2\pi^2}\frac{W^1}{n_e T_e}. \tag{8.24}$$

Dividing (8.24) by (8.16) we find the spectrum in the resonance region (see also (4.29)):

$$W_k^1 = \frac{m_e\omega_{pe}^2}{6\pi^2}. \tag{8.25}$$

This quantity is larger than the thermal level, which is equal to $W_k^l = T_e k^2/2\pi^2 = m_e\omega_{pe}^2 v_{Te}^2/2\pi^2 v_0^2$, by a factor of about $(v_2/v_{Te})^2$. The beam is therefore also smeared out in this stage but with a characteristic time of order $(10/v_e)\,(v_0/v_{Te})^3$, that is, of the order of the time after which the beam is dissolved due to binary collisions (this actual fact must be included in the concept of a total stabilization of a beam instability).

In the first stage of the spectrum formation and in the second stage of the beam stabilization electromagnetic radiation is produced. We shall give an estimate of the intensity of this radiation in a moment but first we shall consider the more general properties of two-mode turbulence involving longitudinal and transverse plasmons.

Longitudinal and transverse plasmon turbulence

The longitudinal plasmons which are in resonance and generated directly by the beam are through non-linear scattering by ions not only transferred to the region of smaller wavenumbers but they also transfer their energy to transverse plasmons. If the optical depth for both the transverse and the longitudinal plasmons is much larger than unity, inside the turbulence region there will be a continuous exchange of energy between the longitudinal and transverse plasmons accompanied by their change to smaller k. To describe this process we write down kinetic equations, using the method for the analysis of turbulence involving longitudinal plasmons only which we considered in detail in § 4.

The set of kinetic equations for the wavenumber range $k_* < k < k_g$ has in the unified scale ($k_p = \sqrt{3}(v_{Te}/c)\,k$) the following form:

$$\frac{dW_k^l}{dt} = Q_k^l - v_e W_k^l - \beta^l\left(\frac{W_k^l}{k}\right) + \alpha^l W_k^l\left[\frac{\partial W_k^l}{\partial k} + k\frac{\partial}{\partial k}\,\frac{W_k^p}{k}\right], \tag{8.26}$$

$$\frac{dW_k^p}{dt} = -v_e W_k^p + \alpha^l\,\frac{W_k^p}{k}\,\frac{\partial}{\partial k}(kW_k^l) \tag{8.27}$$

The difference between the set (8.26), (8.27) and equation (4.5) is connected with the fact that we have taken into account all possible transfer rates for longitudinal and transverse plasmons through scattering by ions (using the growth rates (3.17), (3.19) and (3.20)).

It would be natural to start the study of the set (8.26), (8.27) by looking for a stationary solution. However, it turns out that in the given case there is no stationary solution of equations (8.26) and (8.27) which satisfies the physical boundary conditions in the transfer region. To show this, we consider a simpler set by dropping in (8.26) and (8.27) terms describing losses through fusion (β^l) and through collisions of electrons with ions (v_e). In the transfer range $k_* < k < k_g$, that is, when $Q_k^l = 0$, the solution of the stationary set will be

$$W_k^l = A\frac{k}{k_g}, \quad W_k^p = \frac{A}{2}\left(\frac{k}{k_g} - \frac{k_g}{k}\right) + B\frac{k}{k_g}, \tag{8.28}$$

where A and B are integration constants (the values of W_k^l and W_k^p for $k \approx k_g$). It follows from (8.28) hat one cannot make a reasonable choice of A and B such that W_k^p and W_k^l are both positive for the whole interval from $k = k_g$ to $k = k_* \ll k_g$.

However, the set (8.26) and (8.27) has a quasi-stationary solution in which W_k^l and W_k^p somehow pulsate in time while at the same time shifting along the wavenumber axis. We encountered a similar case already in § 4 when considering Alfvén and magneto-sound turbulence.

This means the following. Let at some time the distributions W_k^l and W_k^p be given. After a while the energy from one range of wavenumbers of longitudinal plasmons has been transferred to a neighbouring interval of transverse plasmon wavenumbers, and from there to an interval of even smaller longitudinal plasmon wavenumbers. Waves are "travelling" along the wavenumber axis and transfer energy to different modes. A characteristic period of these "waves" is

$$\frac{1}{t_n} \approx 2\pi\alpha^l \frac{\partial W_k^l}{\partial k} \approx \omega_{pe} \frac{m_e}{m_i} \left(\frac{v_0}{v_{Te}}\right)^2 \frac{W^l}{n_e T_e}. \tag{8.29}$$

In general, a two-mode plasma turbulence has in the framework of differential transfer a "wavelike" character in contrast to a single-mode turbulence where it is possible to have a

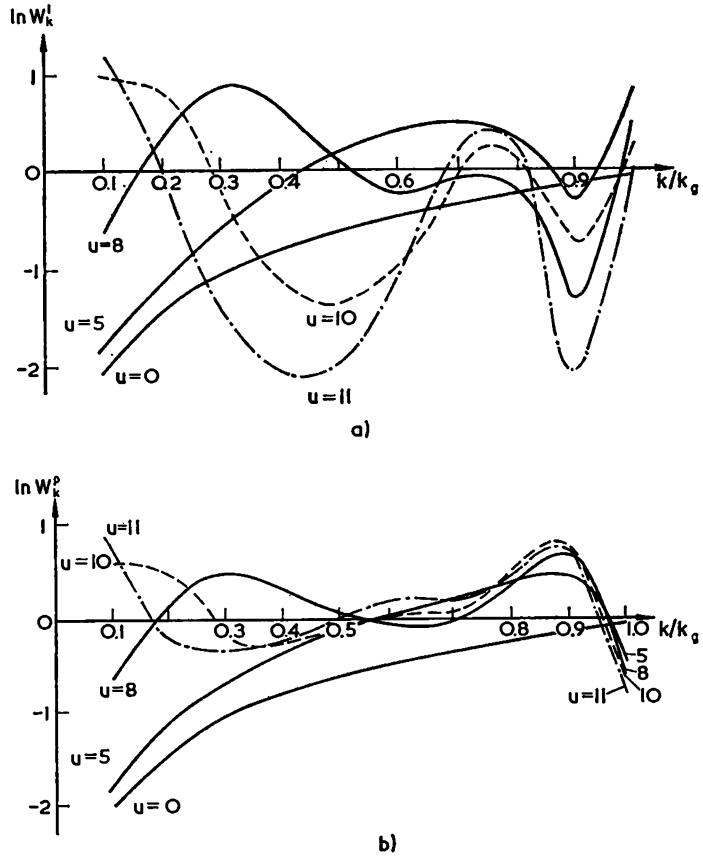

FIG. 39. Spectral energy densities of longitudinal (a) and transverse (b) plasmon turbulence for the case of differential transfer of the turbulence excited by a beam with $v_0 = v_{Te}^2/3v_{Ti}$ at different times $u = 2\gamma^l t$.

stationary solution or a non-stationary one which has the form of a jump on the wavenumber axis.

When the transfer is integral, the wave-like energy transfer effect is absent, as the plasmons change at once to the small wavenumber region of the spectrum. All those peculiarities are illustrated by Figs. 39 to 41 where we give longitudinal and transverse plasmon turbulence spectra obtained from a numerical solution of the kinetic equations taking into account the possibility of both differential and integral transfer (Kaplan and Tsytovich, 1968a).

The main characteristic is the time-averaged spectral energy density of the longitudinal and transverse plasmons. It follows from general considerations based on the fact that the

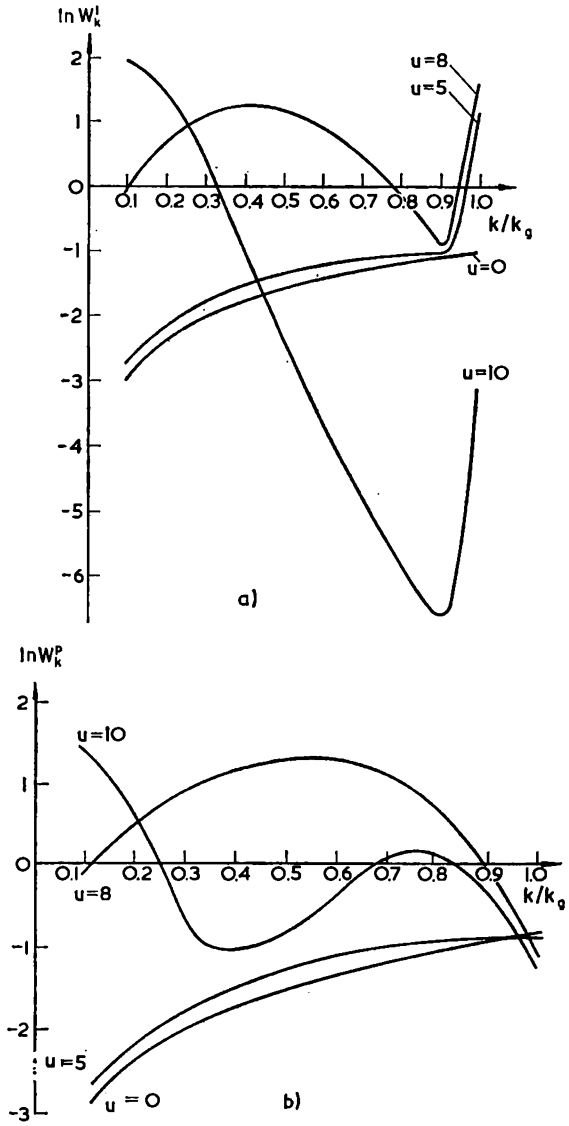

FIG. 40. The same as in Fig. 39 but for a larger beam velocity, $v_0 = v_{Te}^2/v_{Ti}$ which is on the boundary between the differential and integral turbulence transfer.

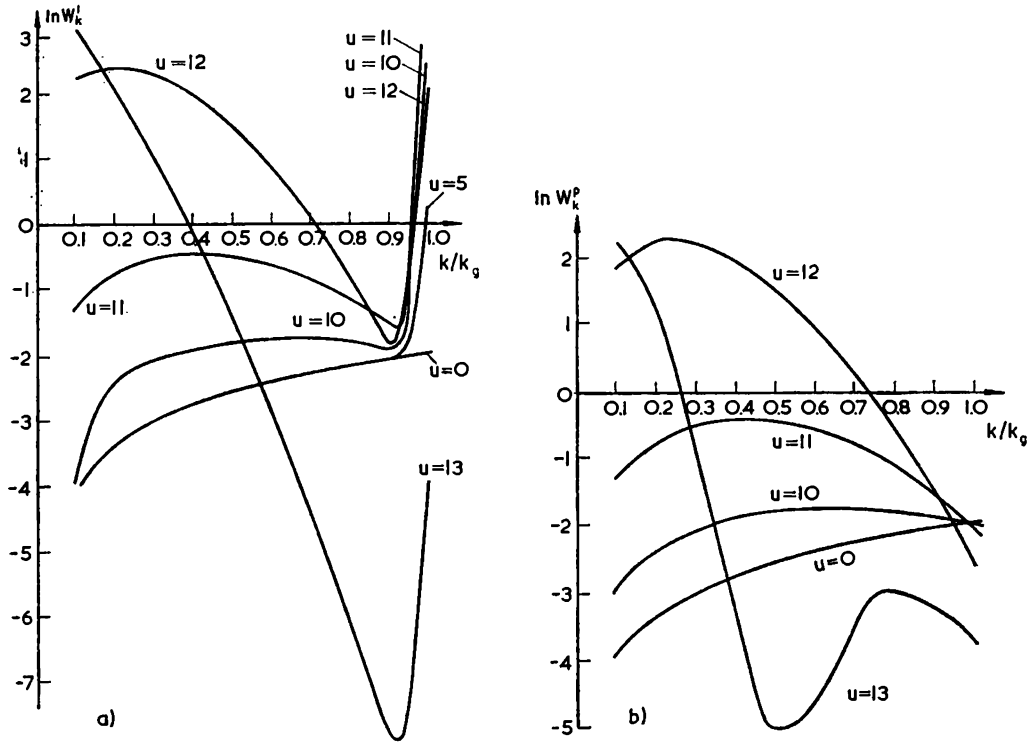

FIG. 41. The same as in Fig. 39 but for the integral transfer case with $v_0 = 3v_{Te}^2/v_{Ti}$.

transfer probabilities from longitudinal to transverse plasmons and vice versa are equal, and also from consideration of Figs. 39 to 41 that one may expect that the average values of W_k^l and W_k^p are more or less the same and correspond to the flat longitudinal plasmon turbulence spectrum already obtained in § 4 (see (4.8) and Fig. 4). For the wavenumber range $k_* < k < k_g$ we have thus:

$$W_k^l \approx W_k^p \approx \sqrt{\frac{Q^l}{\alpha^l}} \approx \frac{\gamma^l \, \Delta k_g}{\alpha^l} \approx \frac{n_* m_e v_*^4}{\omega_{pe} v_0}. \tag{8.30}$$

After substituting Q^l from (8.23) or γ^l from (8.2) we get for the transverse plasmons in their own wavenumber scale

$$W_{k_p}^p \approx \frac{n_* m_e c v_*^4}{\omega_{pe} v_{Te} v_0}. \tag{8.31}$$

The total longitudinal plasmon energy density in the wavenumber interval corresponding to the flat spectrum is equal to

$$W \approx W_k^l k_g \approx n_* m_e v_*^2 \frac{v_*^2}{v_0^2}. \tag{8.32}$$

In the very small wavenumber (main scale-length) region of the spectrum the longitudinal plasmon energy density is determined by the beam stabilization conditions. This problem will be discussed in detail in the next section. Here we can expect that in that region of the

spectrum the longitudinal plasmon energy density will be much larger than the estimate (8.32) (see, for instance, equation (8.17)).

On the other hand, in the large wavenumber region, $k > k_g$ the longitudinal plasmon energy spectrum decreases because the growth rate (8.2) diminishes. Using the same method to determine the plasmon spectrum for a known growth rate we find for the longitudinal and transverse plasmons in a unified wavenumber scale:

$$W_k^l \approx W_k^p \approx \frac{n_* m_e v_0^2}{k} \left(\frac{v_*}{v_0}\right)^4. \tag{8.33}$$

This spectrum can not extend to large wavenumbers as the inverse transfer of plasmons to the region of wavenumbers of order k_g through non-linear scattering by electrons turns out to be important. Substituting into (4.26) the quantity Q^l from (8.23) we find the growth rate for that transfer:

$$\gamma^l(l+e \rightleftharpoons l'+e') \approx \frac{n_*}{n_e} \left(\frac{v_*}{v_0}\right)^4 k v_{Te}, \tag{8.34}$$

that is, it increases with increasing k. Comparing (8.34) with (8.2) we find that the spectrum (8.33) extends to

$$k_{max} \approx \frac{\omega_{pe}}{v_{Te}} \left(\frac{v_0}{v_*}\right)^{2/3} \left(\frac{v_{Te}}{v_*}\right)^{2/3} \approx \frac{\omega_{pe}}{4 v_{Te}} \left(\frac{v_0}{v_*}\right)^{2/3}. \tag{8.35}$$

In the region $k > k_{max}$ the longitudinal plasmon spectrum is described by equations (4.29) or (4.30). This energy is already too low to produce stimulated processes, in particular, to transfer energy into transverse plasmons. Spontaneous emission of transverse plasmons also does not give here an energy density appreciably above the thermal level.

In the first stage of turbulence development in the large wavenumber region we find the transverse plasmon energy spectrum for $k_p \lesssim \omega_{pe}/3c$ in its own scale from (8.33):

$$W_{k_p}^p \approx \frac{n_* m_e v_0^2}{k_p} \left(\frac{v_*}{v_0}\right)^4. \tag{8.36}$$

However, in actual fact equation (8.36) gives overestimated values as here the role of the transfer by ions from longitudinal to transverse plasmons has diminished considerably.

In the second stage of turbulence development when there is stabilization through scattering from the main scale-length region the total generation power falls steeply and the spectra (8.33) and (8.36) are still further decreased.

We can get rough quantitative estimates using the formulae which we have just given but one must stress the desirability of a more thorough analysis of the processes which take place here.

Escape of the radio-emission from turbulence regions

It is clear that longitudinal and transverse plasmon turbulence must generate electromagnetic radiation both at frequencies close to ω_{pe} (escape of transverse plasmons) and at frequencies close to $2\omega_{pe}$ (fusion of longitudinal plasmons). We shall obtain estimates of the intensities of both forms of electromagnetic radiation.

Although the transverse plasmon energy density in plasma turbulence regions is very high, the probability for its escape is very small. Indeed, we have shown in § 7 that only plasmons with relatively large wavenumbers (for instance, $k_p > 0{\cdot}2\omega_{pe}/c$) can escape from the solar corona without being very strongly absorbed. From this we get an upper limit to the radiation intensity at frequencies close to ω_{pe}, if $k_p \approx 0{\cdot}2\omega_{pe}/c$:

$$I_\omega^p \approx \frac{W_{kp}^p}{4\pi} \approx 3\times10^{-3}\frac{c}{\omega_{pe}} n_* m_e v_0^2. \tag{8.37}$$

The observed intensity is less than this value by, probably, two orders of magnitude, if we take into account absorption in the upper layers.

When the turbulence region shifts in the corona the quantity I_ω^p given by (8.37) increases proportional to ω_{pe}^{-1} (for constant n_*) which agrees qualitatively with observations.

The radiation intensity at frequencies close to $2\omega_{pe}$ can be evaluated using equation (3.30). The limitations on the wavenumbers of the electromagnetic waves which can escape from the turbulence region are weaker here than in the case of emission at frequencies close to ω_{pe} but nevertheless the larger probabilities for leaving the medium occur for radiation at the highest wavenumbers. If we retain in (3.30) only the second integral, assuming that W_k^l = constant (flat spectrum) we get

$$J^t = \int J_\omega^t \, d\omega \approx \frac{4\omega_{pe}^3}{n_e m_e c^4}(W_k^l)^2 \approx \frac{4\omega_{pe}^3 Q^l}{n_e m_e c^4 \alpha^l} \approx \frac{4n_*^2 m_e v_*^8}{n_e c^4 v_0^2}\omega_{pe}, \tag{8.38}$$

where we have used equation (8.30) for W_k^l.

The spectral emission coefficient can be estimated as to order of magnitude by putting $J_\omega^t \approx J^t/\omega_{pe}$. Putting also $v_* \approx v_0/2$ we have

$$J_\omega^t \approx \frac{n_*^2 m_e v_0^6}{100 n_e c^4}. \tag{8.39}$$

Plasma turbulence regions are usually optically thin with respect to the decay of a wave at $2\omega_{pe}$ into two longitudinal plasmons. Denoting by R the characteristic size of the region in which fusion takes place we have for the spectral intensity:

$$I_\omega^t \approx \frac{J_\omega^t R}{4\pi} \approx 8\times10^{-4}\frac{n_*^2 m_e v_0^6}{n_e c^4} R. \tag{8.40}$$

When the height of the region in the corona increases the emission at a frequency $\sim 2\omega_{pe}$ increases as $1/n_e \sim 1/\omega_{pe}^2$. Superimposed on this effect is the increase in the radiation intensity with time at a given level when the plasmons accumulate in the main scale-length region (first term in (3.30)). This accumulation, begins, however, to suppress the plasmon density with $k \approx \sqrt{3}.\omega_{pe}/c$ because of scattering by electrons (see (3.30)) and (8.16). The total radiation intensity does not greatly exceed (8.40). The increase in intensity at a frequency $2\omega_{pe}$ with height and its delay with respect to the emission at the frequency ω_{pe} are in qualitative agreement with observations.

INFLUENCE OF A MAGNETIC FIELD ON THE LONGITUDINAL AND TRANSVERSE PLASMON TURBULENCE

The observed polarization of type III bursts indicates that a magnetic field has a well-defined effect on the plasma turbulence properties in the solar corona although it is possible that to some degree the observed polarization is connected with the conditions for propagation of radio-waves in the solar corona.

As a rule the inequality $\omega_{pe} \gg \omega_{He}$ is satisfied in the solar corona. Therefore we shall consider first the influence of a weak magnetic field upon the longitudinal and transverse plasmon turbulence. After that we shall briefly consider also the case of strong magnetic fields when $\omega_{pe} \ll \omega_{He}$. Although the plasma turbulence in the solar corona does not have this character, this case can easily occur for other bodies in the solar system, for instance, in a planetary ionosphere with a strong magnetic field such, as exists, for instance, near Jupiter.

It follows from equation (1.23) that when $\omega_{pe} \gg \omega_{He}$ the influence of the magnetic field on the longitudinal plasmons can be taken into account by replacing the quantity ω_{pe}^2 in the dispersion relation by $\omega_{pe}^2 + \omega_{He}^2 \sin^2 \vartheta$. This apparently small correction leads, however, to a considerable change in the character of the turbulence. Indeed, the main peculiarity of the non-linear energy transfer along the turbulence spectrum consists in the fact that in such a transfer process the frequencies get lower. If there is no magnetic field a decrease in frequency is possible only when the wavenumber decreases. In a magnetic field the longitudinal plasmon frequency can drop also because the angle ϑ decreases, that is, when the plasmon wavevectors are aligning themselves along the magnetic field.

In other words, the following happens in plasma turbulence when $\omega_{pe} \gg \omega_{He}$. Longitudinal plasmons created with a large value of the wavenumber initially go to lower values of k through non-linear transfer processes until the quantities $\omega_{He}^2 \sin^2 \vartheta$ and $3v_{Te}^2 k^2$ become comparable. Non-linear transfer then begins to decrease the magnitude of ϑ so that the turbulence at lower wavenumbers will tend to a state in which all wavevectors are parallel (or antiparallel) to the magnetic field. One can assume to a first approximation that the longitudinal plasmon turbulence retains an isotropic character for wavenumbers larger than

$$k_H \approx \frac{\omega_{He}}{\sqrt{3} v_{Te}} \approx \frac{\omega_{He}}{\omega_{pe}} \frac{1}{\sqrt{3} \cdot d_e}, \tag{8.41}$$

while for smaller wavenumbers it is one-dimensional to a first approximation. Of course over the whole of the spectrum there is transfer to smaller values of k.

If $k_H > k_g$ the plasma turbulence is almost one-dimensional and the plasmons are confined to a small angle $\Delta\theta_1$ around the magnetic field. The equations obtained in the previous section remain valid also here except that the intensity of the emission at $2\omega_{pe}$ is decreased by a factor $(\Delta\theta_1)^2$. The remaining changes practically reduce to small corrections in the numerical coefficients (for instance, non-linear effects increase by a factor 3 because $(\boldsymbol{k} \cdot \boldsymbol{k}_1)^2 / k^2 k_1^2 \approx 1$ while in the isotropic case the average of this expression is $\frac{1}{3}$). The magnetic field strength in the corona is unknown, but if we put $H \approx 1$ Oe, we have $\omega_{He} \approx 1{\cdot}8 \times 10^7$ s^{-1} and $k_H \approx (20 d_e)^{-1}$ which is comparable with k_g, but appreciably less than $k \sim (4 d_e)^{-1}$

corresponding to the emission at a frequency ω_{pe}. The magnetic field therefore suppresses the emission at frequencies $\sim 2\omega_{pe}$ more strongly than the emission at frequencies ω_{pe}. However, the magnitude of the magnetic field affecting the turbulence must, apparently, be larger than 1 Oe as a field of the order of 1 Oe is created by the current of the beam itself.

The dispersion relation for transverse plasmons is (see (1.10))

$$\omega^2 = \frac{\omega_{pe}^2}{1 \pm (\omega_{He}/\omega_{pe}) \cos \vartheta} + c^2 k^2, \tag{8.42}$$

when the magnetic field is taken into account. Decreasing the frequency is accompanied by a change in the orientation of the wavevector with respect to the magnetic field. The ordinary wave (plus sign) tends to turn its wavevector along the magnetic field (increase in $\cos \vartheta$) while the extraordinary wave (minus sign) turns its wavevector at right angles to the field. As the probability for the conversion in a non-linear scattering process is proportional to $([\boldsymbol{k}_p \wedge \boldsymbol{k}_l] \cdot [\boldsymbol{k}_p \wedge \boldsymbol{k}_l])$ it follows from this that extraordinary transverse plasmons have a larger probability of being excited in the case of one-dimensional longitudinal plasmon turbulence.

In general, the whole theory of the polarization of the emission of transverse plasmons generated by one-dimensional turbulence in a relatively weak magnetic field still needs detailed attention taking into account the wave propagation in a magnetized plasma.

We shall now say a few words about the case $\omega_{He} \gg \omega_{pe}$. Tsytovich and Kaplan (1969) have given a detailed study of non-linear processes in such a plasma. Tsytovich (1971a, 1972b) has considered the turbulence of gyro-frequency plasmons. We shall here give only the main results restricting ourselves to the case of one-dimensional turbulence when the wavevectors of all longitudinal (including the gyro-frequency) plasmons are directed along the magnetic field.

The growth rate of the beam instability for the excitation of gyro-frequency plasmons, that is, at a frequency ω_{He}, is determined by the equation

$$\frac{\gamma^h}{\omega_{He}} \approx \frac{n_*}{n_e} \left(\frac{\omega_{pe}}{\omega_{He}}\right)^4 \left(\frac{v_0}{\Delta v_0}\right)^2. \tag{8.43}$$

One can evaluate by the usual methods the transfer of energy along the gyro-frequency plasmon spectrum and from the gyro-frequency plasmons to electromagnetic waves and vice versa. It is best to refer here the spectral functions not to wavenumber intervals, but to frequency (ω) intervals. More precisely, we shall take the functions W^h and W^t with respect to the quantity

$$\Delta\omega = \omega - \omega_{He} - \frac{1}{6} \frac{\omega_{pe}^2}{\omega_{He}}. \tag{8.44}$$

We get for the rate of transfer along the gyro-frequency plasmon spectrum (analogous to (3.17)) the estimate

$$\frac{\gamma^h(\mathrm{h}+\mathrm{i} \rightleftharpoons \mathrm{h}'+\mathrm{i}')}{\omega_{He}} = \frac{\pi}{2(1+T_e/T_i)^2} \frac{\omega_{pe}^4}{n_e m_i v_{Te}^2 \omega_{He}^3} \frac{\partial}{\partial(\Delta\omega)} [\Delta\omega W^h_{\Delta\omega}]. \tag{8.45}$$

The rate at which gyro-frequency plasmons transfer to electromagnetic waves is:

$$\frac{\gamma^{t}(h+i \rightleftharpoons t+i')}{\omega_{He}} = \frac{\pi}{2(1+T_e/T_i)^2} \frac{\omega_{pe}^4}{n_e m_i v_{Te}^2 \omega_{He}^3} \Delta\omega \frac{\partial}{\partial(\Delta\omega)} W_{\Delta\omega}^{h}. \tag{8.46}$$

In the case of a quasi-stationary spectrum the energy is continuously transferred from gyro-frequency plasmons to electromagnetic waves and vice versa. The general scheme is similar to the case $\omega_{pe} \gg \omega_{He}$. One can find the corresponding spectra from the condition $\gamma^{t} \approx \gamma^{h} \approx 0$ whence we get for the gyro-frequency plasmons:

$$W_{\Delta\omega}^{h} = \frac{\text{constant}}{\Delta\omega}, \quad W_{k}^{h} = \frac{\text{constant}}{k}. \tag{8.47}$$

For the electromagnetic waves we get a white spectrum

$$W_{\Delta\omega}^{t} \approx \text{constant}. \tag{8.48}$$

The magnitude of the constants is determined by the power of the sources generating the turbulence.

It turns out in the general analysis of gyro-frequency plasmon turbulence that here all linear and non-linear processes occur with a smaller efficiency than the analogous processes for a plasmon with $\omega \approx \omega_{pe}$. Moreover, the role of cyclotron absorption is here also important.

The following is important. There are always two plasmon branches in a plasma in a strong magnetic field: one with a frequency ω_{He} and a second one with a frequency $\approx \omega_{pe}|\cos\vartheta|$. Apart from factors of order unity all relations obtained for the normal Langmuir waves are valid for the second branch. This means that if an electron beam is incident upon a plasma with a strong magnetic field it will first excite the longitudinal plasmon branch with $\omega \approx \omega_{pe}|\cos\vartheta|$. An unstabilized beam transfers practically all its energy to Langmuir turbulence and the gyro-frequency turbulence level turns out to be low, practically thermal. A stabilized beam can also excite gyro-frequency turbulence to an appreciable level as now the ratio of the growth rate of the beam instability to the transfer rate is of the same order of magnitude as for Langmuir turbulence.

Longitudinal plasmons with a frequency $\omega^{l} \approx \omega_{pe}|\cos\vartheta|$ can decrease their frequency in non-linear transfer by decreasing $\cos\vartheta$, that is, by an orientation at right angles to the field. The decrease in frequency is limited by the condition

$$\omega^{l} \gtrsim \sqrt{(\omega_{He}\omega_{Hi})} \approx \sqrt{\frac{m_e}{m_i}}\,\omega_{He}. \tag{8.49}$$

Such plasmons are rapidly absorbed. In several papers (see, for instance, Kovner, 1960, 1961; Shapiro and Shevchenko, 1968; Gershman and Trakhtengerts, 1966) one can find a detailed study of beam instability and quasi-linear relaxation in a plasma in a strong magnetic field.

§ 9. Electromagnetic radiation from ion-sound turbulence

It was possible to make progress in the theory of type III bursts mainly because for that effect the source of the turbulence is clearly observed: a beam of fast particles. In other words, the particle acceleration and the creation of turbulence by them are in type III bursts spatially separated. The other kinds of sporadic solar radio-emission do not have this property. For them the particle acceleration and the excitation of longitudinal and transverse plasmons occurs in the same region or, at least, in spatially neighbouring regions.

One must bear in mind that in the conditions of the solar corona the generation of strong electromagnetic radiation at frequencies $\sim\omega_{pe}$ must precede the excitation of longitudinal and transverse plasmon turbulence. Moreover, the presence of a rather large number of electrons with velocities exceeding $3v_{Te}$ is necessary for the stimulated emission of longitudinal plasmons. Apparently, the process of accelerating electrons up to such velocities and the subsequent excitation of longitudinal and transverse plasmon turbulence by them also occurs in type II bursts and possibly also in other slow-drift or stationary bursts.

One can distinguish two basic acceleration mechanisms. One of them, the stochastic one, was considered in detail in § 5. Fast particles absorbing and emitting plasma waves increase their velocity while diffusing along the "momentum axis". The second acceleration mechanism is connected with the occurrence in the plasma of a sufficiently large electric field strength. Both mechanisms are important for plasma astrophysics but their action is different.

The stochastic mechanism is effective when charged particles are in a plasma turbulence region for a sufficiently long time. This occurs, for instance, in plasma reactors, which will be considered in detail in later chapters, or in magnetic bottles. The acceleration by an electric field does not require the containment of the particles in a closed region during a long period but, on the other hand, one needs for it rather large gradients of the magnetic field to create the appropriate electric field. In the conditions of the solar corona the stochastic acceleration mechanism is effective only in magnetic trap regions with a sufficiently high level of plasma turbulence energy. The particle acceleration by an electric field is, apparently, more probable in those parts of the solar corona and the chromosphere where there are strong magnetic field gradients.

Fast electrons in shock waves

In a shock wave in the conditions of the solar corona there appears an induction electric field the magnitude of which can be determined as follows. The change in the magnetic field in the shock wave—and in general in any system in which it occurs—is connected with the density of the current $\boldsymbol{j}$ flowing in the plasma by the following equation

$$\operatorname{curl} \boldsymbol{H} = \frac{4\pi}{c}\boldsymbol{j} = \frac{4\pi}{c}\sigma\boldsymbol{E} = \frac{4\pi e n_e}{c}\boldsymbol{u}, \tag{9.1}$$

where $\boldsymbol{u}$ is the drift velocity of the electrons with respect to the ions.

We considered the plasma conductivity in detail in § 4. We saw that if u is larger than the ion sound velocity v_s strong ion-sound turbulence is developed with an energy density which can become 0·1 to 0·2 times the thermal energy density of the plasma.

If for some reason in the chromosphere and the solar corona there is formed a magnetic field gradient such that

$$\frac{\partial H}{\partial l} \gtrsim \frac{4\pi e n_e}{c} v_s, \tag{9.2}$$

ion-sound turbulence will then unavoidably occur and, moreover, there will be an electric field with a field strength determined by equations (4.40) and (9.1):

$$eE = \frac{10^{-2}}{4\pi} \frac{c^2}{v_{Te}\omega_{pe} n_e} \left(\frac{\partial H}{\partial l}\right)^2. \tag{9.3}$$

Equation (9.3) is valid as long as the field strength E satisfies inequality (4.43). This field accelerates electrons. At the same time, the acceleration is impeded by the collisions of the electrons both with the ions and with the ion-sound plasma turbulence waves.

If the turbulence is developed the collisions with ions can, of course, be neglected and the electrons are decelerated only through being scattered by ion-sound waves. The frequency of such collisions was determined by equation (4.39) for a thermal electron while for an electron moving with a velocity $v > v_{Te}$ we have

$$\nu_s(v) = \frac{\omega_{pe}}{100} \frac{u}{v_{Te}} \left(\frac{v_{Te}}{v}\right)^3 = \left(\frac{v_{Te}}{v}\right)^3 \nu_{eff}. \tag{9.4}$$

"Collisions" of charged particles with ion-sound waves have the same velocity dependence as the usual Coulomb collisions. The force acting on the electron due to the field is eE and the friction force is $-m_e v \nu_s(v)$. The electron becomes a "run-away" electron, if

$$m_e v \nu_s(v) = m_e v_{Te} \left(\frac{v_{Te}}{v}\right)^2 \nu_{eff} \leqslant eE. \tag{9.5}$$

We can replace the quantity eE by $m_e u \nu_{eff}$ as the drift velocity is just determined by the condition that the deceleration of the main mass of electrons equals their acceleration. Hence we find that electrons with a velocity larger than the value

$$v_r = v_{Te} \left(\frac{v_{Te}}{u}\right)^{1/2} \tag{9.6}$$

turn out to be "run-aways" (Dreicer, 1960; Gurevich, 1960) not only with respect to the usual electron-ion collisions, but also with respect to collisions with ion-sound turbulence waves.

If there are no other deceleration mechanisms for electrons with $v > v_r$ they will be accelerated, after they have passed through the ion-sound turbulence region, to energies

$$\varepsilon = eER \approx \frac{eEH}{\partial H/\partial l} \approx \frac{10^{-2} c^2}{v_{Te}\omega_{pe} n_e} \frac{\partial}{\partial l}\left(\frac{H^2}{8\pi}\right), \tag{9.7}$$

if the size of the turbulent region is of the order of the scale of the magnetic field gradient.

However, this usually does not happen. The fact is that the electrons considered move predominantly in one direction—along the electric field—and thus form a beam which strongly excites longitudinal plasmon turbulence, as the velocity of the accelerated electrons $v_r > v_{Te}$.

The final result of the whole process is thus the excitation of a rather strong Langmuir turbulence in a region where ion-sound turbulence had already been excited. Let us estimate its parameters. For the growth rate we have $\gamma^l \approx \omega_{pe} n_r/n_e$, where n_r is the density of the "run-away" electrons. We also assumed that $\Delta v_r \approx v_r$. We can neglect collisional damping. If the velocity of the accelerated electrons $v_r \approx v_{Te}(v_{Te}/u)^{1/2} > 3v_{Te}$, that is, $u < v_{Te}/9 \approx 5v_s$, we can also neglect Landau damping.

The main absorption mechanism for Langmuir waves in the field of a strong ion-sound turbulence (when $N_k^l \ll N_k^s$) is the stimulated conversion of Langmuir waves into ion-sound waves through the non-linear scattering by the thermal electrons in the plasma. This conversion is accompanied by a fast heating of the electron gas as almost all the energy of a longitudinal plasmon is transferred to the scattering electron. The damping rate for this process is

$$\frac{\gamma^l(l+e \rightleftharpoons s+e')}{\omega_{pe}} = \frac{-\sqrt{(2\pi)}}{12}\int \frac{\omega_{pe}}{k_1 v_{Te}} \exp\left[-\frac{1}{2}\left(\frac{\omega_{pe}}{k_1 v_{Te}}\right)^2\right] \frac{W_{k_1}^s\, dk_1}{n_e T_e} \approx 10^{-2}\frac{W^s}{n_e T_e}. \quad (9.8)$$

The magnitude of the damping rate is practically independent of the longitudinal plasmon wavenumber but the stimulated scattering process occurs with participation of ion-sound waves which have large wavenumber values and which have about 0·1 of the total ion-sound turbulence energy; this has been taken into account in the numerical estimate.

The excitation of Langmuir turbulence by electrons accelerated by an electric field is thus possible only when the density of those electrons satisfies the inequality

$$\frac{n_r}{n_e} > 10^{-2}\frac{W^s}{n_e T_e} \approx 10^{-3}. \quad (9.9)$$

Unfortunately it is at the present difficult to give a reliable estimate of the quantity n_r. It depends on the fast particle distribution function and under different conditions (shock waves in strong or weak magnetic fields) is determined differently. One can probably for a first, rough estimate assume that the fast particle distribution function remains Maxwellian. In that case

$$\frac{n_r}{n_e} = \exp\left[-\frac{1}{2}\left(\frac{v_r}{v_{Te}}\right)^2\right] = \exp\left(-\frac{1}{2}\frac{v_{Te}}{u}\right). \quad (9.10)$$

This quantity changes from 10^{-9} for $u \approx v_s$ to 0·6 for $u \approx v_{Te}$. For the most probable value $u \approx 0{\cdot}1 v_{Te}$ (neglecting Landau damping) we have $n_r \approx 10^{-2} n_e$. Such a value of the relative density of the fast particles has also been observed in laboratory apparatus in experimental studies of the anomalous resistivity with $u \approx 4$ to $10v_s$. We shall take this estimate, $n_r \approx 10^{-2} n_e$, in what follows.

We shall now show that the unimpeded acceleration of electrons by the field (9.3) is possible only for very large gradients. It is clear that it is necessary for this effect that the characteristic time of acceleration, $t \approx m_e v_r/eE$ be smaller than the characteristic time for the excitation of Langmuir turbulence, $n_e/n_r\omega_{pe} \approx 100/\omega_{pe}$. We get thus from (9.3) an estimate

for the magnetic field gradient which will produce accelerated electrons which in the full sense of the word are "run-aways":

$$\frac{\partial H}{\partial l} > \frac{4\pi}{c} n_e e \sqrt{(v_r v_{Te})}. \tag{9.11}$$

In the actual conditions of the solar corona the right-hand side of (9.11) is of the order of 0·1 Oe/cm which is larger than the steepest observed gradients.

The energy density of the Langmuir-waves can be estimated from the balance of the energy which, on the one hand, is gained by an electron when it is accelerated in the field (9.3) and, on the other hand, is lost through the excitation of plasma waves and through the conversion and absorption of the plasma waves. We shall not consider here this problem, but restrict ourselves to give an estimate, assuming that the whole excess of energy of the run-away electrons above the thermal energy is converted into energy of longitudinal plasmons. Of course, this gives us only a rough estimate of the orders of magnitude involved. In this way we have:

$$W^l \approx \frac{1}{2} n_r m_e (v_r^2 - v_{Te}^2) \approx \frac{1}{2} n_r T_e \left(\frac{v_{Te}}{u} - 1 \right) \approx 3 \times 10^{-2} n_e T_e, \tag{9.12}$$

if we take $v_{Te} \approx 9u$. Apparently, as a rule, the excitation of ion-sound turbulence is accompanied by the generation of Langmuir turbulence with an energy density an order of magnitude smaller than that of the ion-sound turbulence. The longitudinal plasmon wave-numbers are rather large, $k_g \approx \omega_{pe}/v_r \approx (\omega_{pe}/v_{Te})\ (u/v_{Te})^{1/2}$, that is, the phase velocities are only a few times larger than the electron thermal velocities.

Emission at plasma frequencies from regions of magnetic field gradients (shock waves)

The process of energy transfer from longitudinal to transverse plasmons is in the case discussed here appreciably different from the case considered in § 8. A stabilized beam creates a more or less stationary Langmuir plasmon spectrum which exists for a time which is appreciably longer than the inverse of the growth rate. Here we have no stabilization whatever and the particle acceleration is therefore accompanied by a fast quasi-linear relaxation. The excitation of particles and the excitation of longitudinal plasmon turbulence proceeds thus fast, in a time of order $100/\gamma^l \approx 10^4/\omega_{pe}$ in the case considered. In such a short time it is impossible for the stimulated transfer of energy from longitudinal to transverse plasmons and for the total isotropization to get established. A more or less one-dimensional (or rather, a one-sided) plasmon spectrum is formed with the plasmons moving approximately in the direction of the electric field, with a wide spread, however. They form gradually a spread-out "turbulent trail" behind the shock wave—due to the relatively large group velocities $\sim v_{Te}/3$. In this trail turbulence is not greatly generated and the spectrum changes gradually due to non-linear interactions leading to a transfer to the smaller k-region. Isotropization proceeds even faster. The first process decreases the emission coefficient and the second increases it as stimulated emission becomes important. We need therefore a rather complicated evaluation even for a qualitative estimate. In the case of

a one-dimensional spectrum the transverse plasmon emission coefficient is in this case therefore determined by equation (3.13) for spontaneous transfer:

$$J^{\mathrm{p}}_{k_{\mathrm{p}}}(\mathrm{l}+\mathrm{i} \rightleftharpoons \mathrm{p}+\mathrm{i}') \approx 10^{-2}\frac{\omega^3_{\mathrm{pe}}W^{\mathrm{l}}}{n_e c^2}. \tag{9.13}$$

Hence we have for the radiation intensity, accumulated over a length R:

$$I^{\mathrm{p}}_{\omega} \approx \frac{J^{\mathrm{p}}_{k_{\mathrm{p}}}R}{c} \approx 10^{-2}\frac{\omega^3_{\mathrm{pe}}}{n_e c^3}W^{\mathrm{l}}R. \tag{9.14}$$

Taking non-linear transverse plasmon absorption mechanisms, such as their non-linear conversion into ion-sound waves through scattering by the plasma electrons (the process $\mathrm{p} \rightleftharpoons \mathrm{l}\pm\mathrm{s}$) with a damping rate also equal to (9.8), into account is important only in the ion-sound turbulence region, but not in the "turbulent trail".

Let us now determine the radiation intensity at twice the plasma frequency. This radiation mechanism is the same as the one considered in § 8 but we must now take into accout-that the longitudinal plasmon wavenumbers are relatively large (we take $k_{\mathrm{l}} \approx \omega_{\mathrm{pe}}/3v_{Te}$). We then get for the emission coefficient per unit volume from (3.30):

$$J^{\mathrm{t}}(\mathrm{l}+\mathrm{l}' \rightarrow \mathrm{t}) \approx 10^2\frac{\omega_{\mathrm{pe}}v^3_{Te}}{n_e m_e c^5}(W^{\mathrm{l}})^2\frac{v^2_{Te}}{100c^2}, \tag{9.15}$$

where the extra factor $v^2_{Te}/100c^2$ takes into account what fraction of the waves will take part in the fusion due to their one-sided distribution.

Hence we find for the radiation intensity (as before we take for the width of the frequency interval $\Delta\omega \approx \omega_{\mathrm{pe}}$):

$$I^{\mathrm{t}}_{\omega} \approx \frac{J^{\mathrm{t}}R}{\omega_{\mathrm{pe}}} \approx \frac{v^5_{Te}(W^{\mathrm{l}})^2}{n_e m_e c^7}R. \tag{9.16}$$

We have also used here condition (9.12). We shall continue the discussion of these relations in the next section.

We must stress that the change in energy of the electric field generated by the magnetic field gradients into electromagnetic radiation occurs only through several stages. First of all ion-sound turbulence is created, then particles are accelerated, after that Langmuir turbulence is produced, and, finally, the longitudinal plasmons are changed into transverse ones.

The problem arises: might it be possible to reduce the number of these energy conversion stages? Might it not be possible, for instance, for the ion-sound turbulence to be converted directly into electromagnetic radiation.

If such a conversion took place through scattering by thermal plasma electrons, it would follow from the conservation law

$$\omega^{\mathrm{p}} = \omega^{\mathrm{s}} + ([\boldsymbol{k}_{\mathrm{p}} - \boldsymbol{k}_{\mathrm{s}}]\cdot\boldsymbol{v}) \approx k_{\mathrm{s}}v_{Te} \tag{9.17}$$

that in that process only ion-sound waves with the largest wavenumbers can take part and their number is relatively small. One can, however, easily verify that it is totally

impossible to obtain through this process a radiation intensity well above the thermal level without taking the fast particles into account.

Indeed, as the scattering process here must occur with increasing frequency the transfer of energy from ion-sound waves to transverse plasmons through scattering by thermal electrons must be spontaneous. In other words, thermal electrons excited by ion-sound oscillations must emit spontaneously in this effect. It is well known that in such a case the effective temperature of the radiation cannot exceed the kinetic temperature of the particles. The opposite conclusion (about the possible efficient conversion of ion-sound into electromagnetic waves through scattering by thermal fluctuations) was reached in some astrophysical studies, but it was based upon neglecting the reverse conversion of electromagnetic into ion-sound waves which is here very important. It is true that taking into account the electron drift in the electric field makes the stimulated conversion of ion-sound waves into transverse ones possible as transfer with increase of frequency is possible in a non-Maxwellian plasma but a detailed study shows that this effect is also insufficient as long as the drift velocity $u < v_{Te}$.

The stimulated transfer with increase in frequency is made possible by the presence in the ion-sound turbulence region of fast ("run-away") electrons, as their velocity $v_r > v_{Te}$. The fact that the frequency of the electromagnetic waves can then appreciably exceed ω_{pe} is here also important. Indeed, if we substitute into (9.17) instead of v the minimum velocity of the fast electrons, v_r, and put $k_s \approx \omega_{pe}/v_{Te}$ we get for the frequency of the electromagnetic radiation generated through the conversion of ion-sound waves by "run-away" electrons:

$$\omega^t \approx \omega_{pe}\left(\frac{v_{Te}}{u}\right)^{1/2} \lesssim 6{\cdot}5\omega_{pe}. \tag{9.18}$$

The emission coefficient for such a conversion is determined by the fast particle density. We can obtain the following estimate:

$$J_k^t(s+e \to t+e') \approx 10^{-3}\frac{n_r}{n_e}\frac{v_{Te}}{u}\left(\frac{v_{Te}}{c}\right)^2\frac{m_e\omega_{pe}\omega^2}{(2\pi)^3}\frac{W^s}{n_eT_e}. \tag{9.19}$$

The radiation intensity is determined by the time the accelerated electrons exist, that is, by the quantity $10^4/\omega_{pe}$. As a result we get

$$I_\omega^t \approx \frac{10^4 J_k^t}{4\pi\omega_{pe}} \approx 10\frac{v_{Te}}{u}\left(\frac{v_{Te}}{c}\right)^2\frac{W^s}{n_eT_e}\frac{m_e\omega^2}{(2\pi)^3} \approx 10^{-3}\frac{m_e\omega^2}{(2\pi)^3}. \tag{9.20}$$

This quantity is appreciably smaller than the expressions (9.14) and (9.16) given earlier but it is important to note that here a wide frequency spectrum is emitted, $\omega_{pe} < \omega \lesssim 6\,\omega_{pe}$. The intensity can also be larger if the number of fast particles is restocked constantly.

Accumulation of particles in magnetic traps by ion-sound turbulence

The magnetic field configurations in the Sun make it possible to form magnetic traps. The accumulation of particles in traps can explain a number of solar activity phenomena. We shall describe mechanisms which make it possible to fill the traps with particles by strong ion-sound turbulence.

We must first of all note the following. The ion-sound wave frequencies are the same as the whistler frequencies (see Fig. 1 on p. 24). Ion-sound wave energy can thus through non-linear scattering by thermal ions and electrons be changed into whistlers. An estimate for the growth rate of this transfer gives:

$$\frac{\gamma^{w}(s+i \to w+i')}{\omega_{He}} \approx 0{\cdot}1 \frac{4\pi W^{s}}{H^{2}} \approx 0{\cdot}1 \frac{n_e T_e}{H^{2}}. \tag{9.21}$$

The transfer occurs strongly and without obstacles as it is accompanied by a decrease in frequency.

The whistlers in turn are absorbed by particles. It is here important to take into account that the polarization of whistlers is such that their absorption untwists the electron motion at right angles to the magnetic field, in other words, only the transverse particle energy increases. This means that a particle originally moving approximately along the magnetic field and therefore not being contained in the trap, can acquire an appreciable velocity component perpendicular to the field which causes its reflection from the bunching of the field lines at the ends of the trap.

The rate of change in the transverse energy is determined by an equation which follows from a formula analogous to (5.13):

$$\frac{d\varepsilon_{\perp}}{dt} = \frac{D_{\parallel}^{w}}{m_e} \approx \frac{\pi}{6} \frac{\omega_{He}^{2} W^{w}}{n_e \omega_{pe}}. \tag{9.22}$$

The accumulation of particles in traps is obstructed by the ion-sound waves which try to make the particle velocity directions isotropic. The corresponding change in the transverse energy is equal to

$$\frac{d\varepsilon_{\perp}}{dt} = \frac{D_{\perp}^{s}}{m_e} = \frac{\pi}{4} \frac{\omega_{pe} W^{s}}{n_e} \frac{v_{Te}}{v}. \tag{9.23}$$

Comparing (9.22) and (9.23) we find the condition under which the accumulation in traps is possible by means of ion-sound turbulence:

$$\frac{W^{w}}{W^{s}} \gg \frac{\omega_{pe}^{2}}{\omega_{He}^{2}} \frac{v_{Te}}{v}. \tag{9.24}$$

Under the conditions in the solar corona this inequality can be satisfied in regions with a strong magnetic field where $\omega_{pe} \approx \omega_{He}$ and $W^{w} \approx W^{s}$. Of course, the faster the particles, the more easily they are confined in the trap. The condition $\omega_{pe} \approx \omega_{He}$ is, by the way, also necessary in order that there is no appreciable scattering due to diffusion in "pitch"-angle (see Trakhtengerts, 1971).

Assuming that during its flight through the trap of size R the particle changes its transverse energy by an amount of the order of its total energy we get a second criterion for the confinement of the particle in the trap:

$$\frac{\omega_{pe} v^{3}}{\omega_{He}^{2} R v_{Te}^{2}} \lesssim \frac{W^{w}}{n_e T_e} \lesssim \frac{W^{s}}{n_e T_e} \lesssim \frac{v^{4}}{\omega_{pe} R v_{Te}^{3}}. \tag{9.25}$$

We shall give some numerical estimates in the next section.

§ 10. Discussion of the theory of solar radio-bursts

In the preceding two sections we obtained a number of relations which allow us to interpret at least the main observational properties of solar radio-bursts. It is possible that the assumed interpretation is non-unique and that it will have to be reconsidered in the light of further observational data. All the same, the problems considered are of independent interest from the point of view of processes in plasma physics occurring in cosmic conditions. In what follows we shall discuss the correspondence between theory and observations and also briefly dwell upon alternative interpretations of the same phenomena. We shall consider here only some of the more recent research based already on plasma turbulence theory. A detailed analysis of the theory of the solar radio-emission, in particular, a complete study of the regular (non-turbulent) transformation of plasma waves into electromagnetic radiation as it was developed up to 1964 can be found in Zheleznyakov's treatise (1970a).

Type III, V, and U-radio bursts

The first radio-astronomical papers by Shklovskiĭ (1946), Ginzburg (1946), Martin (1947), and Wild (1950) have discussed the assumption considered in § 8 that type III radio-bursts are excited by a beam of fast electrons ejected from a chromospheric flare region. Ginzburg and Zheleznyakov (1959) were the first to develop a quantitative theory. However, at that time the effect of quasi-linear relaxation of beams was not known and the very important problem of the free passage of the beam through the corona was thus not discussed. Taking quasi-linear relaxation effects into account requires a considerable reconsideration of the theory of type III bursts. Sturrock (1966; see also Kaplan and Tsytovich, 1968a) was the first to draw attention to this fact.

Up to the present the development of the theory of type III bursts has proceeded in two directions: on the one hand, the possibility of exciting radio-emission when quasi-linear relaxation is taken into account, that is, assuming a fast dissipation of the beams, has been analysed, while, on the other hand, the possibility for a stabilization of beams in the solar corona has been considered.

In the opinion of the authors of the present book a number of observational data on type III bursts indicate that there is no quasi-linear relaxation. These considerations can be formulated as follows.

1. Even the maximum possible length for quasi-linear relaxation of beams is two to three orders of magnitude smaller than the path which is in fact traversed by beams in the corona.
2. In practically all observed cases the beams pass through the corona with an approximately constant average velocity.
3. No appreciable increase in either the longitudinal size (due to an increase in the velocity spread when a plateau is formed) or the transverse size (due to angular relaxation) of the emission sources has been observed.

The interpretation of given observations is, however, generally not unique and the possibility that quasi-linear relaxation plays an important role can therefore not be excluded completely.

Let us first consider the possibility for beam stabilization. Sturrock (1964) assumed in his first model for type III bursts in which the stabilization effect was taken into account that it may be connected with processes involving the fusion of longitudinal plasmons with ion-sound waves which may be present in active regions of the corona. Incidentally, radar studies of the Sun possibly indicate that there may be a more or less constant level of ion-sound turbulence in the corona solar. The energy level density of ion-sound turbulence necessary to stabilize the beam can be estimated when we compare the growth rate of the beam instability with the pumping rate for the process $l+s \rightleftharpoons l'$, using (9.8). We find in this way

$$\frac{W^s}{n_e T_e} \gtrsim 10^2 \frac{n_*}{n_e} \approx 10^{-5}. \tag{10.1}$$

A disadvantage of this model is the requirement that the ion-sound turbulence level must be constant along the whole path of the beam.

Several authors (Kaplan and Tsytovich, 1968a; Zheleznyakov and Zaĭtsev, 1970; Smith and Fung, 1971) have considered the stabilization through non-linear scattering by ions. We gave the results of the first-mentioned paper supplemented with a few new considerations in § 8. As we emphasized there the characteristic times for quasi-linear relaxation and non-linear stabilization evaluated in some rough models turn out to be of the same order of magnitude in the conditions of the solar corona, and they are considerably shorter than the observed time during which the beam passes through the corona. As we gave earlier in detail the physical picture of the processes occurring in the case of stabilization we shall start here with a discussion of some papers (Zheleznyakov and Zaĭtsev, 1970; Zaĭtsev, Mityakov, and Rappoport, 1972) in which an attempt was made to interpret type III bursts in the framework of assuming an important role for quasi-linear relaxation and neglecting non-linear processes.

Zheleznyakov and Zaĭtsev (1970) assumed the following scheme for generating plasma waves in dense regions of the solar corona ($\nu_e > v_0/R$, where v_0 is the fast electron beam velocity and R the size of the region occupied by these electrons). They assumed that the quasi-linear relaxation had already smeared out the electron beam so that the constant plateau on their distribution function had been reached. Taking into account the "running-away" of the fastest electrons at the leading front of the current leads again to a beam instability which again relaxes rapidly succeeding, however, in exciting a certain energy level of Langmuir waves in the resonance region of the spectrum. As a result there is formed, according to Zheleznyakov and Zaĭtsev (1970), a quasi-stationary state in which the growth rate of the beam instability all the time remains of the order of the effective collision frequency ν_e. They get the following estimate for the plasma wave energy density in that case:

$$W^l \approx n_* m_e v_0^2 \frac{v_0}{\nu_e R}. \tag{10.2}$$

In the corona ν_e decreases rapidly as the beam moves outwards but if then through quasi-linear relaxation the size R increases and the number of fast "run-away" electrons decreases,

we may assume that W^1 is also a slowly varying quantity. One must in this scheme prevent quasi-linear relaxation in angles. This cannot be done in a sufficiently strong magnetic field (Shapiro and Shevchenko, 1968), $\omega_{He} \gg \omega_{pe}$, as the cyclotron instability growth rate describing in this case the relaxation in angles is only $(\omega_{pe}/\omega_{He})^2$ times less than the growth rate along the field. In a weak field ($\omega_{pe} \gg \omega_{He}$) one cannot prevent the angular relaxation, and this relaxation effect in angles just leads to a supression—vanishing—of the beam. One might assume, but this was not discussed by Zheleznyakov and Zaĭtsev (1970), that right from the beginning the beam occurs with such a density that γ is only slightly larger than ν_e in the direction of the beam. However, the growth rate for the generation of lateral waves is somewhat smaller and for them $\gamma < \nu_e$. This point of view is possible, but it is, however, necessary to assume that the density of particles falls all the time as ν_e varies strongly along the beam path.

In regions of the corona with a low density such a quasi-stationary state with $\gamma^1 \approx \nu_e$ does not occur and here a simple expansion of the fast electron region will take place according to Ryutov and Sagdeev (1970) and Zaĭtsev, Mityakov and Rappoport (1972), which can be described in the framework of a similarity solution. In the whole of the expansion region the electron flux generates plasma waves but the maximum of the energy density shifts with a constant velocity depending on their initial average energy. The plasma-wave energy density decreases with increasing distance from the flare. All these considerations are made in the framework of a one-dimensional model (motion of the electrons along the magnetic field and generation of one-dimensional plasma waves by them), but a three-dimensional expansion model apparently contradicts the observations. Moreover, the magnetic fields in the corona are not so strong that such an expansion would become one-dimensional. One can merely emphasize that the quasi-linear relaxation hypothesis in its present form loses all advantages of a beam interpretation and one is already dealing with emission from some turbulent region in which the maximum emission intensity shifts with a constant velocity but not the sources exciting the plasma turbulence.

In the opinion of the authors of the present book much more probable is a scheme in which one considers the non-linear stabilization of the beam instability (we gave earlier in this section the observational data in favour of this idea). We discussed in § 8 the theoretical possibility of such a stabilization; we shall now give a few additional considerations.

As we noted in § 8 the stabilization by non-linear processes gives a steep dependence of the effect on the beam velocity. Therefore, an insignificant change in the beam velocity by, say, a factor 2, changes the estimate of the efficiency of the non-linear effects by a factor 16 or even 64. We must also note that the estimates of the non-linear processes contain factors of order unity. Even in a weak magnetic field the tendency to "one-dimensional" turbulence because of non-linear effects increases their intensity by a factor three for the same total turbulence energy. It turns out that scattering by ions, neglecting the accumulation effect, at once leads to the fact that for the observed beam velocities the non-linear and the quasi-linear effects are at any rate comparable as to order of magnitude. Changing the beam parameters, in particular, for a beam velocity $v \gtrsim 0{\cdot}5c$, we find that with a large margin stabilization by ions occurs (Kaplan and Tsytovich, 1968a) although for lower values of the observed velocities we find that there is no stabilization by ions (Zheleznyakov and Zaĭtsev, 1970). Of course, the existing theory and observations are not very accurate

and it is impossible to trust factors 2 and it is even more impossible to derive categorical conclusions from these estimates. It seems to us that at present one must in the first instance pay attention to qualitative conclusions from the theory and compare them with observations.

For instance, the theory of the stabilization of a beam instability shows that there is a lower threshold of beam velocities (critical stabilization velocity) in clear agreement with observational data. Incidentally, the determination of the observed lower limit of velocities of $\sim 0{\cdot}2c$ is connected with some uncertainties because of insufficiently certain knowledge of the behaviour of the electron density in the active regions of the Sun. There is no lower limit on the velocities in the quasi-linear theory.

It is, for instance, also important that independent of the conditions for the propagation of beams in the corona (for possible different configurations and values of the magnitude of the magnetic field) no spreading of the radio-emission sources has been observed.

Let us now compare the relations from § 8 with the estimated observational parameters of solar type III bursts. It follows from the data of Fig. 23 that type III bursts start at frequencies of 6 to 40×10^8 s^{-1}. This observational result can be interpreted as follows. An electron beam is formed in the deep layers of the chromosphere and the corona in such a way that at the beginning of its path the beam instability is suppressed by the collisional damping of longitudinal plasmons ($\gamma^l \ll \nu_e$). The excitation of longitudinal plasmons and as a result the generation of radio-emission starts when a level is reached where $\gamma^l \approx \nu_e$, that is,

$$\frac{n_*}{n_e}\left(\frac{v_0}{\Delta v_0}\right)^2 \approx \frac{1}{N_D}. \tag{10.3}$$

Putting $(v_0/\Delta v_0)^2 \approx 10$ we find from this condition for the particle density in the electron beams $1 \lesssim n_* \lesssim 10^3$ cm^{-3}. This value agrees with the number of electrons usually measured in interplanetary space after type III bursts.

Often one estimates the beam particle density by comparing the observed intensity with the one evaluated using the appropriate theoretical formulae such as (8.37) or (8.40). It is, however, difficult in this way to obtain a reliable estimate of the magnitude of n_* as in those formulae the very important absorption of the electromagnetic waves is not taken into account. It is therefore better to evaluate I_ω^t and I_ω^p using the above estimates for n_* and to require that the calculated values are not lower than the observed ones. Substituting into (8.37) and (8.40) the parameters of the beams which generate type III bursts we find for $n_* \approx 10\,\text{cm}^{-3}$, $n_e \approx 10^8\,\text{cm}^{-3}$, and $R \approx 10^{10}$ cm the following values: $I_\omega^p \approx 3\times10^{-7}$ erg/cm^2 and $I_\omega^t \approx 2\times10^{-8}$ erg/cm^2; these are, as might have been expected, one to two orders of magnitude larger than the observed values which can be ascribed to the effect of absorption.

The theoretical intensity of radiation at frequencies close to $2\omega_{pe}$ is even slightly larger if we take into account the accumulation of oscillations in the main scale-length region. This discrepancy can be removed by the anisotropy of plasma turbulence in a magnetic field which considerably decreases the probability for fusion processes. The radiation intensity increases according to (8.37) and (8.40) with increasing height, in agreement with observations.

The observations apparently indicate a slight delay of the emission at a frequency $2\omega_{pe}$ with respect to the emission at a frequency ω_{pe}. According to Smerd, Wild and Sheridan (1962) this is connected with the fact that the observed radiation at a frequency $2\omega_{pe}$ does not arrive directly but is reflected from deeper layers; however, it is more probable that this effect is due simply to a retardation in the formation of that part of the longitudinal plasmon spectrum, which is responsible for the fusion processes, as was already explained earlier.

Discontinuities in the dynamic spectrum (vanishing of the radiation at some frequencies and its appearance again at a lower frequency, see Fig. 25) was explained by Zheleznyakov and Zlotnik (1971) by the fact that here, perhaps, a double plasma resonance occurs— the ratio ω_{pe}/ω_{He} turns out to be close to an integer. Strong cyclotron absorption then occurs and the radiation is strongly weakened. The necessary magnetic fields are of the order of a few Oersted.

However, the influence of the magnetic field also leads to more important effects, in particular, to the appearance of polarization of type III bursts. We considered in § 8 in detail the influence of the magnetic field upon longitudinal and transverse plasmon turbulence and showed that when $k_g \lesssim \omega_{He}/v_{Te}$ (for beams generating type III bursts this is as a rule satisfied) the longitudinal plasmon turbulence has in first approximation a one-dimensional character (we remind ourselves that parallel and antiparallel orientations of the wavevectors with respect to the magnetic field are equally probable). In that case the energy transfer occurs mainly into "extraordinary" transverse plasmon. In other words, if we neglect the conditions for the propagation of waves in a magnetic plasma, in such a kind of plasma turbulence polarized emission is generated with a preponderance of extraordinary waves. Of course, if we take the propagation of the waves into account the whole picture may change but in first approximation the agreement with the observational data is completely satisfactory. At any rate, one can explain in this way the qualitative behaviour of the polarization of type III bursts.

Finally, one can also explain through the influence of the magnetic field the behaviour of type V and type U bursts. Type V bursts are apparently produced when part of the particle flux which generates type III bursts is trapped in the external part of the corona in a magnetic trap and is contained there for a period. Type U bursts appear because the flux turns back downwards in its motion along the magnetic field lines with both ends "pinned" in layers of the solar surface.

One can easily estimate the magnitude of the magnetic field necessary to control the flux motion in such a way:

$$\frac{H^2}{8\pi} \gg n_* m_e v_0^2, \tag{10.4}$$

that is, $H \gg 10^{-2}$ Oe; closed magnetic field configurations with such a field strength are completely realistic. It is true that at present there are no data whatever on the detailed structure of the magnetic field but the fact that a complex system of magnetic field lines is possible in the corona follows from its very complicated structure which is observed in the optical emission of the solar corona which consists of streamers, "plumes", "helmets" and other similar shapes.

A theoretical analysis of the radio-emission of a flux of trapped particles is performed

by the same methods as in the case of a straight-lined beam. We note by the way that Weiss and Stewart (1965) made the assumption that the emission of type V bursts was connected with plasma turbulence and that a theoretical scheme was considered by Zheleznyakov and Zaĭtsev (1968) in the framework of the quasi-linear theory.

One must bear in mind that the long duration of type V bursts imposes an increased demand on the stabilization of a beam of trapped particles. One must here "restrain" the beam particles not only from decreasing their velocity in general but also especially from a loss of their velocity component at right angles to the magnetic field as otherwise the beam rapidly "leaks out" of the magnetic trap. Non-linear transfer can arrest the loss both of the longitudinal and of the transverse particle velocities but here one must apparently require that the particle velocities are appreciably larger than v_*: it is probable that the fastest particles of those which are accelerated in a chromospheric flare are trapped. A detailed scheme for the stabilization of a beam in traps and the generation by them of type V bursts which takes all possible instabilities which are typical for magnetic traps into account has not yet been considered.

Type II radio-bursts

According to existing theories type II radio-bursts are generated by a shock wave passing upwards into the corona from a chromospheric flare. This hypothesis was advanced by Pikel'ner and Gintsburg (1964) and was considered by Zaĭtsev (1969). It is based upon the following scheme. Sagdeev (1962) has shown that if a shock wave passes from a chromospheric flare almost at right angles to the magnetic field, an oscillatory structure appears leading to a drift of electrons with respect to the ions. When the magnetic Mach number satisfies the condition

$$2 > M > 1+\frac{3}{8}\left(\frac{8\pi n_e T_e}{H^2}\right)^{1/2} = 1+\frac{3}{4\sqrt{2}}\frac{v_s}{v_A}, \tag{10.5}$$

the drift velocity u becomes larger than the electron thermal velocity. The upper limit for the Mach number in (10.5) restricts the limits of the applicability of the laminar theory on the basis of which this conclusion was reached.

Pikel'ner and Gintsburg, and Zaĭtsev also, assumed that the one condition $u > v_{Te}$ is sufficient for Langmuir turbulence to be excited. The real picture is, however, apparently more complicated. Firstly, the laminar theory which was used to evaluate the drift velocity becomes inapplicable if turbulence with an anomalous resistivity is developed. One must then determine the drift velocity using the method given in §§ 4 and 9. We note incidentally that the range of Mach numbers in which the laminar theory is applicable is too narrow to cover the whole possible range of type II burst velocities (see Fig. 31). Secondly, for shock waves, not Langmuir turbulence, but ion-sound turbulence is the main one and the transformation of its energy into electromagnetic radiation is a many-stage process which is, for instance, described by the scheme considered in § 9. In passing we note that Zaĭtsev (1966, 1967 a, b, 1969; Zaĭtsev and Kaplan, 1967) assumed that ion-sound waves can be transformed into electromagnetic radiation through Raman scattering by the fluctuations in the electron density, in other words, through the fusion of ion-sound waves with Langmuir

plasmons from the thermal background. However, he neglected the reverse process of the decay of an electromagnetic wave into a longitudinal plasmon and an ion-sound wave. Taking that process into account considerably decreases the electromagnetic radiation.

One can rigorously show that no decay or non-linear interactions of the excited ion-sound turbulence with the thermal background of Langmuir waves is able to generate electromagnetic waves at a level appreciably exceeding the thermal background. This is connected with the fact that in all such processes the number of longitudinal and transverse plasmons is conserved while the energy of each of them is changed insignificantly as the energy of ion-sound "quanta" is small.

We gave in § 9 a scheme for the development of plasma turbulence in a shock wave taking the anomalous resistivity into account and we evaluated the intensity of the radiation at the plasma frequency and at twice the plasma frequency. Substituting the numerical values into equations (9.14) and (9.16) and assuming $\omega_{pe} \approx 3\times 10^8$ s^{-1}, $R \approx 10^{10}$ cm we find for the spectral intensities $I_\omega^p \approx I_\omega^t \approx 10^{-9}$ erg/cm^2 which also agrees with the observational data when we take into account the inevitable absorption of the radiation when it escapes from the corona. Of course, the value for R which we have taken may be too large, as the change of the spectrum at the wave front decreases the emission coefficient. On the other hand, as we mentioned in § 9 (just before equation (9.13)) isotropization increases the emission coefficient so that these effects possibly can cancel one another. In any case, the observation of type II and III radiobursts shows that the already excited turbulence is maintained on a time scale of the order of $1/\nu_e \approx c/R$, that is, $R \approx 10^{10}$ cm.

We note that for other values of the parameters numerical estimates generally give larger values for the intensity at $2\omega_{pe}$ as compared with the intensity at ω_{pe} in agreement with the observations which show that often type II bursts emit only at a frequency $2\omega_{pe}$. The increase of intensity with height might be explained by the increase of the steepness of the shock wave and the generation of a larger number of fast electrons. It is possible that also a diminishing absorption with height shows up in this case.

An important confirmation of the scheme described in § 9 can be found in the observed "treelike" structure of type II bursts—the superposition of small type III bursts on large type II bursts. Apparently this structure is connected with the excitation of Langmuir turbulence by "run-away" electrons.

A second property needing explanation is the splitting of the radiation spectra both at the main frequency and at the doubled frequency into two bands which repeat one another even in details. It has been attributed to the influence of the magnetic field or to Doppler splitting (Zheleznyakov, 1967b). In a collisionless shock wave there will be initially a soliton behind which an oscillatory structure follows. On the slopes of the soliton two current layers are formed with opposite directions of electron drift and there can occur a splitting up of the emission into two bands at a distance from one another of $\Delta\omega \approx 2\omega_{pe}u/c$. As the drift velocity $u \leqslant v_{Te}$ the maximum shift is $\Delta\omega \approx 2\omega_{pe}v_{Te}/c \approx 10^7$ s^{-1} which is somewhat less than the observed value.

It is even simpler to assume that the structure of the shock wave is such that there are two regions of strong gradients in it. The splitting is then simply connected with the simultaneous generation of plasma turbulence in two spatially separated regions with correspondingly different frequencies. It is sufficient if the spatial separation of the layers is one-

fifth of the equivalent height in order to obtain the observed splitting of the frequencies which is approximately one-tenth of the frequency itself. The regions of strong magnetic field gradients occur both in the leading and in the trailing parts of the shock wave.

It follows from equation (9.2) that for the onset of plasma turbulence generation in a shock wave one needs a magnetic field gradient $\partial H/\partial l \approx 10^{-4}$ Oe/cm. If the magnetic field strength is of the order of 100 Oe (it is difficult to assume a larger value) the thickness of the current layer is of the order of 10 km. It is well known that the characteristic size of the oscillatory structure of a laminar collisionless shock wave is of the order of $c/\omega_{pi} \approx 3\times 10^3$ cm in the conditions of the solar corona. Of course, the structure of a turbulent shock wave may be much more extended. At any rate, at present there are no indications to contradict the picture considered here. Smith (1971, 1972 a, b) has developed a similar scheme for producing type II bursts. We note here that ion-sound turbulence occurs in shock waves with magnetic Mach numbers in the range $1{\cdot}6 < M < 2{\cdot}9$. The main difference with the scheme given a moment ago is connected with the estimate of the run-away nature of the fast electrons.

The interpretation of "long-lived" stationary bursts

As we noted earlier in § 7 one often observes in the sporadic radio-emission of the Sun, bursts the total duration of which is tens of minutes and even hours (type IV bursts, noise storms). The frequencies of these bursts can lie either in a narrow or in a wide range. As a rule there is no systematic shift in frequency. There is here thus a constantly acting source for the generation of turbulence localized in a small volume or occupying a relatively large region—if, of course, we connect this radiation with the conversion of plasma turbulence. Apparently, type I radiobursts can be generated by small scale turbulence with $W_k \propto k^{-5/2}$ (see p. 62) as was proposed by Gordon (1970).

An alternative possibility is the explanation of these effects through synchrotron emission. However, in that case it is also necessary to contain in a certain volume, regardless of all possible instabilities, a well-defined number of relativistic particles during a rather long time. Apparently, also here one cannot get away without taking plasma effects into account.

Therefore without connecting our considerations of these bursts with a definite emission mechanism (for such an analysis one can easily use the equations of §§ 8 and 9 for the case of plasma mechanisms and those of §§ 12 and 13 for the case of the synchrotron and Compton mechanisms) we concentrate on the more important problem of containing a number of fast particles in restricted regions in the solar corona.

First of all we must bear in mind that the regions of ion-sound turbulence in the solar corona are not restricted by the strong shock waves which occur in the chromospheric flares. It is well known that the heating of the solar corona itself is connected with the dissipation of the energy of relatively weak shock waves which are formed from sound and magneto-sound waves generated by the convective zone immediately under the surface of the solar photosphere. It is possible that here also ion-sound turbulence is an intermediate stage between the weak shock waves and the heating of the corona. There are in this case in the corona relatively stable ion-sound turbulence zones at least during periods of maximum activity. One can reach this conclusion about this turbulence either from radar

measurements (vide infra) or from other data such as the non-monotonic behaviour of the solar brightness temperature (Kaplan and Tsytovich, 1968b): ion-sound turbulence can, by scattering the radio-emission of the Sun, produce either additional maxima or additional minima in its stationary spectrum. So far neither observational data nor theoretical considerations have enabled us to reach convincing conclusions about the extent and nature of the ion-sound turbulence. On the other hand, the hypothesis that ion-sound turbulence plays an important role in many phenomena of solar activity is also not contradicted either by observational data or by theoretical considerations.

If one can fill magnetic traps with fast particles through ion-sound turbulence, one has the possibility to explain also other sporadic emission effects in the framework of the plasma turbulence hypothesis. We shall give the appropriate numerical estimates, using equation (9.25). Let us assume that the size of the trap is of the order of 10^{10} cm (which is the order of magnitude of the size of the sporadic emission in the solar corona) and let us assume that in the trap the magnetic field is such that $\omega_{pe} \approx 10\,\omega_{He}$. We then have for $\omega_{pe} \approx 5\times 10^8 s^{-1}$ the following inequality:

$$10^{-8}\left(\frac{v}{v_{Te}}\right)^3 \lesssim \frac{W^w}{n_e T_e} \lesssim \frac{W^s}{n_e T_e} \lesssim 10^{-10}\left(\frac{v}{v_{Te}}\right)^4, \tag{10.6}$$

which can be satisfied for $v \approx 10^2\, v_{Te}$ and $W^w \approx W^s \approx 10^{-2}\, n_e T_e$. The filling of magnetic traps by near-relativistic particles through ion-sound turbulence is thus fully possible but all these problems still require detailed studies.

The interpretation of radar observations

The main problem of such an interpretation is the explanation of the very large cross-sections for the reflection of radar signals. There are, apparently, two possibilities for such an explanation.

Let us begin by assuming that the reflection occurs at a level where the plasma frequency is close to the frequency of the reflected signal and we shall also assume that in the region of the reflection there is Langmuir turbulence. An appreciable amplification of the signal is then possible both before and after its reflection (Gordon and Tsytovich, 1971). The amplification rate is determined by equation (3.19) which can be simplified by replacing $\partial W_k^l/\partial k$ by W^l/k^2:

$$\frac{\gamma^p}{\omega_{pe}} \approx \frac{m_e}{m_i}\frac{W^l}{n_e T_e}. \tag{10.7}$$

As the amplification must proceed in the confines of the small-sized reflection region where the plasma frequency differs little from the signal frequency we can from this obtain a limit on the quantity γ^p and hence also on the plasma turbulence energy density. Putting the size of the region $\sim 10^{10}$ cm, we get $W^l \approx 10^{-5}\, n_e T_e$, that is, about three orders of magnitude above the thermal level. The reflection itself may be connected either with spontaneous scattering by thermal ions or with stimulated scattering described by equation (3.21). The relative width of the reflected signal is then of order v_{Ti}/c which is a few times smaller than the observed one.

Another possibility is connected with reflection at a level where the frequency of the transmitted signal is larger than the plasma frequency. One needs here not only amplification of the signal but also amplification of the reflection efficiency which can now be connected with decay processes. We remind ourselves that the Langmuir frequency corresponding to the radar frequency of 38·25 MHz, equal to $2{\cdot}4\times10^{7}\ s^{-1}$, is reached at a height of $1{\cdot}6R_{\odot}$ while the reflection is observed from levels at heights up to $2{\cdot}7R_{\odot}$. It is possible that in this case amplification of the reflected signal occurs through the mechanism considered in § 5 (equation (5.37)). The necessary ion-sound turbulence density is $W^{s} \approx 3\times10^{-4}\, n_e T_e$. The reflected signal is here amplified only if the electric field creating the ion-sound turbulence is directed outwards. The reflection process is connected either with scattering by ion-sound waves (Gordon, 1968b) or with four-plasmon interactions with Langmuir waves (Gordon, Liperovskiĭ and Tsytovich, 1971). The reflection process can be a single one (the wavevector turns at once over 180°) or a multiple one. In the first case the spectrum must consist of two components shifted relative to the incident frequency by v_s/c which is, generally speaking, not observed. Multiple scattering can produce a wider uniform spectrum but the number of scattering processes need not be large (probably not more than 3 or 4) since according to the observational data the signal reflection occurs over the whole of the spectrum simultaneously over the whole range of frequencies.

Particle acceleration in chromospheric flares

For many years chromospheric flares have been the subject of intensive observational and theoretical studies. In practically all studies it is assumed that in the chromospheric flare region there occurs a sudden restructuring of the magnetic field and a change of a part of its energy into the energy of plasma motion and the energy of fast particles. In some schemes (see, for instance, Severnyĭ, 1960; Severnyĭ and Shabanskiĭ, 1961 a, b; Syrovatskiĭ, 1961, 1966 a, b) a release of energy was considered when the gas is compressed with

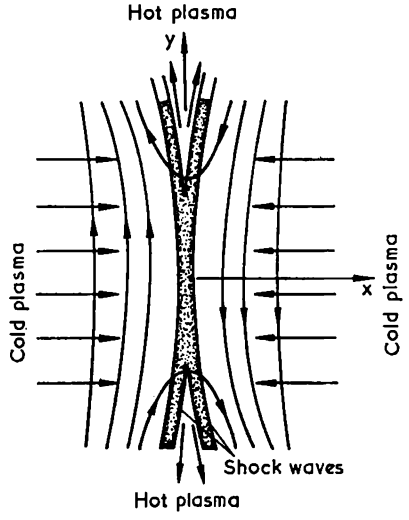

FIG. 42. A possible scheme for forming narrow current layers when two shock fronts intersect.

the magnetic field towards a neutral line. The energy of the magnetic field in that case goes predominantly into kinetic and thermal energy and it is difficult to explain the large number of fast particles. According to other proposals (see, for instance, Severnyĭ, 1960; Sweet, 1969; Parker, 1964, 1968; Alfvén and Carlquist, 1967; Petschek, 1963; Syrovatskiĭ, 1961, 1966 a, b; Friedman and Hamberger, 1969; Tomozov, 1971) the release of the magnetic field energy occurs in a current layer. Let us consider these schemes in somewhat more detail.

It is fully possible that when the conditions change gradually in some region in which subsequently a chromospheric flare is produced, a magnetic field gradient may arise. The current density will then increase according to (9.1) and as the density n_e is bounded the drift (or current) velocity u of the electrons will also rise. According to Syrovatskiĭ (1966 a, b; 1971) one can select such conditions that the current velocity becomes comparable to the light velocity; in the current layer all plasma electrons will then acquire velocities close to the light velocity; Syrovatskiĭ called such a scheme dynamic dissipation. His cal-

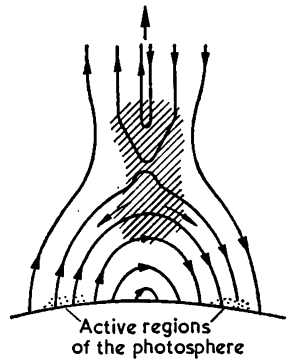

FIG. 43. The most often used scheme for forming chromospheric flares. The closing of field lines (in the shaded region) and their tension eject plasma currents and currents of fast particles in the directions indicated by the arrows.

culations were originally done in the framework of the laminar theory but afterwards he also took into account plasma turbulence.

Friedman and Hamberger (1969) were apparently the first to take the role of plasma turbulence in the formation of chromospheric flares into account; they drew attention to the important value of the anomalous resistivity for calculating the current layer. For concrete estimates they started from a model by Petschek (1963) according to which a narrow current layer is formed when two magnetohydrodynamic discontinuities intersect at a small angle (Fig. 42). However, this model does not give the necessary power of the energy released in a chromospheric flare.

The freezing-in principle is violated in such a narrow current layer because of the decrease in the conductivity, magnetic field lines can close, and according to the authors of several such models their tension can eject plasma with a high velocity (Fig. 43). The chromospheric ejections, shock waves and fast particle currents are formed in this way. The details of the magnetic field configurations assumed here are different in different papers but the essential mechanism is the same (see Coppi and Friedland, 1971).

CHAPTER III

Galactic Nuclei, Radio-galaxies, Quasars

(Turbulent Processes in a Plasma with an Admixture of Ultra-relativistic Electrons)

COMMON to galactic nuclei, radio-galaxies, and quasars is that in all of them we have powerful electromagnetic radiation at all wavelengths which is often ascribed to the synchrotron mechanism. The role played by plasma turbulence in these objects must be considerable, determining, on the one hand, the acceleration of the fast particles and having, on the other hand, a noticeable influence on their radiation.

We shall consider here a number of plasma astrophysics problems connected with collective processes in a normal "cold" plasma which has, however, a significant admixture of ultra-relativistic electrons and ions. The results of the solutions of these problems may be used to interpret at least some of the properties of the above-mentioned objects. We start with a brief summary of the observational data while at the end of the chapter we discuss possible ways of applying the relations obtained.

§ 11. Brief summary of observational data on galactic nuclei, radio-galaxies and quasars

We shall give here only those data which are of interest from the point of view of applying plasma astrophysics methods. The summary has no pretensions whatever of completeness. We refer merely to survey papers.

CHARACTERISTIC PROPERTIES OF GALACTIC NUCLEI

The nuclei of many galaxies—which are in their central parts—are objects with a strikingly expressed activity, the nature of which so far remains unclear. Significant changes in these regions occur over a characteristic time of the order of a year so that the dimensions of the central nuclei of galaxies are of the order of one parsec, that is, $\sim 10^{18}$ to 10^{19} cm or even less.

There are stars, interstellar plasma, a magnetic field, and relativistic particles in the galactic nuclei. In "normal" galaxies the nuclei are more or less "quiescent" but in approximately 1 to 5 per cent of all galaxies they display great activity. It is possible that in all galaxies there is a relatively short ($\sim 10^7$ to 10^8 years) stage of an active state of the nucleus which

might be either a "one-time" stage (for instance, in an early stage of the formation of the galaxy), or one which repeats after a time interval, but with smaller intensity.

We shall first give data on the "quiescent" states of galactic nuclei.

There are about 10^9 stars, or perhaps more, in galactic nuclei. Stars of different spectral classes have been observed, among them both young and old ones. It is possible that there are neutron stars and "collapsars" (stars which have undergone a collapse into the Schwarzschild sphere). From the spectral lines in some nuclei one has detected ionized gas with a mass of the order of $10^6\, M_\odot$, that is 10^{-3} of the mass of the stars. The temperature of the gas is a few thousand degrees and its density $n_e \approx 10^3$ to 10^6 cm^{-3}. So far no neutral gas has been observed in the nuclei but one encounters it in the central parts of galaxies.

The stellar and gas velocities are in the stationary states of nuclei as a rule small, up to 10^7 cm/s; one sometimes meets with somewhat larger values. Fast rotation of the nucleus is, however, possible.

Galactic nuclei usually produce radio-emission which practically always can be described by a spectral intensity distribution of the form

$$I_\omega \propto \omega^{-\alpha}, \tag{11.1}$$

where $\alpha \approx 0{\cdot}7$ for the overwhelming majority of sources; the absolute magnitudes of the intensity I_ω of different objects varies within wide limits. A very rough estimate of the radio-emission intensity of a nucleus in a "normal" state will be of the order of $I_\omega \approx 10^{-8}$ erg/cm^2 (at a frequency $\omega \approx 10^9$ s^{-1}) but deviations from this by two orders of magnitude (or even more) in both directions are possible.

Low and Aumann (1970) observed in 1970 that galactic nuclei (and in particular the nucleus of our Galaxy) produce almost 90 per cent of their luminosity in a narrow band of infra-red radiation with a maximum at a frequency $\omega_{ir} \approx 2{\cdot}5 \times 10^{13}$ s^{-1} (that is, a wavelength $\lambda_{ir} \approx 70\ \mu$) and with a steep drop on both sides of it ($I_\omega \propto \omega^{3{\cdot}5}$ for $\omega < \omega_{ir}$ and $I_\omega \propto \omega^{-3{\cdot}5}$ for $\omega > \omega_{ir}$). It is remarkable that for a whole set of objects which greatly differ in their properties the frequencies of this maximum in the infra-red emission are practically identical.

In our Galaxy this infra-red emission with a total luminosity of the order of 3×10^{41} erg/s is concentrated in three sources with dimensions which are probably considerably less than 10^{19} cm. Its spectral intensity at the maximum is thus larger than $I_\omega \approx 10^{-10}$ erg/cm^2 and even, possibly, of the order of 10^{-6} erg/cm^2. We shall see below that the measurements of active nuclei also give a maximum intensity in the infra-red band, but considerably larger.

Finally, we note that statements have been made about the detection of gravitational radiation from the nucleus of the Galaxy at a frequency $\sim 10^4$ s^{-1}. These results are, however, insufficiently reliable.

Further observations of galactic nuclei are closely connected with the increase in the resolving powers of radio-astronomy instruments (through the building of very long baseline interferometers). In the near future there will probably be many significantly new observational data (see Kellermann, Jauncey, Cohen, Shaffer, Clark, Broderick, Rönnäng, Rydbeck, Matveyenko, Moiseyev, Vitkevitch, Cooper and Batchelor, 1971).

ACTIVITY OF GALACTIC NUCLEI, QUASARS

As we have noted already very active nuclei are observed in a fraction of the galaxies.

First of all, this shows up in a steep increase in the velocity of the gas motions. Galaxies in the nuclei of which motions with velocities from 10^7 to 10^8 cm/s are observed are called Seyfert galaxies. The velocities determined from different spectral lines often turn out to be very different. Violent motions sometimes comprise regions of the galaxies outside the limits of the nucleus and extend to distances of up to $\sim 10^{21}$ cm.

Some of the Seyfert galaxies produce strong radio-emission. In two Seyfert galaxies a strong infra-red radiation has been detected at a frequency $\omega_{ir} \approx 2{\cdot}5 \times 10^{13}$ s^{-1} with an intensity $I_\omega \approx 3 \times 10^{-4}$ erg/cm^2 and a luminosity $\sim 10^{46}$ erg/s.

The so-called N-type galaxies are another type of galaxies with active nuclei. They are encountered at least as often as Seyfert galaxies and have a larger brightness. Violent gas motions occur also in them in the central regions although at somewhat lower velocities. This kind of galaxies produces as a rule a strong radio-emission and a continuous optical spectrum which for some of them shows variations in a time of the order of a year so that the size of their nuclei, or at least details in the nucleus, are not larger than $\sim 10^{18}$ cm.

Rarer, but brighter objects with active nuclei are known: quasars. Apparently one can indicate a sequence: Seyfert galaxies, N-type galaxies, quasars. There are many common features, but also many systematic differences. The role of the emission spectrum (determined by the gas motions) along this sequence decreases while the role of the continuous spectrum (determined by the presence of relativistic particles) increases. Along this sequence the average cosmological red shift increases (indicating the decreasing fraction of the corresponding objects and the relative increase in brightness). In Seyfert galaxies the nuclei are clearly visible as well as the galaxy itself while there are everywhere stars, whereas in N-type galaxies there is a bright nucleus in which there are probably no stars and weakly radiating surroundings in which there are, perhaps, also stars; finally, in quasars there are almost certainly no stars of the usual kind in the nucleus while any surrounding stellar galaxy is altogether unnoticeable, although it is possible that there is gas accumulation in them.

We shall consider the quasi-stellar objects in somewhat more detail. They are denoted by QSO (quasi-stellar objects; quasars) or, if only the radio-emission is observed, QSS (quasi-stellar sources). As in other active nuclei a plasma is detected in them but with somewhat higher electron density of the order of 10^6 cm^{-3} and a temperature $\sim 2 \times 10^4$ °K. Apart from emission lines a set of absorption lines are observed in them with a red shift which in general is not the same as the red shift of the emission lines. It is possible that the relative violet shift of the absorption lines is connected with the incidence of matter onto the quasar while the relative red shift is connected with its separation or with the absorption of light in closer objects (with smaller cosmological red shifts).

The very strong continuous emission of quasars in the optical spectrum and in the radio-band can be described by the power spectrum (11.1). The time variation of the intensity in both bands enables us to limit the size of the quasars to $\sim 10^{18}$ to 10^{20} cm, and perhaps even less, of the order of 10^{16} to 10^{17} cm. The nature of these variations is different at different wavelengths. We give in Figs. 44 and 45 data which show how the intensity of the quasar emission (Fig. 44) and of the Seyfert galaxy emission (Fig. 45) changes.

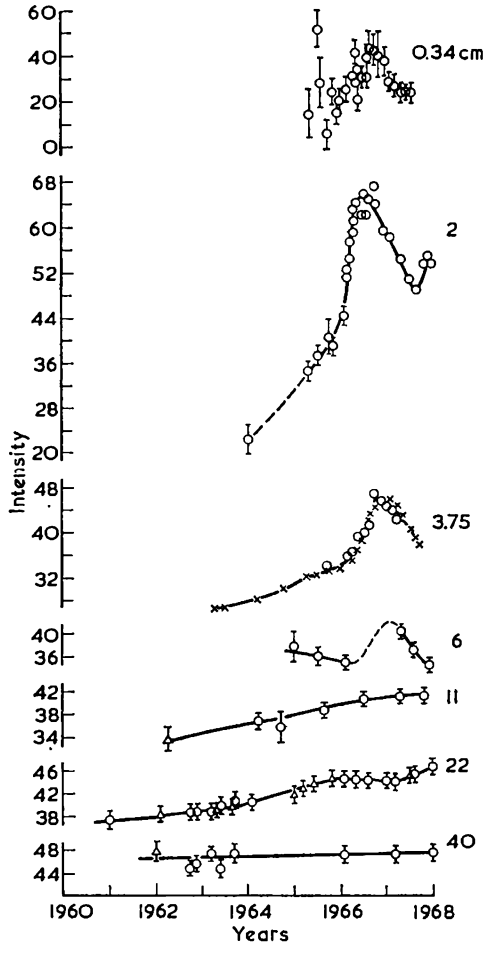

FIG. 44. Change with time in spectral intensity at different wavelengths (in cm) of the closest quasar 3C 273.

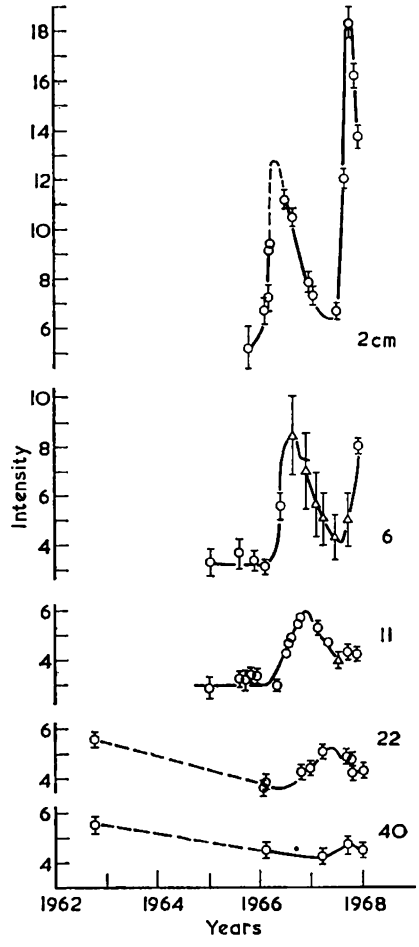

FIG. 45. The same as in Fig. 44 for the galaxy 3C 120.

Observations have only just started and one needs more data. We stress only that the smaller the wavelength the stronger the time variations in intensity. This is also clear from Fig. 46 where we show the change in time in spectra of a number of sources (both quasars and Seyfert galaxies).

One can only estimate very roughly the characteristic spectral intensity. In the optical band $I_\omega \approx 10^{-8}$ erg/cm^2 while in the radio-band it can apparently reach values of $I_\omega \approx 10^{-3}$ erg/cm^2. The maximum intensity occurs here also in the infra-red region ($\omega \approx 2{\cdot}5 \times 10^{13}$ s^{-1}) where, possibly, $I_\omega \approx 10^{-2}$ erg/cm^2. The values given here can only serve as order of magnitude estimates.

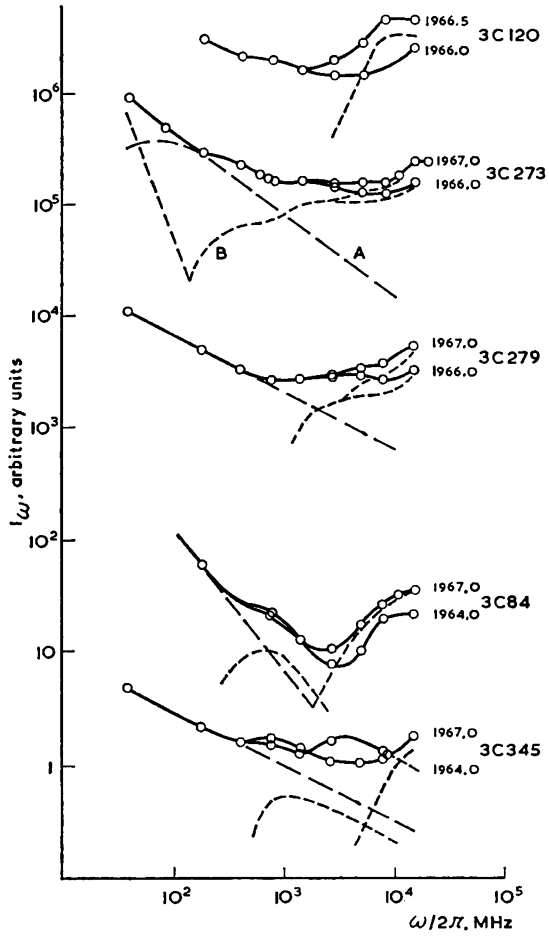

FIG. 46. Change in time of the radio-emission spectra of some radio-galaxies and quasars. The intensity scales are not the same for different objects. Full-drawn curves correspond to observed spectra. Dashed curves correspond to spectra of details, resolved in the case of 3C 273 and assumed in the case of the other sources.

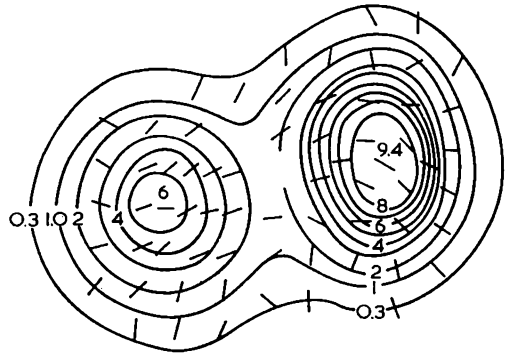

FIG. 47. Radio-emission intensity distribution in the source For A. The full-drawn lines correspond to radio-isophots. The dashes show the direction of the polarization of the radiation.

RADIO-GALAXIES, OUTBURSTS AND THEIR RADIO-SPECTRA

The radio-emission of galaxies is very varied in intensity. It is usual to distinguish and call radio-galaxies those objects for which the radio-luminosity in the frequency range $\omega \approx 6\times 10^8$ to 6×10^{10} s^{-1} exceeds 10^{41} erg/s. The dimensions of the radio-emitting regions of these objects often exceed considerably the dimensions of the galaxies themselves, visible in the optical band.

It has been possible in a number of cases to give a detailed analysis of the radio-brightness distribution in the plane of the figure. The radio-emission turns out to be concentrated in two (sometimes more) regions on both sides of the galaxy. It is clear that here a cloud (or beam) of relativistic electrons is ejected into intergalactic space. It is probable that these clouds are also ejected from galactic nuclei (Rees, 1967).

We give in Figs. 47 to 49 examples of observed radio-brightness distributions. The cloud of relativistic electrons can be very spread out (Fig. 47) or, on the other hand, remains very

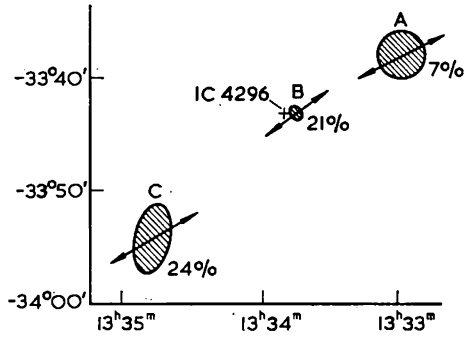

FIG. 48. Structure of the radio-emission source MSH 13–33. The shaded regions indicate radio-emission regions, and the numbers give the degree of polarization while the arrows show its direction. The optical galaxy IC 4296 is indicated by a cross.

compact (Fig. 49). One must note that in the latter case two clouds left the galaxy almost on a straight line and are at approximately the same distance from it.

The emission from these ejections is, as a rule, significantly polarized. An interesting case is the source Virgo A (Fig. 50) where the ejection is still close to the nucleus of the galaxy and emits polarized optical radiation. The ejection consists of different parts positioned as a chain and for neighbouring links the polarization vectors of the radiation are mutually perpendicular.

Probably the most interesting property of radio-galaxies, and of almost all cosmic radio-sources, including also the quasars, consists in the fact that notwithstanding the different conditions for the generation of radio-emission and the very different scales of the various objects, their spectra turn out to be astonishingly similar, and describable by equation (11.1) with practically the same spectral index $\alpha \approx 0{\cdot}7$.

FIG. 49. The radio-source 3C 33. Two very compact emission sources are situated at identical distances from a galaxy visible in the optical band.

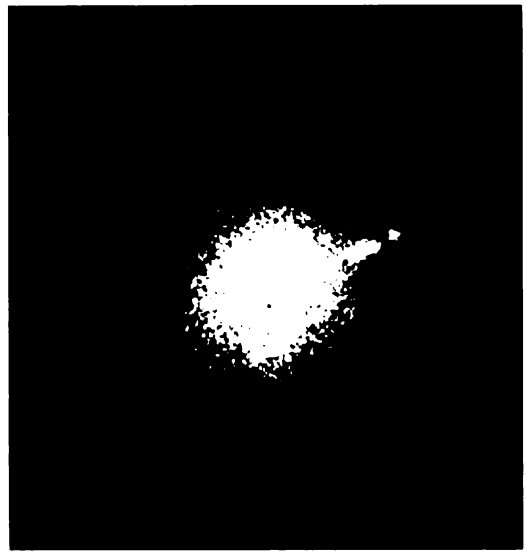

FIG. 50. The radio-galaxy Virgo A with ejections of ultra-relativistic electrons.

We give in Figs. 51 and 52 the spectra of various sources. The differences between them, although noticeable, are small. In Fig. 53 we show the distribution of the observed spectral indices at different frequencies. Practically always $0{\cdot}5 \lesssim \alpha \lesssim 1{\cdot}2$ and, generally, $\alpha \approx 0{\cdot}7$.

The widening of the range of frequencies for radio-astronomical observations has led to the detection also of deviations from this simple behaviour. In particular, in the decametre band (see, for instance, the work by Braude, Lebedeva, Megn, Ryabov and Zhouck, 1969) a steeper rise in the spectrum ($\alpha \approx 3$ to 4) and drops of a different kind from the ones shown in Fig. 52 have been observed. We shall in the following sections discuss possible interpreta-

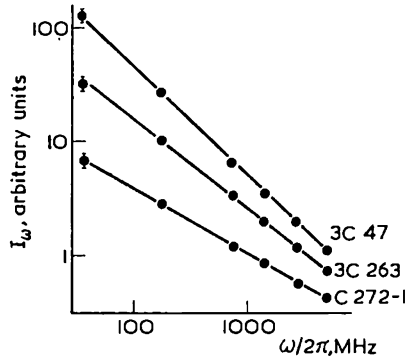

FIG. 51. Radio-galaxy spectra: the case of a constant spectral index in a very wide range of frequencies.

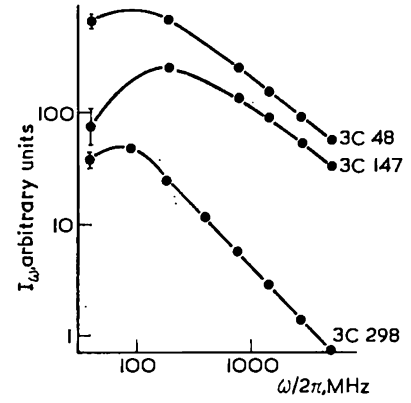

FIG. 52. Radio-galaxy spectra: the case of a drop in the spectrum at low frequencies.

tions of this behaviour of radio-spectra. We stress, however, that probably the main problem for the theory is the explanation of the universality of the spectrum, $I_\omega \propto \omega^{-0{\cdot}7}$.

For detailed introductions about these objects we refer to books by Shklovskiĭ (1960a), Ginzburg and Syrovatskiĭ (1964a), Kaplan and Pikel'ner (1970) and by Burbidge and Burbidge (1967).

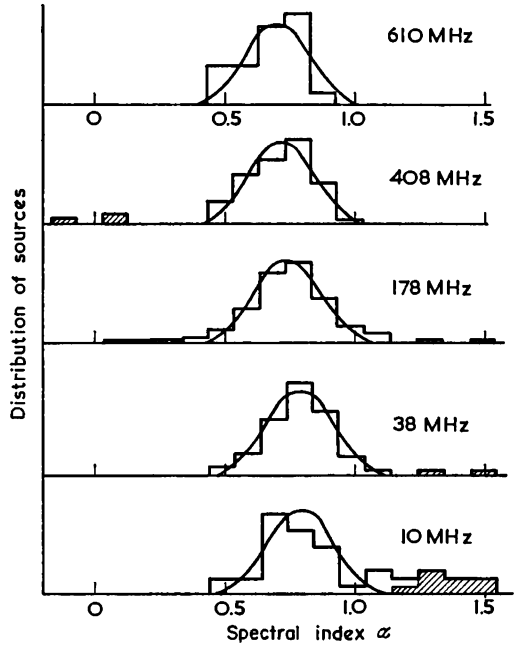

FIG. 53. Distribution of radio-sources with respect to spectral indices (at different frequencies).

§ 12. Emission by ultra-relativistic particles in a plasma

For a long time already one has invoked ultra-relativistic particles in astrophysics for the explanation of various phenomena. However, up to recently one has not paid proper attention to the fact that they emit in a plasma and not *in vacuo*. It is true that a few effects of the influence of the plasma on the radiation by relativistic particles have been taken into account: it was shown, for instance, that the plasma "suppresses" the emission at low frequencies and this leads to the observed drop in the spectrum (Tsytovich, 1951; Razin, 1960). After that the synchrotron instability effect in a plasma was discovered (Zheleznyakov, 1967a; McCray, 1966) which we shall consider in detail in what follows. However, even in those cases the plasma played essentially a passive role: its influence reduces merely to changing the index of refraction for electromagnetic waves.

One needs hardly show now that the collective properties of a plasma significantly affect all that is connected with it, including the conditions for emission by ultra-relativistic particles in the plasma. Relativistic particles emit all kinds of plasma waves, produce new possibilities for plasma instabilities, and are themselves accelerated by plasma turbulence. The "collective" coupling between ultra-relativistic particles and a plasma is the most important aspect of plasma astrophysics which should never be let out of sight.

THE DISTRIBUTION FUNCTION OF THE RELATIVISTIC PARTICLES

It is clear that relativistic particles under cosmic conditions do not satisfy the Maxwell distribution function if only because collisions of such particles are extremely rare. The problem of determining the total distribution function of ultra-relativistic particles under

cosmic conditions is a difficult one and at present as yet unsolved. We can only approximate this function on the basis of rather qualitative theoretical considerations and observational data.

Under cosmic conditions both isotropic and anisotropic velocity distributions of ultra-relativistic particles are apparently possible although the first are probably encountered much more often. We start thus with a determination of the isotropic energy distribution function of the ultra-relativistic particles, the quantity f_ε, already introduced in § 2.

Numerous observational data† show that in the region of high energies the distribution function of the relativistic electrons almost always decreases with increasing energy according to a power law,

$$f_\varepsilon \propto \varepsilon^{-\gamma}, \tag{12.1}$$

where the index γ turns out to be astonishingly constant in very different circumstances and is approximately equal to $\gamma = 2{\cdot}5$ to $2{\cdot}7$. It is true that sometimes discontinuities in the distribution function are observed when for some energy the index γ decreases or increases with a jump after which the distribution function again is described by equation (12.1) but with a different value for γ.

One of the main problems of plasma astrophysics is an explanation of this power-law spectrum and a theoretical determination of the value of the index γ. We shall show in § 14 how one can solve this problem.

In the low energy region it is difficult to observe f_ε. It is natural to expect that the function f_ε reaches a maximum for some value ε_* and after that decreases with decreasing energy. If we assume that the phase density at low energies is constant—the usual condition for equilibrium or quasi-equilibrium distributions which are established through interactions, which in some way or other are responsible for the isotropy of the velocities—we must have for small energies $f_\varepsilon \propto \varepsilon^2$ (we assume here as before that $\varepsilon \gg mc^2$).

Apparently with an accuracy which is sufficient for all astrophysical estimates one can take for the energy distribution function of fast electrons the following approximate formula:

$$f_\varepsilon = \frac{\gamma(\gamma^2-1)n_*\varepsilon_*^{\gamma-1}\varepsilon^2}{2(\varepsilon_*+\varepsilon)^{\gamma+2}}. \tag{12.2}$$

This function behaves as (12.1) when $\varepsilon \gg \varepsilon_*$, while for $\varepsilon \ll \varepsilon_*$ we have $f_\varepsilon \propto \varepsilon^2$. Moreover, f_ε in (12.2) is normalized so that the total density of relativistic electrons is equal to n_*.

Equation (12.2) is convenient for applications and it has three parameters, n_*, ε_*, and γ, to get it to agree with observational data and, what is very important, it is satisfactory from the point of view of having a physical meaning. For instance, one often assumes in astro-

† Most of these are obtained on the basis of the assumption that the emission by relativistic electrons is synchrotron radiation and that the connection between the spectral index α and the index γ follows the equation $\alpha = (\gamma-1)/2$ (see equation (12.16)). It will be shown in what follows that most plasma mechanisms for the emission through relativistic electrons in a cold plasma also give the same connection $\alpha = (\gamma-1)/2$. The reduced "observational" values of γ are thus in fact not connected with the hypotheses about concrete emission mechanisms. However, this does not refer to the case of very strong magnetic fields and to a totally ultra-relativistic plasma (for instance, close to a pulsar) where the connection between α and γ can be different (for details see Chapter 4).

physics the condition that f_ε has a cut-off at some ε_1 so that $f_\varepsilon = 0$ when $\varepsilon < \varepsilon_1$; this, however, contradicts both simple statistical considerations as well as well-known physical mechanisms which may be responsible for producing f_ε.

The distribution function (12.2) is a peculiar analogue of the equilibrium Maxwell distribution in the sense that it is stable with respect to stimulated excitation of practically any waves in the plasma (this is, of course, not true in the case of very strong magnetic fields when the particle distribution becomes anisotropic; see Chapter 4), that is, for this function the absorption always dominates over the stimulated emission. The distribution (12.2) may thus be called a quasi-equilibrium one. However, it is better to call it a standard distribution as all subsequent evaluations of emission coefficients of ultra-relativistic electrons will be obtained with the help of equation (12.2).

We must bear in mind that (12.2) refers only to an isotropic velocity distribution established through the interaction of particles with the surrounding medium. Anisotropic distribution functions of ultra-relativistic particles have a completely different character. There are then no general considerations whatever (such as, for instance, would require that $f_\varepsilon \propto \varepsilon^2$ for small ε) and the distribution function is determined both by the condition for producing the anisotropy and by the action of oscillations due to the development of instabilities.

An anisotropy in the velocities of ultra-relativistic particles is, for instance, produced in surface layers of astrophysical objects where particles moving outwards have a larger chance to escape from the object than particles moving inwards. A convenient approximation may then be given by the distribution function

$$f_\varepsilon = f_\varepsilon^{(0)}(1+\eta \cos \theta), \tag{12.3}$$

where $f_\varepsilon^{(0)}$ is an isotropic distribution function, such as (12.2), θ the angle with a chosen direction, such as the normal to the surface of the object, and η the anisotropy parameter.

Another striking case of anisotropy is given by relativistic beams consisting of particles with a possible spread in energy, but moving with the same velocity $v \approx c$ in one direction It is best to assume for them a Gaussian energy distribution with a maximum at some value ε_* and a spread $\Delta\varepsilon_*$. Of course, a real beam will also have a spread $\Delta\theta$ in the velocity directions.

Another possibility is the one where under cosmic conditions the solid angles in which the beam motions are directed are very small. It is well known that when charged particles move in a magnetic field the adiabatic invariant which is the ratio of the transverse particle energy to $\sqrt{H}$, where H is the magnetic field strength, is conserved. If a particle therefore enters from a strong magnetic field region into a weak field region a particle beam is stretched along the magnetic field lines.

The loss of transverse energy through synchrotron emission also leads to the same effect. We may thus expect that there will occur under cosmic conditions rather narrow, more or less monoenergetic beams of ultra-relativistic electrons which move almost strictly along the magnetic field.

SPONTANEOUS SYNCHROTRON EMISSION OF ELECTROMAGNETIC AND LANGMUIR WAVES

The main property of ultra-relativistic electrons, which is valued so highly in astrophysics, is their ability to produce a powerful non-thermal emission through the synchrotron mechanism. It is well known (see, for instance, Ginzburg and Syrovatskiĭ, 1964a; Bekefi, 1966; Ginzburg, Sazonov and Syrovatskiĭ, 1968) that an ultra-relativistic electron moving along a spiral—at an angle θ to the magnetic field—emits electromagnetic waves in a wide frequency band. The maximum intensity of the emission occurs at the frequency (2.16). If we take into account the motion along spirals a more exact formula for the frequency of the maximum of the emission spectrum of a single electron is

$$\omega_{\max} = 0{\cdot}29\omega_{He} \sin\theta \left(\frac{\varepsilon}{m_e c^2}\right)^2. \tag{12.4}$$

We shall in what follows include the factor $\sin\theta$ in the quantity ω_{He}, that is, we shall mean in the present section by ω_{He} the quantity $eH \sin\theta/m_e c$.

Let us now determine the emission probability. This quantity was at the maximum of the spectrum determined by equation (2.17). We must now take into account a dimensionless function which characterizes the distribution of the radiation energy over the spectrum. The exact formula for the synchrotron emission probability with a given wave number $k = \omega/c$ and taking the influence of the plasma into account has the form (Tsytovich, 1951)

$$u_k = \sqrt{3}\cdot\pi \frac{e^2 c^2 \omega_{He}}{2\omega^3} \Psi(\omega, \varepsilon), \tag{12.5}$$

where

$$\Psi(\omega, \varepsilon) = \frac{\omega}{\omega_c}\left[\int_{\omega/\omega_c}^{\infty} K_{5/3}(\eta)\, d\eta\right]\left[1+\frac{\omega_{pe}^2}{\omega^2}\left(\frac{\varepsilon}{m_e c^2}\right)^2\right]^{-1/2}. \tag{12.6}$$

Here $K_{5/3}(\eta)$ is a MacDonald function and

$$\omega_c = \frac{3}{2}\omega_{He}\left(\frac{\varepsilon}{m_e c^2}\right)^2\left[1+\frac{\omega_{pe}^2}{\omega^2}\left(\frac{\varepsilon}{m_e c^2}\right)^2\right]^{-3/2}. \tag{12.7}$$

It is not very convenient to use this exact formula but it is not necessary either, as its application would exceed the astrophysical accuracy of all calculations. For our purpose it is completely sufficient to use the approximate formula

$$\Psi(\omega, \varepsilon) = \sqrt{3}\cdot q^{1/3} \exp\left[-\frac{2}{3} q\left(1+\left(\frac{\zeta}{q}\right)^2\right)^{3/2}\right], \tag{12.8}$$

where

$$q = \frac{\omega}{\omega_{He}}\left(\frac{m_e c^2}{\varepsilon}\right)^2, \quad \zeta = \frac{\omega_{pe}}{\omega_{He}}\frac{m_e c^2}{\varepsilon}. \tag{12.9}$$

One obtains equation (12.8) from asymptotic approximations of the exact formula. When q changes from 0 to 15 the deviation of (12.8) from the exact formula (12.6) is less than 5 per cent.

We now turn to the problem of the dependence of the emission probability on the angle θ. Diminishing this angle while the other conditions remain the same decreases ω_{He} in (12.4) which is equivalent to increasing q for a given emission frequency and particle energy. This in turn means that the function $\Psi(\omega, \varepsilon)$ for decreasing θ and given ε enters a region where it decreases exponentially. Only particles for which $\sin\theta$ is not too small will radiate significantly. We shall consider the case of small θ-values in the next chapter.

It is also easy to estimate the influence of the plasma on the synchrotron emission probability if we use equation (12.8) where this influence is described by the parameter ζ. It is clear that the emission probability will only not be exponentially small if both q and ζ are not larger than of order unity. Hence it follows that one can certainly neglect the influence of the plasma if

$$1 \ll \frac{\omega_{pe}}{\omega_{He}} \ll \frac{\varepsilon}{m_e c^2}. \tag{12.10}$$

If one has the inverse inequality to (12.10) the radiation will be strongly suppressed at practically all frequencies. One can practically assume that particles with an energy $\varepsilon > m_e c^2 \omega_{pe}/\omega_{He}$ will emit electromagnetic waves without particular obstruction from the plasma while the emission by particles with an energy $\varepsilon < m_e c^2 \omega_{pe}/\omega_{He}$ will be strongly suppressed.

The total power—integrated over all frequencies—of the synchrotron emission of a single relativistic electron, neglecting the influence of the plasma, is

$$P_0 = \frac{2}{3}\frac{e^2}{c}\omega_{He}^2\left(\frac{\varepsilon}{m_e c^2}\right)^2. \tag{12.11}$$

If the influence of the plasma is important the total radiating power will for small ε be equal to (Zheleznyakov and Trakhtengerts, 1966)

$$P^t = \frac{3\sqrt{3}}{4}\frac{e^2}{c}\omega_{He}\omega_{pe}\frac{\varepsilon}{m_e c^2}\exp\left[-\sqrt{3}\frac{\omega_{pe}}{\omega_{He}}\frac{m_e c^2}{\varepsilon}\right], \tag{12.12}$$

that is, there is a steep decrease in the emission power for $\varepsilon < m_e c^2 \omega_{pe}/\omega_{He}$ (Fig. 54, curve labelled t).

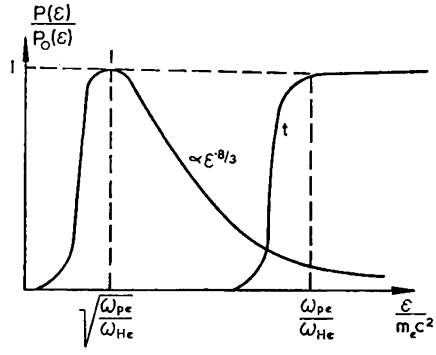

FIG. 54. Total synchrotron radiation power in a plasma as function of the energy of the relativistic electron. Here $P_0(\varepsilon) = \frac{2}{3}(e^2/c)\omega_{He}^2(\varepsilon/m_e c^2)^2$ is the emission power *in vacuo;* the curve labelled t is the radiation power for electromagnetic waves, and the curve labelled l the power for Langmuir waves.

We shall now evaluate the emission probability per unit volume and unit frequency:

$$J_\omega^t = \frac{J_k^t}{c} = \frac{\omega^3}{\pi^2 c^3} \int_0^\infty u_k f_\varepsilon \, d\varepsilon. \tag{12.13}$$

Substituting here (12.2), (12.5) and (12.8) leads to a rather complicated expression which we shall not write down here. From the point of view of applying the theory to astrophysical problems it is sufficient to obtain approximate expressions in the above defined frequency ranges. Moreover, unless specifically stated otherwise, we shall henceforth drop numerical factors of order unity which depend on the parameter γ in all emission and absorption coefficients. We discuss one important exception in § 14.

One can split the whole emission spectrum into two parts, the low- and the high-frequency parts. One verifies easily that the maximum intensity occurs at the frequency

$$\omega_{\max}^* \approx \omega_{He} \left(\frac{\varepsilon_*}{m_e c^2} \right)^2. \tag{12.14}$$

In the low-frequency part of the spectrum, that is, when $\omega \ll \omega_{\max}^*$ we have for the emission coefficient:

$$J_\omega^t \approx \frac{e^2}{c} \omega_{He} n_* \left(\frac{\omega}{\omega_{\max}^*} \right)^{1/3}. \tag{12.15}$$

At high frequencies, that is, when $\omega \gg \omega_{\max}^*$, we have (Getmantsev, 1952; Korchak and Terletskiĭ, 1952; Ginzburg and Syrovatskiĭ, 1964a)

$$J_\omega^t \approx \frac{e^2}{c} \omega_{He} n_* \left(\frac{\omega_{\max}^*}{\omega} \right)^{(\gamma-1)/2} = \frac{e^2}{c} \omega_{He} n_* \left(\frac{\varepsilon_*}{m_e c^2} \right)^{\gamma-1} \left(\frac{\omega_{He}}{\omega} \right)^{(\gamma-1)/2}. \tag{12.16}$$

In both cases we have neglected the influence of the plasma, that is, we put $\zeta = 0$.

One can easily understand the qualitative behaviour of the spectra (12.15) and (12.16). At low frequencies almost all ultra-relativistic electrons emit in that part of their spectrum where one can take the exponent in (12.8) to be equal to unity. Here each electron emits according to the law $J_\omega \propto \omega^{1/3}$ and the whole emission spectrum therefore has the same form for $\omega < \omega_{\max}^*$. In the high-frequency part of the spectrum, that is, when $\omega > \omega_{\max}^*$, the position is different. Here it is important that at high frequencies only particles with a high energy can emit, as follows from (12.8) and (12.9). The contribution to the emission at a frequency ω of particles with energies less than $m_e c^2 (\omega/\omega_{He})^{1/2}$ is exponentially small (because $q > 1$). The emission coefficient at high frequencies is thus determined by the number of particles with the appropriate energies.

What we have said about the behaviour of the spectrum of the electromagnetic synchrotron emission is valid as long as we can neglect the influence of the plasma. It is not difficult to take this now into account. Indeed, as the plasma suppresses the emission when $\varepsilon < m_e c^2 \omega_{pe}/\omega_{He}$ it follows at once that if

$$\frac{\varepsilon_*}{m_e c^2} \gg \frac{\omega_{pe}}{\omega_{He}}, \tag{12.17}$$

the plasma has practically no effect on any part of the synchrotron emission spectrum, except that it slightly lowers it in the region where (12.15) holds. On the other hand, if the inequality which is the opposite of (12.17) holds, the plasma very strongly influences the situation. As it cuts off the emission by particles with energies around $m_e c^2 \omega_{pe}/\omega_{He}$ this means that the whole section of the spectrum with frequencies less than

$$\omega_{cr} \approx \frac{\sqrt{3}}{\pi(\gamma-1)}\,\omega_{He}\left(\frac{\omega_{pe}}{\omega_{He}}\right)^2 \approx \frac{4\sqrt{3}}{\gamma-1}\,\frac{en_e c}{H} \tag{12.18}$$

will also be cut off (Razin, 1960). The emission at higher frequencies ($\omega > \omega_{cr}$) is as before approximately determined by equation (12.16) while for frequencies less than (12.18) there is a steep drop in the spectrum. The character of the electromagnetic spectrum of synchrotron emission with a drop agrees with the spectrum shown in Fig. 54.

The physical explanation of such an influence of the plasma on the synchrotron emission consists in the following. Both the number of the harmonic at which a maximum occurs in the radiation intensity and the directional diagram (concentration of the emission in a narrow solid angle) are determined by the parameter $(1-v^2/v_p^2)^{1/2}$, where v is the particle velocity and v_p the phase velocity of the wave. These specifically relativistic effects occur for any radiation and not only in the generation of electromagnetic waves. *In vacuo* for electromagnetic waves $v_p = c$ and $(1-v^2/v_p^2)^{1/2} \approx m_e c^2/\varepsilon \ll 1$. The phase velocity of electromagnetic waves in a plasma is $v_p = \omega/k = c(1+\omega_{pe}^2/2\omega^2) > c$ and therefore the quantity $(1-v^2/v_p^2)^{1/2}$ can be not very small even for ultra-relativistic electrons. Now we have

$$\sqrt{\left(1-\frac{v^2}{v_p^2}\right)} \approx \sqrt{\left(1-\frac{v^2}{c^2}+\frac{v^2}{c^2}\,\frac{\omega_{pe}^2}{\omega^2}\right)} \approx \sqrt{\left\{\left(\frac{m_e c^2}{\varepsilon}\right)^2+\frac{\omega_{pe}^2}{\omega^2}\right\}} \gtrsim \frac{\omega_{pe}}{\omega}. \tag{12.19}$$

If ω_{pe}/ω is not too small the emission even by ultra-relativistic particles is distributed over a wider solid angle and occurs at a harmonic of a lower number.

It follows from this, incidentally, that there is yet another important conclusion: the synchrotron mechanism can produce only those waves for which the phase velocity is very close to the velocity of light. Apart from electromagnetic waves only Langmuir waves can have such phase velocities in a plasma. We must thus consider also the synchrotron emission of longitudinal plasmons with wavenumbers around $k_c = \omega_{pe}/c$, that is, with a phase velocity very close to the light velocity.

The emission probability of a longitudinal plasmon through the synchrotron mechanism is determined by the equation (Kaplan and Trakhtengerts, 1967; Kaplan and Tsytovich, 1972b):

$$u_k = \frac{2\pi}{\sqrt{3}}\,\frac{e^2}{k^2}\int_{\xi}^{\infty} K_{1/3}(\eta)\,d\eta, \tag{12.20}$$

where $K_{1/3}(\eta)$ is a MacDonald function. The lower limit of the integral is equal to

$$\xi = \frac{2}{3}\frac{\omega_{pe}}{\omega_{He}}\left(\frac{m_e c^2}{\varepsilon}\right)^2\left[1+2\frac{\omega_{pe}-kc}{\omega_{pe}}\left(\frac{\varepsilon}{m_e c^2}\right)^2\right]^{3/2}. \tag{12.21}$$

It is convenient to replace equation (12.20) by the approximate expression

$$u_k = \frac{2\pi^2}{3}\frac{e^2c^2}{\omega_{pe}^2}\exp\left\{-\frac{2}{3}q\left[1+2\frac{\omega_{pe}-kc}{\omega_{pe}}\left(\frac{\varepsilon}{m_e c^2}\right)^2\right]^{3/2}\right\}, \tag{12.22}$$

where q is defined by (12.9). First of all it follows from this that only electrons, for which

$$\frac{\varepsilon}{m_e c^2} \gtrsim \sqrt{\frac{\omega_{pe}}{\omega_{He}}}, \tag{12.23}$$

will show appreciable synchrotron emission ($q \lesssim 1$). Moreover, the larger the particle energy, the smaller the wavenumber interval in which they emit. From (12.22) we have for the range of wavenumbers of longitudinal plasmons which emit through the synchrotron mechanism:

$$\frac{\Delta k_c}{k_c} \approx \left(\frac{\omega_{He}}{\omega_{pe}}\right)^{2/3}\left(\frac{m_e c^2}{\varepsilon}\right)^{2/3} \lesssim \frac{\omega_{He}}{\omega_{pe}}. \tag{12.24}$$

The maximum width of this interval is reached when (12.23) is an equality.

Integrating the emission probability over all wavenumbers we find the power of the synchrotron emission of longitudinal plasmons by a single relativistic electron (see Fig. 54, curve labelled l):

$$P^l = \frac{\Gamma(\frac{2}{3})3^{1/6}}{4\pi}\frac{e^2\omega_{pe}^2}{c}\left(\frac{\omega_{He}}{\omega_{pe}}\right)^{2/3}\left(\frac{m_e c^2}{\varepsilon}\right)^{2/3} \lesssim 0{\cdot}1\frac{e^2}{c}\omega_{pe}\omega_{He}. \tag{12.25}$$

Comparing (12.25) with (12.11) and (12.12) we must bear in mind that when $\varepsilon/m_e c^2 < \omega_{pe}/\omega_{He}$ the synchrotron emission of electromagnetic waves is strongly suppressed. We can thus say the following. When $\varepsilon/m_e c^2 \gg \omega_{pe}/\omega_{He}$ we get through the synchrotron mechanism almost exclusively the emission of electromagnetic waves. When

$$\sqrt{\frac{\omega_{pe}}{\omega_{He}}} < \frac{\varepsilon}{m_e c^2} < \frac{\omega_{pe}}{\omega_{He}}, \tag{12.26}$$

Lagmuir waves are preponderantly emitted by this mechanism. And, finally, when $\varepsilon < m_e c^2(\omega_{pe}/\omega_{He})^{1/2}$ through the influence of the plasma all emission by the relativistic electrons is suppressed.

Cherenkov emission of Langmuir and Alfvén waves by relativistic particles

Of course, ultra-relativistic particles can by means of the Cherenkov mechanism emit all modes of plasma waves with phase velocities less than the velocity of light. In particular, Langmuir waves are also emitted. Earlier we computed the synchrotron emission of Langmuir waves and showed that then only waves in a narrow range of wavenumbers near a phase velocity close to the velocity of light are generated.

Through the Cherenkov mechanism relativistic particles produce longitudinal plasmons with phase velocities from $3v_{Te}$ to c, that is, with wavenumbers from $(3d_e)^{-1}$ to $k_c = \omega_{pe}/c$. The emission probability is as before described by equation (2.24) with $v = c$, that is,

$$u_k = 2\pi^2 \frac{e^2\omega_{pe}}{k^3 c} . \tag{12.27}$$

It is important that this probability is independent of the particle energy. The total power for the emission of Langmuir waves by a single relativistic particle is equal to

$$P^l = \int_{k_c}^{(3d_e)^{-1}} \frac{k^2\omega_{pe}}{2\pi^2} u_k \, dk = \frac{e^2\omega_{pe}^2}{c} \ln\left(\frac{c}{3v_{Te}}\right), \tag{12.28}$$

and is also independent of the particle energy. The power (12.28) is larger than (12.25), (12.12) and even (12.11), provided

$$\frac{\varepsilon}{m_e c^2} < \frac{\omega_{pe}}{\omega_{He}} \sqrt{\left(\frac{3}{2} \ln \frac{c}{3v_{Te}}\right)} . \tag{12.29}$$

The upper limit for the predominance of Langmuir wave emission over transverse wave emission in the inequality (12.26) must thus be increased by yet an extra factor. We note by the way that in general for these energy ranges the ionization losses during the motion of the particles in the plasma are larger than the loss through synchrotron and Cherenkov emission.

As the emission probability is independent of the energy the emission coefficient is determined solely by the quantity n_*. We have

$$J_k^l = \frac{e^2\omega_{pe}^2}{ck} n_* . \tag{12.30}$$

The emission coefficient increases with decreasing wavenumber (Fig. 55). When $k \gg \omega_{pe}/c$ the particle emits almost at right angles to the direction of its motion. When v_p approaches the velocity of light the emission cone gets smaller and the Cherenkov emission becomes concentrated in the forward direction along the motion of the particle.

It is also clear that other plasma waves can also be produced by relativistic particles. Of particular interest here are Alfvén waves because of their relatively low absorption coefficient.

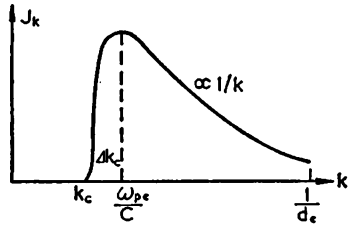

FIG. 55. Coefficient for the emission of Langmuir waves by a relativistic particle. The region $k - \omega_{pe}/c \gg \Delta k_c$ corresponds to the Cherenkov emission mechanism and $k < \omega_{pe}/c$ to the synchrotron mechanism. The quantity Δk_c decreases with increasing energy $\propto (mc^2/\varepsilon)^{2/3}$; see (12.24).

Under the usual conditions in a cosmic plasma the density of relativistic particles is small and the emission coefficient, determined by n_*, is small compared to the absorption coefficient which depends on n_e. Only for Alfvén waves can the effect of emission by relativistic particles in a cosmic plasma be noticeable.

We must, however, first consider the limitation on wavenumbers. Indeed, the wavenumbers of Alfvén waves are small, that is, the wavelengths are long. Moreover, the Cherenkov mechanism "works" when the wavelength of the emitted wave is less than the gyroradius. There is thus a lower limit for the wavenumber of Alfvén waves generated by the Cherenkov mechanism. There is also an upper limit, equal to ω_{Hi}/v_A for them.

Thus

$$\frac{\omega_{Hi}}{v_A} = \frac{\omega_{pi}}{c} > k > k_{min} = \frac{\omega_{He}}{c}\,\frac{m_e c^2}{\varepsilon} = \frac{eH}{\varepsilon}. \tag{12.31}$$

Cherenkov emission of Alfvén waves is thus produced only by particles with an energy

$$\varepsilon > m_i c v_A. \tag{12.32}$$

Relativistic ions can always generate Alfvén waves through the Cherenkov mechanism, but condition (12.32) can also be easily satisfied by relativistic electrons.

The probability of Cherenkov emission of Alfvén waves by relativistic particles, averaged over the angles, is equal to

$$u_k = \frac{\pi^2 e^2 v_A (2c^2 + 3v_A^2)}{4(v_A^2 + c^2)^{3/2} k^2}. \tag{12.33}$$

The total power of the emission of Alfvén waves is

$$P^a = \frac{e^2 c \omega_{Hi}^2 (6v_A^2 + 5c^2)}{30(v_A^2 + c^2)^2} \approx \frac{e^2 \omega_{Hi}^2}{6c}. \tag{12.34}$$

The last equation is valid in the limit $v_A \ll c$. As one would expect, the total power for the emission of Alfvén waves, as well as of other low-frequency waves, is very small as compared to the power of emission of electromagnetic and Langmuir waves.

One can easily determine also the spectrum of the emission of Alfvén waves. We have when $v_A \ll c$:

$$J_k^a = \frac{e^2 v_A^2 k}{6c} \int_{eH/k}^{\infty} f_\varepsilon \, d\varepsilon = \frac{e^2 v_A^2 k_* n_*}{6c} S(k_*/k), \tag{12.35}$$

where $k_* = eH/\varepsilon_*$ while the function $S(k_*/k)$ which determines the spectrum as function of the dimensionless wavelength $\lambda = k_*/k$ has the form

$$S(\lambda) = \frac{\gamma(\gamma+1)\lambda^2 + 2(\gamma+1)\lambda + 2}{\lambda(\lambda+1)^{\gamma+1}}. \tag{12.36}$$

Figure 56 illustrates the form of the spectrum. This spectrum characterizing the small-scale fluctuations of the magnetic field can occur when one observes the fluctuations in intensity

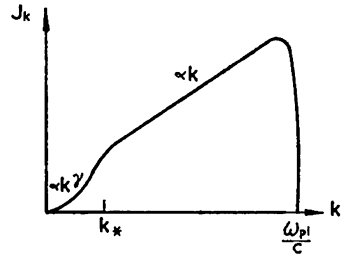

FIG. 56. Coefficient of emission of Alfvén waves by a relativistic particle as function of the wavenumber.

of cosmic radio-sources. More important then is the stimulated excitation of Alfvén waves with their subsequent non-linear conversion into magnetohydrodynamic waves (Scheuer and Tsytovich, 1970).

STIMULATED EMISSION AND REABSORPTION OF RELATIVISTIC ELECTRONS

The growth rate for the stimulated emission or the damping (reabsorption) rate both of electromagnetic and of plasma waves for the case of relativistic particles can be evaluated using formulae from § 2, such as equations (2.33) and (2.34). It is more convenient in the present subsection to change from f_p to f_ε.

We start with the case of an isotropic distribution. It follows from (2.27), (2.33) and (2.34) that

$$\gamma = \int u_k \omega \frac{\partial f_p}{\partial \varepsilon} \frac{4\pi\varepsilon^2 \, d\varepsilon}{(2\pi c)^3} = \omega \int_0^\infty u_k \varepsilon^2 \frac{\partial}{\partial \varepsilon} \left(\frac{f_\varepsilon}{\varepsilon^2} \right) d\varepsilon . \tag{12.37}$$

Integrating by parts and using the fact that $f_\varepsilon(0) = f_\varepsilon(\infty) = 0$, we have

$$\frac{\gamma(\omega)}{\omega} = -\int_0^\infty \frac{f_\varepsilon}{\varepsilon^2} \frac{\partial}{\partial \varepsilon} (\varepsilon^2 u_k) \, d\varepsilon . \tag{12.38}$$

Each of these two formulae is convenient for qualitative estimates in the case when one can judge the sign of one or other of the derivatives. For instance, in the case of the standard distribution function (12.2) the derivative $(\partial/\partial\varepsilon)(f_\varepsilon/\varepsilon^2)$ is always negative. In that case any emission mechanism leads only to absorption, as always $u_k > 0$ and hence, then, $\gamma < 0$. This was just the reason why we called the function (12.2) a quasi-equilibrium function, as it is stable with respect to the excitation of any kind of waves in the plasma. If, on the other hand, $(\partial/\partial\varepsilon)(\varepsilon^2 u_k) > 0$, in general, no isotropic distribution function can produce an amplification of the radiation field, however much we cut it off at low energies.

Equation (12.38) enables us to determine the damping or growth rate per particle. We introduce a quantity which we shall call the reabsorption coefficient:

$$\varkappa = \frac{\omega}{\varepsilon^2} \frac{\partial}{\partial \varepsilon} (\varepsilon^2 u_k), \tag{12.39}$$

and we shall evaluate it for different mechanisms of emission by relativistic particles.

Substituting formula (12.5) into (12.39) we find for the absorption in the synchrotron mechanism:

$$\varkappa^t(\omega) = \sqrt{3}\cdot\pi \frac{e^2c^2}{\omega^2} \frac{\omega_{He}}{\varepsilon^2} \frac{\partial}{\partial\varepsilon}[\varepsilon^2\Psi(\omega, \varepsilon)]. \tag{12.40}$$

Differentiating the function (12.6) or, which is more convenient and also sufficiently accurate, the function (12.8) with respect to ε for constant ω we find

$$\varkappa^t = \frac{2\pi}{\sqrt{3}} \frac{e^2}{m_e\omega_{He}} \left(\frac{\omega_{He}}{\omega}\right)^{5/2} \Psi'(\omega, \varepsilon), \tag{12.41}$$

where the dimensionless function Ψ' (the prime does not indicate differentiation) is

$$\Psi'(\omega, \varepsilon) = \sqrt{3}q^{5/6}\left\{1+q\left[1+\left(\frac{\zeta}{q}\right)^2\right]^{3/2} - \frac{3}{2}\frac{\zeta^2}{q}\left[1+\left(\frac{\zeta}{q}\right)^2\right]^{1/2}\right\}\exp\left\{-\frac{2}{3}q\left[1+\left(\frac{\zeta}{q}\right)^2\right]^{3/2}\right\}. \tag{12.42}$$

We can at once draw some important conclusions from (12.42). Firstly, in the case when the influence of the plasma is unimportant ($\zeta = 0$) the reabsorption coefficient is always positive. In other words, the synchrotron emission of relativistic electrons *in vacuo* can never lead to an amplification of the electromagnetic field (we remember that so far we have considered only an isotropic particle velocity distribution).

The influence of the plasma gives in the reabsorption coefficient a negative part. This effect was discovered† by Zheleznyakov (1967a, b) and McCray (1966) (see also Zheleznyakov and Suvorov, 1968) who obtained for the reabsorption coefficients an expression corresponding to substituting into (12.39) equation (12.6) in the limiting cases $\zeta \gg 1$ and $\zeta \ll 1$. Equations (12.41) and (12.42) are convenient as they are the same for the whole frequency range.

Equation (12.42) show that the negative part of the reabsorption coefficient dominates the positive part when $\zeta^3/q^2 \gtrsim 2$. For larger values of this quantity the exponential function is small so that the reabsorption coefficient is negative and not exponentially small only in a narrow frequency range around the value corresponding to the condition $\zeta^3 \approx 4q^2$, that is,

$$\omega_n \approx \left(\frac{\omega_{pe}^3}{4\omega_{He}} \frac{\varepsilon}{m_ec^2}\right)^{1/2}. \tag{12.43}$$

The width of the range where the reabsorption is negative is of the order $0{\cdot}3\omega_n$ on both sides of the value (12.43). When $\omega \lesssim 0{\cdot}7\omega_n$ the reabsorption coefficient is still negative, but exponentially small, while for $\omega \gtrsim 1{\cdot}3\omega_n$ the reabsorption coefficient is positive (Fig. 57).

We note that a negative part of the synchrotron reabsorption coefficient, that is, synchrotron build-up, occurs in general in a medium with a refractive index which is different from unity. The criterion for the frequencies at which the synchrotron instability occurs for an

† A quantity corresponding to $\Psi'(\omega, \varepsilon)$ was found earlier by Gaĭlitis and Tsytovich (1963) and they found that taking the influence of the plasma into account led to its being negative for some frequencies.

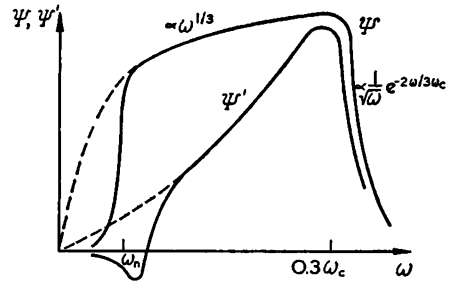

FIG. 57. Frequency dependence of the synchrotron emission coefficient (Ψ) and the reabsorption coefficient (Ψ') for a particle of a given energy. The dashed curves correspond to the case when the influence of the plasma is neglected.

arbitrary form of the refractive index $n(\omega)$, but under the condition that $n(\omega) < 1$, has the form (Kaplan and Tsytovich, 1972b)

$$\frac{\omega\varepsilon}{ecH}\left[1-n(\omega)-\left(\frac{m_ec^2}{\varepsilon}\right)^2\right]\Big/\left\{2[1-n(\omega)]+\left(\frac{m_ec^2}{\varepsilon}\right)^2\right\} \gtrsim 1.$$

Synchrotron instability can occur also when $n(\omega) > 1$, but for separate, very narrow frequency intervals.

We now turn to evaluating the synchrotron damping rate. We obtain the expression we want by substituting (12.41) and (12.42) into (12.38). The general formula is cumbersome but for astrophysical problems we can fully manage with an approximate formula which describes both the low- and the high-frequency parts of the spectrum. In it we shall again omit numerical γ-dependent factors which differ little from unity:

$$\gamma^t(\omega) = -\frac{n_*}{n_e}\frac{\omega_{pe}^2}{\omega_{He}}\left(\frac{\omega_{He}}{\omega^*_{max}}\right)^{5/2}\frac{(\omega^*_{max}/\omega)^{(\gamma+4)/2}}{1+(\omega^*_{max}/\omega)^{(3\gamma+2)/6}}. \tag{12.44}$$

The damping rate decreases with increasing frequency, first slowly (as $\omega^{-5/3}$), when $\omega < \omega^*_{max}$ and then faster (as $\omega^{-(\gamma+4)/2}$) when $\omega > \omega^*_{max}$.

Taking the influence of the plasma into account changes this picture somewhat. In the case of the standard distribution function (12.2) the role of the plasma is, as before, small when $\varepsilon_* \gg m_ec^2\omega_{pe}/\omega_{He}$ but when $\varepsilon_* \ll m_ec^2\omega_{pe}/\omega_{He}$ there is a drop in the damping rate for frequencies less than (12.18) similar to the drop in the emission coefficient.

There arises, however, a new possibility here of the damping in principle changing into amplification. It is true that for this it is necessary that the energy distribution function of the particles is such that there is a positive slope at those energies for which the reabsorption coefficient is negative, that is, that in that energy interval $(\partial/\partial\varepsilon)(f_e/\varepsilon^2) > 0$. Moreover, the distribution function must in the remaining range of energies be small, as otherwise the large positive part of the integrand in (12.37) will swamp the small negative part of this expression. In other words, one can obtain negative reabsorption only when the distribution function has a maximum for an energy determined by the equation

$$\frac{\varepsilon_*}{m_ec^2} \approx \frac{\omega_{pe}}{\omega_{He}}, \tag{12.45}$$

and decreases steeply, almost being cut-off at both sides of this maximum (more steeply than ε^2 at the low-energy side and more steeply than ε^{-2} on the high-energy side). The excitation of the radiation then takes place in a narrow frequency range near the frequency $\omega_n \approx \omega_{pe}^2/2\omega_{He}$. The growth rate is of order

$$\gamma^t \approx \frac{n_*}{n_e} \frac{\omega_{He}^4}{\omega_{pe}^3}. \tag{12.46}$$

Figure 58 (Kaplan, 1968) illustrates the sensitivity of the excitation effect to the choice of the energy of the relativistic electrons and to the determination of the optical depth at which the excitation becomes noticeable; in this figure we give the total (integrated over angles) intensity of the emission by a system of mono-energetic relativistic electrons as function of their energy ε_* and of the optical depth $\tau_0 = \gamma^t R/c$, where γ^t is given by (12.46)

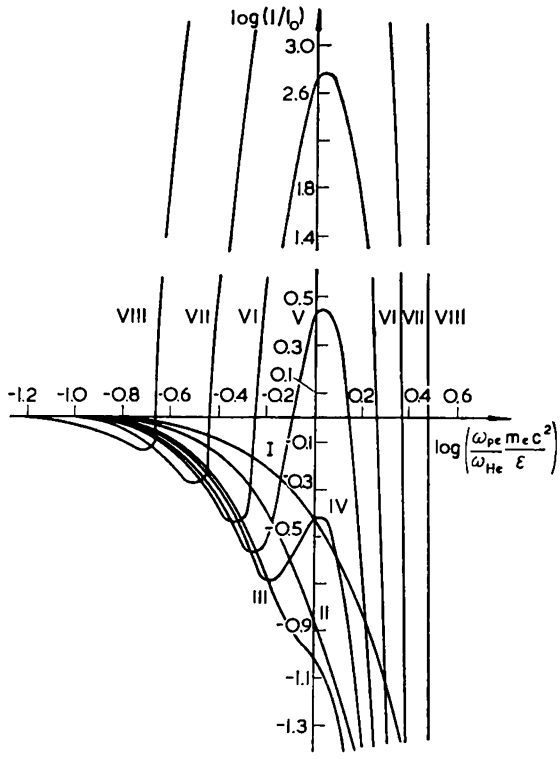

FIG. 58. The intensity of the electromagnetic radiation excited by the synchrotron instability of transverse waves, taking the influence of the plasma into account, as function of the particle energy and of the optical depth τ_0 of the system: I: $\tau_0 = 1$; II: 100; III: 600; IV: 800; V: 1000; VI: 1500; VII: 3000; VIII: 10 000.

and R is the size of the system. Non-linear effects are neglected; I_0 is the intensity for $\tau_0 \ll 1$. From the curves in Fig. 58 it is clear that amplification is possible only if $\tau_0 \gtrsim 10^3$ and for the following range of electron energies

$$0{\cdot}9 \lesssim \frac{\omega_{pe}}{\omega_{He}} \frac{m_e c^2}{\varepsilon_*} \lesssim 1{\cdot}5. \tag{12.47}$$

We emphasize that if the energy of the mono-energetic electrons ε_* lies outside the limits (12.47) or if the condition $\tau_0 \gtrsim 10^3$ is not satisfied there will be no amplification of the synchrotron emission. It is also important that for particle energies given by the condition (12.47) Langmuir waves are more efficiently emitted than electromagnetic ones by the synchrotron mechanism. It is natural to expect that the growth rate for Langmuir waves will also be much larger than (12.46) (Goreva and Suvorov, 1972; Kaplan and Tsytovich, 1972b).

Substituting expression (12.22) into (12.39) we find the coefficient for synchrotron reabsorption of Langmuir waves:

$$\varkappa^l = \frac{2\pi^2 e^2}{3} \frac{q^{1/2}(\omega_{He}/\omega_{pe})^{3/2}}{m_e \omega_{He}} \left\{ 2 + \frac{2}{3} q \left[1 + \left(\frac{\zeta}{q} \right)^2 \right]^{3/2} - 2 \frac{\zeta^2}{q} \left[1 + \left(\frac{\zeta}{q} \right)^2 \right]^{1/2} \right\} \times \exp\left\{ -\frac{2}{3} q \left[1 + \left(\frac{\zeta}{q} \right)^2 \right]^{3/2} \right\}, \tag{12.48}$$

where

$$\left(\frac{\zeta}{q} \right)^2 = 2 \frac{\omega_{pe} - kc}{\omega_{pe}} \left(\frac{\varepsilon}{mc^2} \right)^2, \quad q = \frac{\omega_{pe}}{\omega_{He}} \left(\frac{m_e c^2}{\varepsilon} \right)^2.$$

We have specially written (12.48) in a form which makes it resemble (12.42). The analysis of relation (12.48) runs parallel to the one given earlier for (12.42). The reabsorption coefficient is positive everywhere except a small interval where $\zeta^3 \gtrsim q^2$. It follows from (12.48) that for k exactly equal to $k_c = \omega_{pe}/c$ the waves are only damped, with a damping rate

$$\gamma^l = -\frac{2\pi^2(\gamma - 1)}{3} \frac{e^2 c^2}{\omega_{pe} \varepsilon_*} n_* \approx -\frac{n_*}{n_e} \frac{m_e c^2}{\varepsilon_*} \omega_{pe}. \tag{12.49}$$

When k decreases the reabsorption coefficient decreases and when

$$\frac{k_c - k_1}{k_c} \approx \frac{1}{4} \left(\frac{\omega_{He}}{\omega_{pe}} \frac{m_e c^2}{\varepsilon_*} \right)^{2/3}, \tag{12.50}$$

it becomes negative. Its maximum absolute value is reached approximately when

$$\frac{k_c - k_2}{k_c} \approx \frac{1}{2} \left(\frac{\omega_{He}}{\omega_{pe}} \frac{m_e c^2}{\varepsilon} \right)^{2/3}, \tag{12.51}$$

that is, in the middle of the range of wavenumbers of the plasma waves generated by the synchrotron mechanism. If we assume that there is here also a distribution function with a maximum at the energy (12.45) and with steep drops at both sides, there will be a synchrotron instability also for the longitudinal waves but now with a growth rate

$$\gamma^l \approx \frac{n_*}{n_e} \frac{m_e c^2}{\varepsilon_*} \omega_{pe}. \tag{12.52}$$

The ratio of the electromagnetic to the Langmuir growth rate is

$$\frac{\gamma^t}{\gamma^l} \approx \left(\frac{\omega_{He}}{\omega_{pe}} \right)^3 \ll 1. \tag{12.53}$$

This inequality indicates that the synchrotron instability is more difficult for electromagnetic waves. Indeed, as soon as the slope corresponding to the synchrotron instability is formed on the distribution function, in first instance, Langmuir waves are excited. This is caused by the fact that, firstly, their growth rate is much larger; secondly, that the group velocity of the longitudinal plasmons is much lower than the group velocity of the electromagnetic waves so that for them the optical depth becomes earlier larger than unity; thirdly, that although the collisional damping rate of the Langmuir waves is also larger than that of the transverse waves, their ratio is equal to $(\omega_{pe}/\omega_{He})^2$—in other words, one can even realize conditions where γ^t is less than and γ^l larger than the corresponding damping rate.

In equation (12.53) we compared the growth rates for electrons with the same energy $\varepsilon_* \approx m_e c^2 \omega_{pe}/\omega_{He}$, where the growth rate of the electromagnetic waves is a maximum. However, the growth rate for Langmuir waves is a maximum for electrons with a lower energy, namely, when $\varepsilon_* \approx m_e c^2 \sqrt{(\omega_{pe}/\omega_{He})}$, for which we have $\gamma^l_{max} \approx (n_*/n_e)\sqrt{(\omega_{pe}\omega_{He})}$. The width of the range of wavenumbers for which Langmuir waves are excited is then larger than (12.51) and approximately equal to $k_c - k \approx \omega_{He}/c$.

Moreover, it is easier to stabilize the synchrotron instability for longitudinal plasmons through non-linear processes than the synchrotron instability for electromagnetic waves so that the case is also possible when a considerable amplification of electromagnetic waves only occurs.

Longitudinal plasmons with a phase velocity less than c undergo also Landau damping on relativistic particles. The corresponding evaluation of the reabsorption coefficient is elementary as u_k is here independent of the energy:

$$\varkappa^l = \frac{2u_k\omega_{pe}}{\varepsilon} = \frac{\pi\omega_{pe}^4}{c^3k^3n_e}\frac{m_e c^2}{\varepsilon}. \tag{12.54}$$

Integrating, using the distribution function (12.2), we find the damping rate:

$$\gamma^l \approx -\frac{n_*}{n_e}\frac{\omega_{pe}^2}{(ck)^3}\frac{m_e c^2}{\varepsilon_*}. \tag{12.55}$$

The damping rate decreases with increasing wavenumber. Its maximum value, at $k \approx k_c$ is equal to

$$\gamma^l \approx -\frac{n_*}{n_e}\frac{m_e c^2}{\varepsilon_*}\omega_{pe}, \tag{12.56}$$

that is, is of the same order as the synchrotron damping rate. The general behaviour of the damping rate of Langmuir waves is shown in Fig. 59. We stress that when $k > k_c$ Landau damping occurs and when $k < k_c$ synchrotron damping in a narrow interval. One must, of course, take both kinds of damping into account when the corresponding damping rate is larger than the collision frequency, that is, when $\gamma^l \gg \nu_e$.

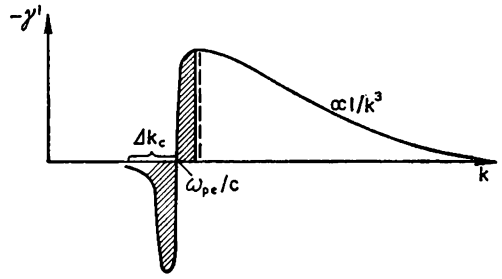

FIG. 59. Damping and excitation of Langmuir waves by isotropic relativistic particles. The shaded region corresponds to the synchrotron mechanism, the unshaded one to the Cherenkov emission mechanism. Excitation occurs for $k < \omega_{pe}/c$ in an interval Δk_c (see equation (12.24)).

We shall also give the expression for the Landau damping rate for Alfvén waves by relativistic particles with the distribution function (12.2):

$$\gamma^a = -\frac{\pi^2 e^2 v_A^2}{3ck} \int_{eH/k}^{\infty} f_\varepsilon \frac{d\varepsilon}{\varepsilon} = -\frac{(\gamma-1)\pi^2}{6} \frac{e^2 v_A^2 n_*}{c\varepsilon_* k_*} \frac{\lambda[\lambda(\gamma+1)+1]}{(1+\lambda)^{\gamma+1}}, \tag{12.57}$$

where we have the reduced Alfvén wavelength $\lambda = k_*/k$, $k_* = eH/\varepsilon_*$. The maximum value of the damping rate is reached for $k \approx k_*$:

$$\gamma^a \approx -0{\cdot}1 \frac{n_*}{n_e} \omega_{Hi}. \tag{12.58}$$

This damping rate can in the case of a relatively large concentration of relativistic particles be larger than the collisional damping rate.

ANISOTROPIC DISTRIBUTIONS AND EXCITATION OF PLASMA WAVES BY RELATIVISTIC PARTICLES

At the start of this section we reminded ourselves that under cosmic conditions one rather often encounters anisotropic particle velocity distributions and, possibly, rather narrow beams. The condition $(\boldsymbol{k}\cdot[\partial f_p/\partial \boldsymbol{p}]) > 0$ can here certainly be satisfied.

However, we noted already that the synchrotron excitation of the electromagnetic radiation field is also then under astrophysical conditions rather improbable. We are dealing in this case with the effect of the amplification of an electromagnetic field occurring thanks to the anisotropy in the electron distribution when there is no cold plasma present (Sazonov and Tsytovich, 1968). We bear in mind that isotropic electrons in that case always lead to absorption and that for amplification it is necessary that the degree of anisotropy in the electron distribution exceeds a certain critical value which can be found from the following simple considerations.

The synchrotron radiation of ultra-relativistic electrons is concentrated in a narrow solid angle of the order of $(m_e c^2/\varepsilon)^2$. Only if the distribution function changes significantly within the limits of these angles can the repopulation of the energy levels, connected with the anisotropy, affect the radiation. The degree of anisotropy must thus be very large to produce amplification: $\theta \ll m_e c^2/\varepsilon$.

Such a stimulation of the electromagnetic synchrotron radiation might, for instance, occur if all electrons move strictly in one plane perpendicular to the magnetic field. However, such a distribution is unstable against many plasma instabilities and it is improbable that such a particle velocity distribution will often be realized under cosmic conditions.

A strongly anisotropic particle velocity distribution with $\theta \ll m_e c^2/\varepsilon$ can, generally speaking, be realized in the important case when a particle beam moves almost strictly along the magnetic field. However, in that case there is in general no synchrotron emission in the usual sense of the word. This case will be considered separately in the next chapter.

There is thus little probability for an anisotropic synchrotron instability under cosmic conditions. However, for a number of astrophysical objects the beam instability of relativistic particles with respect to the excitation of other plasma waves has a large value (Ginzburg, 1966; Tsytovich, 1966b; Lerche, 1966, 1967, 1968).

Let us consider the excitation of longitudinal plasmons by an anisotropic beam of relativistic electrons where we shall assume that the spread in the directions of the particle velocities satisfies the inequality

$$1 > (\Delta\theta)^2 > \left(\frac{m_e c^2}{\varepsilon_*}\right)^2. \tag{12.59}$$

The magnitude of the growth rate depends significantly on $(\Delta\theta)^2$. One checks easily that if $(\Delta\theta)^2$ is not too small compared with unity, one must use for the evaluation of the growth rate the averaged probability of the Cherenkov emission and the magnitude of the growth rate will be of the same order of magnitude as the damping rate (12.56), that is,

$$\frac{\gamma^1}{\omega_{pe}} \approx \frac{n_*}{n_e}\frac{m_e c^2}{\varepsilon_*} \qquad (1 \gtrsim (\Delta\theta)^2). \tag{12.60}$$

It is important that now the growth rate is practically independent of the spread in energy $\Delta\varepsilon_*$, but equation (12.60) is valid as long as this spread and the angular spread $\Delta\theta$ are not too small (we give the appropriate criterion in a moment).

In the case of narrow beams the derivative of the distribution function with respect to the directions contributes more to the growth rate than the derivative with respect to the energy. To estimate the growth rate one can use here the formula $[(\Delta\theta)^2 \gg (m_e c^2/\varepsilon_*)]$:

$$\frac{\partial f_p}{\partial \boldsymbol{p}} \approx \frac{n_*}{(p\,\Delta\theta)^2} \approx \frac{n_* c^2}{\varepsilon_*^2 (\Delta\theta)^2}. \tag{12.61}$$

We thus have for the growth rate the estimate

$$\frac{\gamma^1}{\omega_{pe}} \approx \frac{n_*}{n_e}\frac{m_e c^2}{\varepsilon_*}\frac{1}{(\Delta\theta)^2}. \tag{12.62}$$

A more exact expression contains instead of $(\Delta\theta)^{-2}$ the expression

$$\frac{(\Delta\theta)^2 + (m_e c^2/\varepsilon_*)^2}{[(\Delta\theta)^2 + (m_e c^2/\varepsilon_*)^2 (\Delta\varepsilon_*/\varepsilon_*)]^2},$$

which for $(\Delta\theta)^2 \gg (m_e c^2/\varepsilon_*)^2$ is practically the same as $(\Delta\theta)^{-2}$. Tsytovich (1969c) and Rudakov (1971; see also Breĭzman and Ryutov, 1971) have given a rigorous derivation of

the last formula. The maximum value of the growth rate is reached in the case when $(\Delta\theta)^2 \lesssim (m_e c^2/\varepsilon_*)^2(\Delta\varepsilon_*/\varepsilon_*)$:

$$\frac{\gamma^1}{\omega_{pe}} \approx \frac{n_*}{n_e}\frac{\varepsilon_*}{m_e c^2}\left(\frac{\varepsilon_*}{\Delta\varepsilon_*}\right)^2. \tag{12.63}$$

This growth rate depends on the energy spread $\Delta\varepsilon_*$, but this dependence vanishes when $(\Delta\theta)^2 \gg (m_e c^2/\varepsilon_*)^2(\Delta\varepsilon_*/\varepsilon_*)$.

When the spread in energy and angle is small (a so-called monoenergetic beam) there occurs a hydrodynamic instability with a growth rate (Faĭnberg, Shapiro and Shevchenko, 1970):

$$\frac{\gamma^1}{\omega_{pe}} \approx \left(\frac{n_*}{n_e}\right)^{1/3}\left(\frac{m_e c^2}{\varepsilon_*}\right)^{1/3}\left[\left(\frac{m_e c^2}{\varepsilon_*}\right)^3 + (\Delta\theta)^2\right]^{1/3}. \tag{12.64}$$

Equation (12.64) is valid until the growth rate becomes less than $(\boldsymbol{k}\cdot\Delta\boldsymbol{v}) \approx \omega_{pe}[(\Delta v/c)+(\Delta\theta)^2]$ where Δv is the spread in velocities corresponding to the energy spread $\Delta\varepsilon_*$. From this we get a criterion for the spread in the energy in a beam in which the hydrodynamic instability is larger than the kinetic one:

$$(\Delta\theta)^2 + \left(\frac{m_e c^2}{\varepsilon_*}\right)^2\frac{\Delta\varepsilon_*}{\varepsilon_*} \lesssim \left(\frac{n_*}{n_e}\right)^{1/3}\left(\frac{m_e c^2}{\varepsilon_*}\right)^{1/3}\left[\left(\frac{m_e c^2}{\varepsilon_*}\right)^2 + (\Delta\theta)^2\right]^{1/3}. \tag{12.65}$$

As a rule, hydrodynamic instability is not stabilized. Therefore, if for a newly formed beam the spread in energy and angle is very small, less than (12.65), it increases the spread in energy to reach a value of the order (12.65) very rapidly with a growth rate (12.64). After that the stage of kinetic instability starts (growth rates (12.60) to (12.63)). We shall show below that a kinetic instability of relativistic particles is fairly easily stabilized by non-linear processes so that the parameters of the beam can be conserved over a considerable period under conditions determined by the spread in energy and direction given by (12.65).

Anisotropically distributed particles can excite also other types of waves, in particular, Alfvén waves (Lerche, 1967). The maximum growth rate is then of the order of the quantity (12.58), but with the other sign.

Stabilization of relativistic particle instabilities

The role of the stabilization of instabilities is very large under cosmic conditions where the characteristic times are much larger than the reciprocal of any growth rates. If there were no stabilization, it would in general be difficult to detect the development and maintenance of a well-defined level of plasma turbulence. The role of the stabilization of beams in the determination of the sporadic solar emission was considered in detail in § 8.

Still more important is the stabilization of the instability of relativistic particles. This is explained by several reasons. Firstly, the growth rates of the instabilities of relativistic particles are relatively small (one must bear in mind that the growth rates are determined by

the number of relativistic particles while the rates for the stabilizing non-linear processes are determined by the particle density in the main plasma). Secondly, the characteristic time for changing the energy through quasi-linear relaxation of relativistic particles is also rather long. Thirdly, the range of wavenumbers of longitudinal plasmons excited by an ultra-relativistic beam,

$$\frac{\Delta k}{k} \approx \left(\frac{m_e c^2}{\varepsilon_*}\right)^2 \frac{\Delta\varepsilon_*}{\varepsilon_*} + (\Delta\theta)^2, \tag{12.66}$$

is, as a rule, very narrow and non-linear transfer rapidly removes longitudinal plasmons from resonance with beam particles. Moreover, if Δk in (12.66) is less than $k_* \approx \frac{1}{3}(m_e/m_i)^{1/2}(\omega_{pe}/v_{Te})$ given by (3.8) with $T_e = T_i$, a longitudinal plasmon will escape from resonance with the beam at once in a single non-linear scattering process and if then the magnitude of the growth rate is less than the rate of the non-linear transfer, a beam of ultra-relativistic particles will practically not excite any Langmuir waves (since in the resonance wavenumber region $W_k^l \approx 0$). Such a beam is practically completely stabilized with respect to the excitation of longitudinal plasmons (Tsytovich and Shapiro, 1965; Tsytovich, 1966b, 1969c). Using equation (3.17) where $\partial W_k^l/\partial k$ is replaced by $W^l/k_* k$, we find for the criterion for such an "absolute" stabilization:

$$\frac{W^l}{n_* \varepsilon_*} \gtrsim \frac{10}{(\Delta\theta)^2} \left(\frac{m_i}{m_e}\right)^{1/2} \left(\frac{v_{Te}}{c}\right)^3 \left(\frac{m_e c^2}{\varepsilon_*}\right)^2 \tag{12.67}$$

under the necessary condition following from the inequality $k_* > \Delta k$:

$$\frac{1}{3}\sqrt{\frac{m_e}{m_i}}\frac{c}{v_{Te}} > \left(\frac{m_e c^2}{\varepsilon_*}\right)^2 \frac{\Delta\varepsilon_*}{\varepsilon_*} + (\Delta\theta)^2. \tag{12.68}$$

In equation (12.67) W^l is the energy density of the plasmons which are not in resonance with the beam particles; there are here practically no resonance plasmons.

Inequality (12.68) is under cosmic conditions for ultra-relativistic beams as a rule satisfied. One may assume that any ultra-relativistic beam in a cosmic plasma excites longitudinal plasmon turbulence to the level determined by equation (12.67) and then after that, remaining stabilized, maintains the turbulence at that level, if the dissipative processes lead to a diminishing number of longitudinal plasmons.

If $k_* < \Delta k$ there is no "absolute" stabilization, but the usual stabilization through quasi-linear relaxation of the kind considered in § 8 occurs. The characteristic rate for the beam relaxation when longitudinal plasmon turbulence is excited is then:

$$\frac{\gamma_*^l}{\omega_{pe}} \approx \frac{c^2 D_\perp^l}{\omega_{pe}\varepsilon_*^2(\Delta\theta)^2} \approx 30\frac{m_i}{m_e}\frac{n_*}{n_e}\left(\frac{v_{Te}}{c}\right)^4 \left(\frac{m_e c^2}{\varepsilon_*}\right)^3 \ll \frac{\gamma^l}{\omega_{pe}}, \tag{12.69}$$

where we have assumed that $\Delta\theta > (m_e c^2/\varepsilon_*)^2(\Delta\varepsilon_*/\varepsilon_*)$. Here $1/\gamma_*^l$ is the decay time of the beam and $1/\gamma^l$ is the time for an appreciable increase in the energy level of the longitudinal plasmons.

The synchrotron instability of the Langmuir waves produced by isotropically distributed relativistic electrons can also be stabilized. The whole difference with the beam stabilization case considered a moment ago reduces to the fact that now there is another range of wave-

numbers of longitudinal plasmons which are in resonance with the unstable part of the particle distribution function and that now we must consider (12.51) instead of (12.66). Using the method of § 8 we find the spectral energy density of the longitudinal plasmons in the resonance wavenumber range:

$$W_k^l \approx \frac{\gamma^l \Delta k}{\alpha^l} \approx 30 \frac{m_i}{m_e} \left(\frac{\omega_{He}}{\omega_{pe}}\right)^{2/3} \left(\frac{m_e c^2}{\varepsilon_*}\right)^{8/3} \left(\frac{v_{Te}}{c}\right)^4 \frac{c}{\omega_{pe}} n_* \varepsilon_* . \tag{12.70}$$

Substituting this quantity into the expression for the diffusion coefficient (5.6) we find the characteristic relaxation rate of relativistic electrons due to instabilities involving longitudinal waves:

$$\frac{\gamma_*^l}{\omega_{pe}} \approx \frac{c^2 D_{\parallel}^l}{\omega_{pe} \varepsilon_*^2} \approx 30 \frac{n_*}{n_e} \frac{m_i}{m_e} \left(\frac{\omega_{He}}{\omega_{pe}}\right)^{4/3} \left(\frac{m_e c^2}{\varepsilon_*}\right)^{13/3} \left(\frac{v_{Te}}{c}\right)^4 . \tag{12.71}$$

Comparing (12.71) and (12.52) we verify that indeed the relaxation rate of the synchrotron instability is practically always several orders smaller than the growth rate γ^l for longitudinal plasmons for the same synchrotron instability. This means that the stabilization effect usually occurs, that is, that the time of relaxation of relativistic electrons due to the instability is much larger than the characteristic time of the growth of the Langmuir oscillations. We must, of course, bear in mind that the plasma turbulence intensity in the region of its direct action on relativistic electrons must be evaluated, taking into account the non-linear transfer which stabilizes this instability.

We can consider similarly also the stabilization of the synchrotron instability involving electromagnetic waves. Stimulated Compton scattering or the decay process $t \to t' + l$ remove electromagnetic waves from resonance with the synchrotron instability and in this way stabilize it. Both non-linear effects lead to approximately the same result and we therefore take for the growth rate of the stabilized process equation (3.37) which here we write in the form

$$\frac{\gamma^t}{\omega_{pe}} \approx \frac{\pi}{3} \frac{\omega_{pe}^3}{n_e m_e c^2} \frac{\partial}{\partial \omega} \left(\frac{W_\omega^t}{\omega}\right) \approx \frac{W^t}{n_e m_e c^2} \left(\frac{\omega_{pe}}{\omega}\right)^3 . \tag{12.72}$$

Substituting now $\omega \approx \omega_n$ from (12.43) with $\varepsilon \approx m_e c^2 \omega_{pe}/\omega_{He}$, that is, putting $\omega_n \approx \omega_{pe}^2/\omega_{He}$ and comparing the non-linear transfer rate with the growth rate for electromagnetic waves due to the synchrotron instability (12.46), we find the energy density of the electromagnetic waves for which stabilization occurs:

$$\frac{W^t}{n_e m_e c^2} \approx \frac{n_*}{n_e} \frac{\omega_{He}}{\omega_{pe}} \approx \frac{n_*}{n_e} \frac{m_e c^2}{\varepsilon_*} . \tag{12.73}$$

Such an energy density is established as the result of a balance between the excitation of electromagnetic waves and their transfer from resonance with the synchrotron instability. It follows from (12.73) that always $W^t \ll n_* \varepsilon_*$; this means that the relaxation of the synchrotron instability involving electromagnetic waves also proceeds relatively slowly. Equation (12.72) determines also the spectrum of the electromagnetic radiation in the non-resonance region, $W_\omega^t \propto \omega$ (Fig. 60).

We shall obtain in § 14 the coefficient for the diffusive acceleration of relativistic electrons when they absorb electromagnetic waves in the synchrotron mechanism (equation (14.16)).

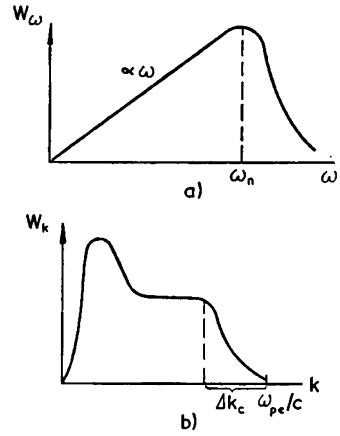

FIG. 60. The spectra of transverse (a) and longitudinal (b) waves excited by synchrotron instability for the case of an isotropic distribution of radiating particles.

Using this formula we can estimate also the relaxation time of the synchrotron instability. We have for the diffusion coefficient of the synchrotron acceleration:

$$D_{\parallel}^{t} \approx \frac{\pi^2 e^2}{\omega}\left(\frac{m_e c^2}{\varepsilon_*}\right)^2 W^t \approx \frac{m_e \omega_{He}}{n_e}\left(\frac{\omega_{He}}{\omega_{pe}}\right)^2 W^t. \tag{12.74}$$

The relaxation rate is thus

$$\frac{\gamma_*^t}{\omega_{pe}} \approx \frac{c^2 D_{\parallel}^t}{\omega_{pe}\varepsilon_*^2} \approx \frac{n_*}{n_e}\left(\frac{\omega_{He}}{\omega_{pe}}\right)^6. \tag{12.75}$$

The ratio of the relaxation rate involving electromagnetic waves to the relaxation rate involving longitudinal waves in the synchrotron instability is

$$\frac{\gamma_*^t}{\gamma_*^l} \approx 10^{-5}\left(\frac{c}{v_{Te}}\right)^4\left(\frac{\omega_{He}}{\omega_{pe}}\right)^{1/3}. \tag{12.76}$$

This ratio can be both larger than and smaller than unity so that in different actual conditions the decay of the synchrotron instability can proceed in a different way. We have compared here γ_*^l and γ_*^t for the same electron energy $\varepsilon_* \approx m_e c^2 \omega_{pe}/\omega_{He}$ while γ_*^l reaches a maximum for $\varepsilon_* \approx m_e c^2 (\omega_{pe}/\omega_{He})^{1/2}$.

The equations of the present subsection, in particular the estimates of the energy density of the longitudinal plasmons excited by the beam instability of the ultra-relativistic electrons (equation (12.67)) and the estimate of the energy density of electromagnetic waves excited by the synchrotron instability (equation (12.73)), can be used to interpret the observational data. We shall consider this problem in § 15.

§ 13. Plasma and Compton emission mechanisms of ultra-relativistic particles

The conversion of plasma waves into electromagnetic waves through Compton or non-linear scattering of them by the particles in the plasma we called earlier plasma emission mechanisms. However, such a conversion occurs also through scattering by relativistic

particles (Gaĭlitis and Tsytovich, 1964a). We shall now consider these mechanisms, including here also the normal Compton scattering of electromagnetic waves. The particular interest of plasma emission mechanisms of relativistic particles for astrophysicists consists in that there occurs then a considerable increase in frequency. We remember that the conversion in a cold plasma occurs practically without change in frequency. In the simplest form this effect shows up by means of the energy and momentum conservation laws in scattering.

Analysis of the conservation laws for scattering

It is well known that the parameters of the incident and the scattered waves are connected by the conservation laws (equations (3.2) and (3.3)). We shall indicate the parameters of the incident wave by primes.

Let ω be the frequency of an electromagnetic wave with the dispersion relation $\omega^2 = \omega_{pe}^2 + c^2k^2$ and we shall assume that $\omega \gg \omega_{pe}$. We can rewrite equation (3.3) in the following form:

$$\omega(\boldsymbol{k}) = \omega'(\boldsymbol{k}') + ([\boldsymbol{k}-\boldsymbol{k}']\cdot\boldsymbol{v}) = \frac{\omega'(\boldsymbol{k}')-(\boldsymbol{k}'\cdot\boldsymbol{v})}{1-\frac{v}{c}\cos\vartheta+\frac{1}{2}\frac{v}{c}\frac{\omega_{pe}^2}{\omega^2}} \approx \frac{2[\omega'(\boldsymbol{k}')-(\boldsymbol{k}'\cdot\boldsymbol{v})]}{\left(\frac{mc^2}{\varepsilon}\right)^2+\vartheta^2+\frac{\omega_{pe}^2}{\omega^2}}, \quad (13.1)$$

where m is the particle (electron or ion) mass.

Equation (13.1) is valid also for Compton scattering by electromagnetic waves, for instance, by thermal photons (Ginzburg and Syravatskiĭ, 1964b). If ω' corresponds also to an electromagnetic wave (let, for the sake of simplicity, $\omega' \gg \omega_{pe}$ and $\vartheta' \gg mc^2/\varepsilon$), we have

$$\omega = \omega' \frac{2(1-\cos\vartheta')}{(mc^2/\varepsilon)^2+\vartheta^2}. \quad (13.2)$$

Depending on the angles ϑ and ϑ' emission in the scattering process takes place at different frequencies from $\omega = \omega_{pe}$ to

$$\omega_{max} = 4\omega'\left(\frac{\varepsilon}{mc^2}\right)^2. \quad (13.3)$$

The probability for emission at different frequencies varies, however, greatly in this broad spectrum. In accordance with the general properties of emission of relativistic particles we find that the maximum of the emission which is very steep occurs at frequencies close to the one given by (13.3) as they correspond to the case $\vartheta \to 0$, that is, concentration of the radiation within a solid angle smaller than $(mc^2/\varepsilon)^2$.

In the Compton effect a relativistic particle can also absorb high-frequency radiation moving almost exactly along the direction of motion of the particle ($\cos\vartheta' \approx 1$) and being re-emitted in the form of waves at a lower frequency

$$\omega_{pe} \leqslant \omega \leqslant \omega' \frac{(mc^2/\varepsilon)^2+(\omega_{pe}/\omega')^2}{2(1-\cos\vartheta)+(\omega_{pe}/\omega)^2}, \quad (13.4)$$

more or less uniformly in all directions. It is necessary to retain here the terms $(\omega_{pe}/\omega)^2$ and $(\omega_{pe}/\omega')^2$ as the frequency of the scattered waves is low and the case $\cos\vartheta = 1$ is also possible.

The Compton scattering of Langmuir waves accompanied by their conversion into electromagnetic waves occurs clearly only with an increase in frequency. It is convenient to distinguish here two cases: the phase velocities of the longitudinal plasmons are much higher or much lower than the velocity of light.

In the first case we can drop in equation (13.1) $(\boldsymbol{k}'\cdot\boldsymbol{v})$ and $(\omega_{pe}/\omega)^2$. We then get for the frequency of the electromagnetic wave when a longitudinal plasmon is converted.

$$\omega = \frac{2\omega_{pe}}{(mc^2/\varepsilon)^2+\vartheta^2}\,. \tag{13.5}$$

The largest probability occurs here for the emission at a frequency close to the upper limit $\omega = 2\omega_{pe}(\varepsilon/mc^2)^2$.

If longitudinal plasmons with a low phase velocity are scattered we have instead of (13.5)

$$\omega = \frac{2k'v\,|\cos\vartheta'|}{(mc^2/\varepsilon)^2+\vartheta^2} \leqslant 2k'c\left(\frac{\varepsilon}{mc^2}\right)^2, \tag{13.6}$$

and here only those plasmons are converted into electromagnetic waves for which the wavevector makes an angle $\vartheta' > \pi/2$ (as we must have $(\boldsymbol{k}'\cdot\boldsymbol{v}) < 0$) with the velocity vector. We note that as for longitudinal plasmons $k' \leqslant \frac{1}{3}(\omega_{pe}/v_{Te})$, emission of electromagnetic waves with frequencies up to $\frac{2}{3}(c/v_{Te})(\varepsilon/mc^2)^2\omega_{pe}$ is possible in the conversion process.

Finally, other plasma waves can also be converted into electromagnetic waves. As for all these waves the phase velocity is usually less than the velocity of light we have for the emitted frequencies

$$\omega = \frac{2k'c\,|\cos\vartheta'|}{(mc^2/\varepsilon)^2+\vartheta^2+(\omega_{pe}^2/\omega^2)}\,. \tag{13.7}$$

We must retain here the term $(\omega_{pe}/\omega)^2$ as now the emitted frequencies are not necessarily much larger than ω_{pe}. In particular, if electromagnetic waves are emitted in the scattering of very-low-frequency plasma waves by ultra-relativistic particles we have for the emitted frequencies:

$$\omega = \frac{\omega_{pe}^2}{2k'c\,|\cos\vartheta'|}\,. \tag{13.8}$$

We must note yet another case. When gyro-frequency plasmons with $\omega' \approx \omega_{He}$ are scattered one produces also electromagnetic waves with frequencies

$$\omega = 2\omega_{He}\frac{c}{v_p}\left(\frac{\varepsilon}{mc^2}\right)^2, \tag{13.9}$$

if the plasmon phase velocity is less than the velocity of light, and

$$\omega = 2\omega_{He}\left(\frac{\varepsilon}{mc^2}\right)^2, \tag{13.10}$$

if their phase velocity is larger than the velocity of light (this is possible in a plasma with $\omega_{He} \gg \omega_{pe}$).

We can compare the frequencies (13.9) and (13.10) with the synchrotron emission frequency (12.4). Of course, a relativistic electron always emits at the synchrotron frequency while it only emits at the frequencies (13.9) and (13.10) if there are plasmons present with a frequency ω_{He} but on the other hand the increase in frequency for the same particle energy is appreciably larger. For a relativistic ion the equation for the synchrotron emission would contain ω_{Hi} while emission occurs at the frequency (13.10) in scattering of gyro-frequency plasmons by an ion. The important consequence of this analysis is that plasma and synchrotron emission mechanisms give the same dependence of the maximum emitted frequency on the energy of the relativistic particle, $\omega_{\max} \propto (\varepsilon/mc^2)^2$.

The conservation laws given here are valid both for Compton scattering by ultra-relativistic electrons and for non-linear scattering by polarization "coats" of the same electrons or of relativistic ions. Differences occur only when we turn to the directional properties of the scattered radiation.

The Compton scattering by ultra-relativistic electrons is always accompanied by a concentration of the radiation in a narrow solid angle $(mc^2/\varepsilon)^2$ and conversion therefore occurs preferably in favour of the highest frequencies.

The polarization "coat" of ultra-relativistic particles also moves with a velocity close to the velocity of light, but the particles in the "coat" themselves have only thermal velocities. There is here no concentration of the radiation in a narrow solid angle, but it is more or less isotropically distributed. Therefore, although the non-linear scattering by the "coat" of the ultra-relativistic particles also can lead to conversion into high-frequency electromagnetic waves, the probability of such a conversion is small.

There is special interest to calculate the non-linear scattering by the "coat" of relativistic ions as we have no other means to observe them. Moreover there are grounds to expect that there may be many more relativistic ions than electrons in some objects as ions lose their energy more slowly. At least in cosmic rays the observed ion density is by two orders of magnitude larger than the electron density for the same range of energies. The conversion in the non-linear scattering by the "coats" of relativistic electrons is appreciably smaller than the analogous conversion in Compton scattering.

Non-linear scattering of electromagnetic and Langmuir waves by relativistic ions

It follows from an analysis of the conservation laws that the non-linear scattering of Langmuir and electromagnetic waves by a "coat" leads to the production of high-frequency electromagnetic waves even though the intensity is relatively low. A detailed calculation for the case of conversion in the non-linear scattering of any kind of plasma wave both by relativistic and by non-relativistic ions was given by Kaplan and Tsytovich (1969c). We shall give here only the results for the scattering of Langmuir waves: firstly, we can in that case obtain the largest increase in frequency and, secondly, the energy density of longitudinal plasmons can be comparatively large.

In objects with a large electromagnetic radiation density one may expect also appreciable non-linear scattering of electromagnetic waves by the "coats" of relativistic ions. We shall give the corresponding approximate formula for this effect.

One can evaluate in the usual way the emission probability (see Appendix IV). To obtain a simpler expression one can average this probability both over the direction of the plasma

turbulence wavevectors and over the wavevectors of the scattered electromagnetic waves. In that case we have, for the energy range $m_i c^2 \ll \varepsilon \ll m_i^2 c^2/m_e$,

$$u_{k,k'} = \frac{(2\pi)^3}{8} \frac{Z^2 \omega_{pe} e^4}{m_e^2 \omega^4} \frac{k'}{k} \Phi(\omega, k', \varepsilon). \tag{13.11}$$

Here $k = \omega/c$, $k' \gg \omega_{pe}/c$ is a plasma turbulence wavenumber, and the function $\Phi(\omega, k', \varepsilon)$ determines the result of the averaging of the probability over the angle ϑ between the vectors $\boldsymbol{k}$ and $\boldsymbol{v}$. It has the form

$$\Phi(\omega, k', \varepsilon) = \ln\left[\frac{2ck'}{\omega}\left(\frac{\varepsilon}{m_i c^2}\right)^2\right] + \frac{\frac{1}{2}\omega(m_i c^2/\varepsilon)^2 - ck'}{ck'}. \tag{13.12}$$

One can now easily find all conversion parameters. The spontaneous emission coefficient is

$$J_\omega^t(1+i \to t+i') = \frac{\omega^3}{2\pi^2 c^3} \int u_{k,k'} N_{k'}^l f_e \frac{d^3k'\, d\varepsilon}{(2\pi)^3}. \tag{13.13}$$

We substitute in this integral instead of N_k^l the spectral energy density of isotropic plasma turbulence and take for f_e the decreasing part of the distribution function (12.2)—which corresponds to the largest increase in frequency—while taking for n_* the relativistic ion density. We then get

$$J_\omega^t = \frac{\pi\gamma Z^2 e^4 n_*}{(\gamma-1) m_e^2 c^3 \omega} \int \left(\frac{2ck'}{\omega}\right)^{\frac{1}{2}(\gamma+1)} \left(\frac{\varepsilon_*}{m_i c^2}\right)^{\gamma-1} W_{k'}^l\, dk'. \tag{13.14}$$

We assume here that for all plasma wavenumbers the minimum energy of ions free t emit at a frequency ω, is larger than ε_*, that is,

$$\varepsilon_{min} = m_i c^2 \left(\frac{\omega}{2ck'}\right)^{1/2} \gg \varepsilon_*. \tag{13.15}$$

As one should expect, the largest contribution to the emission comes from longitudinal plasmons with the maximum wavenumber $k' \approx k_g$. Omitting numerical (γ-dependent) factors of order unity and assuming that $\varepsilon_* \approx m_i c^2$, we get the estimate

$$J_\omega^t \approx \frac{n_*}{n_e} \frac{Z^2 \omega_{pe}^3}{n_e c^3} \left(\frac{\omega_{pe}}{\omega}\right)^{\frac{1}{2}(\gamma+3)} \left(\frac{k_g c}{\omega_{pe}}\right)^{\frac{1}{2}(\gamma+1)} W^l. \tag{13.16}$$

In agreement with what was said in the preceding section the emission spectrum decreases steeply with increasing frequency proportional to $\omega^{-(\gamma+3)/2}$, that is, for cosmic-ray ions approximately as ω^{-3}.

One can easily explain the physical meaning of this behaviour. The radiation power at a given frequency is primarily determined by the number of ions free to produce such radiation, provided the conservation laws are satisfied. A similar picture occurs also for synchrotron emission. However, in that case the spectrum decreases more slowly, proportional to $\omega^{-(\gamma-1)/2}$. The steeper fall with increasing frequency is connected with the fact that the polarization of the "coat" decreases with increasing frequency proportional to $\varepsilon - 1 \approx \approx \omega_{pe}^2/\omega^2$. This gives the extra factor $\propto \omega^{-2}$.

It is at present difficult to say whether one can distinguish the emission just considered from the background of other mechanisms. However, in a turbulent plasma with a large density of multiply charged relativistic ions it may play a significant role.

One can also evaluate by standard methods the damping rate of the electromagnetic waves when they are inversely converted into plasma waves, a process which is connected with non-linear scattering by relativistic ions. For a power-law distribution we find in the same approximation as we used for equation (13.16):

$$\frac{\gamma^{t}(1+i \rightleftharpoons i'+t)}{\omega_{pe}} \approx -\frac{n_*}{n_e}\frac{Z^2W^1}{n_e m_e c^2}\left(\frac{\omega_{pe}}{\omega}\right)^{\frac{1}{2}(\gamma+8)}. \tag{13.17}$$

The damping rate decreases very fast with increasing frequency. An anisotropy in the ion-velocity distribution leads to an amplification of electromagnetic waves with a growth rate of the order of (13.17). Finally, we give an estimate for the emission coefficient for electromagnetic waves arising from the non-linear scattering of lower-frequency, but still electromagnetic waves by polarization "coats" of relativistic ions. As before we assume that the ion energy distribution follows a power law. We have

$$J^{t}_{\omega}(t'+i \to t+i') \approx \frac{n_*}{n_e}\frac{\omega_{pe}^4}{n_e c^3}\int\limits_{\frac{1}{4}\omega(m_i c^2/\varepsilon)^2}^{\infty}\left(\frac{\omega'}{\omega}\right)^{\frac{1}{2}(\gamma+3)} I_{\omega'}\frac{d\omega'}{\omega'}, \tag{13.18}$$

where $I_{\omega'}$ is the intensity of the low-frequency electromagnetic radiation. We have here the same spectrum of the scattered radiation as in the case of scattering of longitudinal plasmons. The scattering of Langmuir waves is more efficient than that of electromagnetic waves, if

$$W^1 \gtrsim \left(\frac{\omega_*}{k_g c}\right)^{\frac{1}{2}(\gamma+1)} W^t, \tag{13.19}$$

where ω_* is the frequency at the maximum of the scattered electromagnetic radiation and W^t its density. We give in the Appendix the probabilities for conversion of all the most important types of plasma turbulence through scattering by relativistic ions.

Compton scattering of electromagnetic and Langmuir waves by relativistic electrons

An ultra-relativistic electron moving along a spiral in a constant external magnetic field oscillates with a frequency $\omega_{He} m_e c^2/\varepsilon$ and produces synchrotron radiation. If, moreover it finds itself in a field of strong plasma turbulence and intense electromagnetic waves, its motion will be further modulated with the plasma wave frequency (for instance, ω_{pe} or the radiation frequency ω'). As a result the electron emits a very wide spectrum of frequencies with several maxima. One of them is at the frequency $\omega_{He}(\varepsilon/m_e c^2)^2$, another one at the frequency $\omega_{pe}(\varepsilon/m_e c^2)^2$, and a third one at the frequency $\omega'(\varepsilon/m_e c^2)^2$. The relative magnitude of the maxima is determined by the magnetic field, plasma, and electromagnetic wave energy densities.

The whole spectrum is thus essentially produced by a single emission mechanism; the differences are solely connected with the causes for the non-uniform motion of the electron. Of course, the emission coefficients for comparable accelerations will be approximately the same.

We start with the simplest case, that of the conversion of longitudinal and transverse plasmons with a frequency ω_{pe} and a phase velocity much higher than the velocity of light, which was considered in detail by Tsytovich and Chikhachev (1969, 1971).

We note that for longitudinal plasmons we can have either $v_p < c$ or $v_p > c$. The case $v_p \gg c$ is of interest because then the scattering probability is in first approximation independent of the plasmon wavenumber and the emission parameters are thus determined only by the total energy density of the plasma turbulence with $v_p > c$.

The probability for the conversion of longitudinal and transverse plasmons, averaged over the directions of both the scattered plasmons and the high-frequency waves which are produced is the same for both processes and equal to

$$\bar{u}_k = \frac{1}{2}\int u_{k,k'} \sin\vartheta' \, d\vartheta' = \frac{\pi}{12}\frac{\omega_{pe}}{n_e^2}\left(\frac{\omega_{pe}}{\omega}\right)^2\left(\frac{m_e c^2}{\varepsilon}\right)^2 \times\left\{1+\frac{1}{2}\left(\frac{\omega}{\omega_{pe}}\right)^2\left(\frac{m_e c^2}{\varepsilon}\right)^2\left[\left(\frac{m_e c^2}{\varepsilon}\right)^2 - 2\frac{\omega_{pe}}{\omega}\right]\right\}. \quad (13.20)$$

As this probability is the same for all plasmons with $k \ll \omega_{pe}/c$, according to what was said earlier, the emission coefficient will simply be proportional to the total plasmon energy density, $W^l + W^p$, in that part of the spectrum. For isotropically distributed relativistic electrons we have

$$J_\omega^t\begin{pmatrix} p+e \to t+e' \\ l+e \to t+e' \end{pmatrix} = \frac{\omega^3}{\pi^2 c^3}\frac{W^l+W^p}{\omega_{pe}}\int\limits_{m_e c^2(\omega/2\omega_{pe})^{1/2}}^{\infty} \bar{u}_k f_\varepsilon \, d\varepsilon. \quad (13.21)$$

In the subsequent calculations we shall retain also the numerical factors which depend on the index of the relativistic electron energy distribution, assuming that this distribution is described by equation (12.2); we need them in the next section. We shall not write down the general expressions for the whole of the spectrum but rather restrict ourselves to asymptotic formulae for the low and high-frequency regions.

We substitute (13.20) and (12.2) into (13.21). At low frequencies when $\omega \ll 2\omega_{pe}(\varepsilon_*/m_e c^2)^2$ we have

$$J_\omega^t = \frac{\gamma(\gamma-1)}{24\pi}\frac{\omega_{pe}^2}{c^3 n_e}\left(\frac{m_e c^2}{\varepsilon_*}\right)^2\frac{n_*}{n_e}\frac{\omega}{\omega_{pe}}(W^l+W^p). \quad (13.22)$$

In the high-frequency region when $\omega \gg 2\omega_{pe}(\varepsilon_*/m_e c^2)^2$ we have

$$J_\omega^t = \frac{2^{(\gamma-1)/2}\gamma(\gamma-1)(\gamma^2+4\gamma+11)}{12\pi(\gamma+3)(\gamma+5)}\frac{\omega_{pe}^3}{c^3 n_e}\left(\frac{\varepsilon_*}{m_e c^2}\right)^{\gamma-1}\frac{n_*}{n_e}\left(\frac{\omega_{pe}}{\omega}\right)^{\frac{1}{2}(\gamma-1)}(W^l+W^p). \quad (13.23)$$

We have for the damping rate of the electromagnetic waves when they are changed into longitudinal and transverse plasmons through Compton scattering

$$\gamma^t\begin{pmatrix} l+e \rightleftarrows t+e' \\ p+e \rightleftarrows t+e' \end{pmatrix} = (W^l+W^p)\frac{\omega}{\omega_{pe}}\int\limits_{m_e c^2(\omega/2\omega_{pe})^{1/2}}^{\infty} \bar{u}_k \varepsilon^2 \frac{\partial}{\partial\varepsilon}\left(\frac{f_\varepsilon}{\varepsilon^2}\right) d\varepsilon. \quad (13.24)$$

At low frequencies, when $\omega \ll 2\omega_{pe}(\varepsilon_*/m_ec^2)^2$ we find

$$\frac{\gamma^t}{\omega_{pe}} = -\frac{\pi\gamma(\gamma^2-1)}{24}\frac{n_*}{n_e}\frac{W^l+W^p}{n_e\varepsilon_*}\left(\frac{m_ec^2}{\varepsilon_*}\right)^2\frac{\omega_{pe}}{\omega_*} \approx -\frac{n_*}{n_e}\frac{W^l+W^p}{n_em_ec^2}\left(\frac{m_ec^2}{\varepsilon_*}\right)^3\frac{\omega_{pe}}{\omega}. \quad (13.25)$$

In the high-frequency region when $\omega \gg 2\omega_{pe}(\varepsilon_*/m_ec^2)^2$ we have

$$\frac{\gamma^t}{\omega_{pe}} = -\frac{\pi 2^{\gamma/2}\gamma(\gamma^2-1)(\gamma^2+6\gamma+16)}{12(\gamma+4)(\gamma+6)}\frac{n_*}{n_e}\frac{W^l+W^p}{n_e\varepsilon_*}\left(\frac{\varepsilon_*}{m_ec^2}\right)^\gamma\left(\frac{\omega_{pe}}{\omega}\right)^{\frac{1}{2}(\gamma+4)}$$

$$\approx -\frac{n_*}{n_e}\frac{W^l+W^p}{n_em_ec^2}\left(\frac{\varepsilon_*}{m_ec^2}\right)^{\gamma-1}\left(\frac{\omega_{pe}}{\omega}\right)^{\frac{1}{2}(\gamma+4)}. \quad (13.26)$$

We must particularly emphasize that both the emission coefficient and the damping rate in the high-frequency region of the spectrum depend on the frequency in the same way as the synchrotron emission coefficient.

The low-frequency region of the emission spectrum is different for the cases of the Compton and of the synchrotron mechanism. This is connected with the different behaviour of the emission probability at low frequencies. We shall not give here a detailed exposition of all emission mechanisms, leaving that problem for later sections.

One can evaluate in a similar manner the conversion of longitudinal plasmons with small phase velocities $v_p < c$ into transverse electromagnetic waves. This problem has been considered by Gaĭlitis and Tsytovich (1964a) and by Kaplan and Tsytovich (1969a). In particularly, the conversion of different types of plasma waves was studied. We refer the reader for details to the above-mentioned papers and the Appendix and give here only the final formulae for the most important case of the conversion of Langmuir waves into electromagnetic waves through the scattering by relativistic and by non-relativistic, but epithermal ($v \gg v_{Te}$) electrons. The emission coefficient is equal to

$$J^t_\omega(\mathrm{l}+\mathrm{e} \to \mathrm{t}+\mathrm{e}') = \frac{\omega_{pe}^4}{16\pi n_e^2c^4}\int_{\omega_{pe}/c}^{k_g} W^l_{k'}\frac{dk'}{k'}\int_{q\leq 1}^{\infty} f_\varepsilon\Phi(\omega,\varepsilon)\frac{d\varepsilon}{q}, \quad (13.27)$$

where the function $\Phi(\omega, \varepsilon)$ is similar to the function $\Psi(\omega, \varepsilon)$ in equation (12.5) and determines the spectrum of the scattered radiation as function of the electron energy and the plasma turbulence wavenumber k',

$$q = \frac{\omega}{k'v}\left(1-\frac{v}{c}\right) \approx \frac{\omega}{2k'c}\left(\frac{m_ec^2}{\varepsilon}\right)^2. \quad (13.28)$$

Equation (13.27) can in such a general form be applied not only to relativistic but also to non-relativistic electrons; the two cases differ in the form of the function $\Phi(\omega, \varepsilon)$.

In the general case this function is described by a very cumbersome expression. It simplifies for ultra-relativistic electrons because the Compton scattering turns out to be the only important one:

$$\Phi(\omega, \varepsilon) = \tfrac{8}{3}q[(1-q)^3-3q^2(1-q+\ln q)]. \quad (13.29)$$

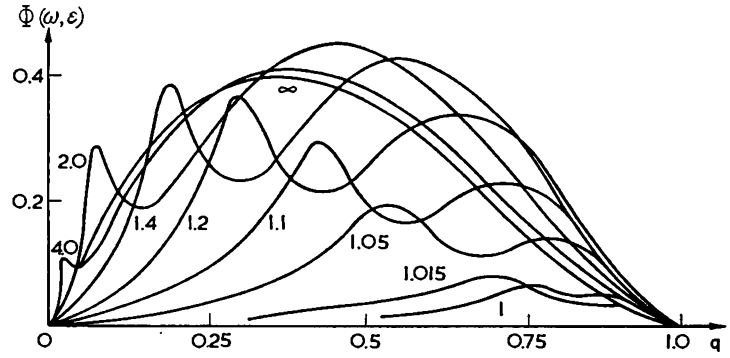

FIG. 61. The emission spectrum when Langmuir waves with $v_p \ll c$ are scattered by relativistic particles. Along the abscissa we plot the quantity $q = (\omega/2k_1c)\,(m_ec^2/\varepsilon)^2$, the number characterizing the curves is the quantity ε/m_ec^2.

We show in Fig. 61 the behaviour of the function $\Phi(\omega, \varepsilon)$ for electrons with intermediate and relativistic energies, where the compensation of the Compton and non-linear scattering at low energies is taken into account. The conversion effect decreases with decreasing energy.

Substituting (12.2) and (13.29) into (13.28) we have for the decreasing part of the spectrum that is, for $\omega \gg 2\omega_{pe}(\varepsilon_*/m_ec^2)^2$:

$$J^t_\omega = \frac{\gamma(\gamma^2+4\gamma+11)2^{(\gamma-1)/2}}{\pi(\gamma+3)^2(\gamma+5)} \frac{n_*}{n_e} \frac{\omega_{pe}^3}{c^3 n_e} \left(\frac{\varepsilon_*}{m_ec^2}\right)^{\gamma-1} \left(\frac{\omega_{pe}}{\omega}\right)^{\frac{1}{2}(\gamma-1)} \int \left(\frac{k'c}{\omega_{pe}}\right)^{\frac{1}{2}(\gamma-3)} W^1_{k'}\, dk'. \tag{13.30}$$

As in actual cases $\gamma \approx 3$ and $k'c/\omega_{pe}$ usually does not differ from unity by more than one or two orders of magnitude, we can put instead of (13.30)

$$J^t_\omega \approx \frac{n_*}{n_e} \frac{\omega_{pe}^3}{c^3 n_e} \left(\frac{\varepsilon_*}{m_ec^2}\right)^{\gamma-1} \left(\frac{\omega_{pe}}{\omega}\right)^{\frac{1}{2}(\gamma-1)} W^1. \tag{13.31}$$

The difference between (13.31) and (13.23) lies in a numerical factor of order unity. However, in (13.31) we must understand by W^1 the total energy of plasma waves with $v_p \lesssim c$ while in (13.23) we meant the usually appreciably larger quantity of the energy of plasma waves with $v_p \gtrsim c$.

For the damping rate we have

$$\gamma^t\begin{pmatrix} t+e = l+e' \\ t+e = p+e' \end{pmatrix} = -\frac{\pi\omega_{pe}^4}{8cn_e^2\omega^2} \int W^1_{k'} \frac{dk'}{k'} \int \frac{f_\varepsilon\, d\varepsilon}{\varepsilon} \Phi'(\omega, \varepsilon), \tag{13.32}$$

where for ultra-relativistic electrons:

$$\Phi'(\omega, \varepsilon) = \tfrac{8}{3}\,[(1-q)^3 + 3q(1-q^2+q\,\ln q)]. \tag{13.33}$$

Under the same conditions as we had for (13.30), we find

$$\frac{\gamma^t}{\omega_{pe}} = -\frac{2^{\gamma/2}\pi(\gamma^2-1)(\gamma^2+6\gamma+16)}{\gamma(\gamma+4)^2(\gamma+6)} \frac{n_*}{n_e} \left(\frac{\varepsilon_*}{m_ec^2}\right)^{\gamma-1} \left(\frac{\omega_{pe}}{\omega}\right)^{\frac{1}{2}(\gamma+4)} \int \left(\frac{ck'}{\omega_{pe}}\right)^{\frac{1}{2}(\gamma-2)} \frac{W^1_{k'}\, dk'}{n_e m_e c^2}. \tag{13.34}$$

In a somewhat worse approximation than was used for the derivation of (13.31) we get the estimate:

$$\frac{\gamma^{\mathrm{t}}}{\omega_{\mathrm{pe}}} \approx -\frac{n_*}{n_{\mathrm{e}}}\left(\frac{\varepsilon_*}{m_{\mathrm{e}}c^2}\right)^{\gamma-1}\left(\frac{\omega_{\mathrm{pe}}}{\omega}\right)^{\frac{1}{2}(\gamma+4)}\frac{W^{\mathrm{l}}}{n_{\mathrm{e}}m_{\mathrm{e}}c^2}, \tag{13.35}$$

and this expression differs from (13.26) only by a numerical factor.

The effect of the Compton scattering, which is the transformation of electromagnetic waves with a lower frequency into high-frequency waves, was analysed by Tsytovich and Chikhachev (1969). The scattering probability, averaged over the angles, can be evaluated by a method similar to the calculation of Compton scattering by Langmuir waves:

$$\bar{u}_k = \frac{\pi}{2}\frac{\omega'}{n_{\mathrm{e}}}\left(\frac{\omega_{\mathrm{pe}}}{\omega'}\right)^2\left(\frac{\omega_{\mathrm{pe}}}{\omega}\right)^2\left(\frac{m_{\mathrm{e}}c^2}{\varepsilon}\right)^2\left\{\left[1-\frac{\omega}{4\omega'}\left(\frac{m_{\mathrm{e}}c^2}{\varepsilon}\right)^2\right]\left[1+\frac{\omega}{2\omega'}\left(\frac{m_{\mathrm{e}}c^2}{\varepsilon}\right)^2\right]\right.$$
$$\left.+\frac{\omega}{2\omega'}\left(\frac{m_{\mathrm{e}}c^2}{\varepsilon}\right)^2\ln\left[\frac{\omega}{4\omega'}\left(\frac{m_{\mathrm{e}}c^2}{\varepsilon}\right)^2\right]\right\}. \tag{13.36}$$

Here ω' and ω are the frequency of the incident and the scattered electromagnetic waves.

The coefficient for the spontaneous Compton scattering follows from the formula:

$$J^{\mathrm{t}}_{\omega}(\mathrm{t}+\mathrm{e}\to\mathrm{t}'+\mathrm{e}') = \frac{\omega^3}{\pi^2c^3}\int W^{\mathrm{t}}_{\omega'}\frac{d\omega'}{\omega'}\int\limits_{m_{\mathrm{e}}c^2(\omega/4\omega')^{1/2}}^{\infty}\bar{u}_k f_{\varepsilon}\,d\varepsilon. \tag{13.37}$$

We remember that here $W^{\mathrm{t}}_{\omega'}/\omega'$ is the number of electromagnetic waves per unit frequency.

Substituting the probability (13.36) and the distribution function (12.2) into (13.37) we find for the decreasing part:

$$J^{\mathrm{t}}_{\omega} = \frac{2^{\gamma}\gamma(\gamma-1)(\gamma^2+4\gamma+11)}{\pi(\gamma+3)^2(\gamma+5)}\frac{n_*}{n_{\mathrm{e}}}\frac{\omega^3_{\mathrm{pe}}}{c^3n_{\mathrm{e}}}\left(\frac{\varepsilon_*}{m_{\mathrm{e}}c^2}\right)^{\gamma-1}\left(\frac{\omega_{\mathrm{pe}}}{\omega}\right)^{\frac{1}{2}(\gamma-1)}\int\left(\frac{\omega'}{\omega_{\mathrm{pe}}}\right)^{\frac{1}{2}(\gamma-3)}W^{\mathrm{t}}_{\omega'}\,d\omega'. \tag{13.38}$$

We must draw attention to the similarity of equations (13.38) and (13.30). Formula (13.38) can best be rewritten in somewhat different form. Let the spectral density of the waves ω' have a maximum at some frequency ω_*. We can then write, bearing in mind that γ is usually close to 3:

$$\int\left(\frac{\omega'}{\omega_*}\right)^{\frac{1}{2}(\gamma-3)}W^{\mathrm{t}}_{\omega'}\,d\omega' \approx W^{\mathrm{t}}, \tag{13.39}$$

and instead of (13.38) we get the estimate

$$J^{\mathrm{t}}_{\omega} \approx \frac{n_*}{n_{\mathrm{e}}}\frac{\omega^4_{\mathrm{pe}}}{\omega_*c^3n_{\mathrm{e}}}\left(\frac{\varepsilon_*}{m_{\mathrm{e}}c^2}\right)^{\gamma-1}\left(\frac{\omega_*}{\omega}\right)^{\frac{1}{2}(\gamma-1)}W^{\mathrm{t}}. \tag{13.40}$$

The difference between (13.40) and the exact relations lies also only in a numerical factor. The quantity $\omega_{\mathrm{pe}}/\omega_*$ occurs because the scattering probability is proportional to the number of scattered waves and not to their energy density.

The damping rate is similarly determined:

$$\frac{\gamma^t}{\omega_*} = -\frac{\pi 2^{\gamma+1}\gamma(\gamma^2-1)(\gamma^2+6\gamma+16)}{(\gamma+4)^2(\gamma+6)}\left(\frac{\varepsilon_*}{m_e c^2}\right)^{\gamma}\frac{n_*}{n_e}\left(\frac{\omega_{pe}}{\omega_*}\right)^4\left(\frac{\omega_*}{\omega}\right)^{\frac{1}{2}(\gamma+4)}\int\left(\frac{\omega'}{\omega_*}\right)^{\frac{1}{2}(\gamma-2)}\frac{W^t_{\omega'}\,d\omega'}{n_e\varepsilon_*}, \tag{13.41}$$

or, approximately,

$$\frac{\gamma^t}{\omega_*} \approx -\frac{n_*}{n_e}\left(\frac{\omega_{pe}}{\omega_*}\right)^4\left(\frac{\omega_*}{\omega}\right)^{\frac{1}{2}(\gamma+4)}\left(\frac{\varepsilon_*}{m_e c^2}\right)^{\gamma-1}\frac{W^t}{n_e m_e c^2}. \tag{13.42}$$

We have paid special attention to the decreasing part of the spectra of the electromagnetic radiation. This is related to the fact that that part of the spectrum is usually observed.

At the beginning of this subsection we have pointed out that in the general case the radiation spectrum can have several maxima. If the reabsorption also plays an important role the picture is still more complicated. It is well known that indeed in most radio-sources the decreasing short-wavelength part of the spectrum has a more or less standard form, corresponding to the law $\omega^{-(\gamma-1)/2}$. However, in some cases a complicated spectrum has also been observed.

We note in concluding this subsection that we can add yet another maximum in the spectrum to all those which are connected with different mechanisms. If conversion into electromagnetic waves occurs for the very-low-frequency plasma turbulence which is such that the conservation laws lead to equation (13.8), the emission coefficient will have a maximum at the frequency $\sqrt{3}\omega_p$ where the quantity ω_p is determined by (13.8) with a maximum value for the wavenumber of the low-frequency turbulence. Near this frequency the spectrum is $I_\omega \propto (\omega-\omega_p)^{(\gamma+3)/2}$.

Amplification and stabilization of the electromagnetic radiation generated in Compton scattering of Langmuir and electromagnetic waves

In the previous section we showed that it is difficult in astrophysical objects to realize conditions under which an amplification of the electromagnetic radiation takes place when it is produced by the synchrotron mechanism of ultra-relativistic particles. In this respect Compton scattering is slightly more efficient and it is completely attainable to find amplification of electromagnetic waves in cosmic conditions.

We shall consider the following problem, to begin with, as an example (Kaplan and Tsytovich 1969a). Let a beam of ultra-relativistic electrons with energy ε_* and density n_* pass through a plasma with a strong Langmuir wave turbulence. The spread in energy of the particles in the beam $\Delta\varepsilon_*$ is unimportant for what follows: the results are valid also when $\Delta\varepsilon_* \lesssim \varepsilon_*$. For the present we shall assume that the spread in the direction of the velocities is small.

Plasma waves undergo Compton scattering by the beam particles. They are then converted into electromagnetic waves the frequency of which is determined, in accordance with the conservation law (13.5) or (13.6) by the scattering angle. The larger the angle between the beam axis and the wavevector of the electromagnetic wave, the lower the frequency of this wave.

The presence of a beam produces a repopulation of the levels with respect to this "lateral" emission. One can thus expect that at an angle $\vartheta > m_e c^2/\varepsilon_*$ with the beam axis a

"maser effect" will appear. Such an effect did not occur in the synchrotron emission as an exponentially small fraction of the energy was produced in the "lateral" directions; here, on the other hand, although there is far less energy emitted than when $\vartheta \lesssim m_e c^2/\varepsilon_*$, it is all the same an appreciable fraction of the total energy. This makes a build-up possible.

The growth rate can be calculated using the usual formulae. One must merely take into account the angular dependence of the scattering probability. In what follows we shall drop numerical factors of order unity. We shall also introduce some other simplifications of the formulae (as compared with the original papers).

The electromagnetic radiation produced in Compton scattering along a direction of the beam motion within an angle $\vartheta < m_e c^2/\varepsilon_*$ is not amplified. The situation is here analogous to the case of synchrotron radiation, that is, the reabsorption coefficient, per particle, is always positive. For the damping rate of waves moving along the beam axis caused by particles in the beam itself we get (the turbulent waves have a rather low phase velocity $v_p < c$):

$$\frac{\gamma^t}{\omega_{pe}} \approx -\frac{n_*}{n_e}\frac{\varepsilon_*}{m_e c^2}\left(\frac{\omega_{pe}}{\omega}\right)^2 \int_{(\omega/2c)(m_e c^2/\varepsilon_*)}^{(d_e)^{-1}} \frac{W^l_{k'}\,dk'}{n_e m_e c^2}\frac{\omega_{pe}}{k'c} \approx -\frac{n_*}{n_e}\left(\frac{\varepsilon_*}{m_e c^2}\right)^3\left(\frac{\omega_{pe}}{\omega}\right)^3 \frac{W^l}{n_e m_e c^2}\,. \tag{13.43}$$

According to what we said earlier the radiation which moves at an angle $\vartheta > m_e c^2/\varepsilon_*$ to the beam axis is amplified. When $m_e c^2/\varepsilon_* \ll \vartheta \ll 1$, $\omega < 2ck_g/\vartheta^2$ we have the following expression for the growth rate (Kaplan and Tsytovich, 1969a):

$$\begin{aligned}\frac{\gamma^t(\vartheta)}{\omega_{pe}} &\approx \frac{4\pi}{\vartheta^4}\frac{n_*}{n_e}\left(\frac{m_e c^2}{\varepsilon_*}\right)^3\left(\frac{\omega_{pe}}{\omega}\right)^2 \int \frac{W^l_{k'}\,dk'}{n_e m_e c^2}\frac{\omega_{pe}}{k'c}\left[1-\left(\frac{\omega\vartheta^2}{2k'c}\right)^2\right] \\ &\approx \frac{8\pi}{\vartheta^6}\frac{n_*}{n_e}\left(\frac{m_e c^2}{\varepsilon_*}\right)^3\left(\frac{\omega_{pe}}{\omega}\right)^3 \frac{W^l}{n_e m_e c^2}\frac{\omega\vartheta^2}{2ck_g}\left[\ln\frac{2k_g c}{\omega\vartheta^2}-\frac{1}{2}\right].\end{aligned} \tag{13.44}$$

It is clear that the decrease of the growth rate with increasing angle ϑ is here not exponential although it is determined by a high power, ϑ^4. We note that the absolute magnitude of the damping rate for $\vartheta \lesssim m_e c^2/\varepsilon_*$ and the growth rate for $\vartheta \approx m_e c^2/\varepsilon_*$ are close to one another (Fig. 62).

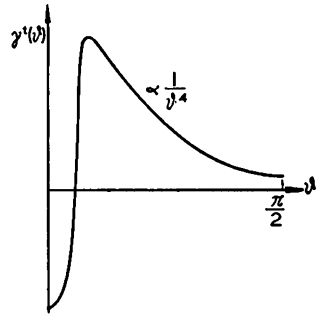

FIG. 62. The growth rate of electromagnetic waves produced by a beam of relativistic electrons as function of the angle between the beam axis and the direction of the wavevector.

In this case also it is rather difficult to obtain an appreciable amplification effect. The question is that apart from the Compton scattering by ultra-relativistic particles in the same turbulent plasma medium there occurs also the decay process $t \to t' \pm l$ which changes the angle ϑ for electromagnetic waves and thereby removes them from resonance with the beam. The elastic scattering coefficient determined by equation (5.27) is as a rule much larger than (13.44). Its minimum value is determined by the expression ($\omega > ck_g$)

$$\frac{\sigma_\perp^l}{\omega_{pe}} \approx \left(\frac{\omega_{pe}}{\omega}\right)^4 \frac{W^l}{n_e m_e c^2} \frac{k_g^3 c^3}{10\omega_{pe}^3}. \tag{13.45}$$

The growth rate therefore exceeds the stabilizing effect of the scattering of electromagnetic waves in a turbulent medium for angles

$$\vartheta \lesssim \vartheta_{cr} = \left(\frac{m_e c^2}{\varepsilon_*}\right)^{3/8} \left(\frac{n_*}{n_e}\right)^{1/8}. \tag{13.46}$$

This means that only a narrow beam with $\Delta\theta \lesssim \vartheta_{cr}$ leads to amplification of electromagnetic radiation in the Compton scattering of plasma waves.

If such an amplification, indeed, takes place one can obtain a limit on the energy density of the excited electromagnetic waves by comparing the growth rate (13.44) with the decay rate of the process $t \to t' \pm l$ determined by equations (3.37) or (12.72); from this comparison follows

$$W^t \approx W^l \left(\frac{m_e c^2}{\varepsilon_*}\right)^3 \frac{n_*}{n_e} \frac{8\pi}{\vartheta^6}. \tag{13.47}$$

A very narrow beam of relativistic electrons passing through a turbulent plasma is thus able to produce a rather strong electromagnetic emission at high frequencies. The plasma wave energy density can be relatively low as the electromagnetic radiation gets its energy from the relativistic particles. Moreover, the excitation is possible only when the optical thickness is larger than unity; this condition imposes a lower limit on the plasma wave energy density.

A characteristic relaxation rate for the beam, connected with the transfer of energy from the relativistic electrons to the electromagnetic waves is of order ($ck_g \approx \omega_{pe}$)

$$\frac{\gamma_*^t}{\omega_{pe}} \approx \left(\frac{m_e c^2}{\varepsilon_*}\right)^4 \frac{W^t W^l}{(n_e m_e c^2)^2} \approx \frac{n_*}{n_e} \left(\frac{m_e c^2}{\varepsilon_*}\right)^7 \frac{8\pi}{\vartheta^6} \left(\frac{W^l}{n_e m_e c^2}\right)^2. \tag{13.48}$$

We note that beams with angular spread in their velocities larger than (13.46) do not relax as a result of Compton scattering.

Let us mention here yet another problem, that is, the scattering of longitudinal plasmons by anisotropically distributed relativistic electrons. Substituting in all equations of the preceding sections instead of the isotropic distribution function (12.2) the anisotropic function (12.3) one can again complete the calculations. In particular, such a calculation of the damping rate (13.35) leads to the appearance of an extra factor

$$1 - \frac{2(\gamma+6)(\gamma^2+\gamma-8)}{(\gamma+2)(\gamma^2+6\gamma+16)} \eta \cos\vartheta, \tag{13.49}$$

which for certain choices of the parameters γ and η can become negative, that is, the damping can become amplification. However, such a weak anisotropy and relatively small growth rate can hardly, for the reasons given earlier, lead to an appreciable amplification.

Let us now turn to the problem of the emission when low-frequency electromagnetic waves are scattered by a beam of relativistic electrons and are converted into high-frequency electromagnetic waves (Kaplan, Tsytovich and Chikhachev, 1970). We shall assume here that the longitudinal plasmon energy density is small. Strictly speaking, this assumption is not obvious: the presence itself of a beam of fast particles inevitably leads to the production of Langmuir turbulence and only when there is considerable damping of longitudinal plasmons can we neglect W^l. However, the relations obtained below are applicable also when there is no plasma at all and a beam of relativistic electrons passes *in vacuo* through an electromagnetic radiation field; we can then, indeed, assume that all effects depend solely on W^t.

Consider a beam of electrons with a very small angular spread $\Delta\theta$—which we shall estimate below—passing through a region of space filled with electromagnetic radiation with a spectral density W^t_ω, which may be either isotropic or anisotropic. As in the case of longitudinal plasmons the electromagnetic waves scattered by the beam particles can excite emission at an angle $\vartheta > m_e c^2/\varepsilon_*$ to the beam axis.

Let ω_* be the frequency of the maximum of the spectral emission density; we assume that the spectral energy density decreases at the high-frequency end with a spectral index α, as usually, that is, $W^t_\omega \propto (\omega_*/\omega)^\alpha$; in the low-frequency region this decrease is steeper than ω^2. One can then show that in the frequency range

$$\omega_* < \omega < \frac{4\omega_*}{(m_e c^2/\varepsilon_*)^2 + \vartheta^2} \tag{13.50}$$

the growth rate is in first approximation independent of frequency and depends only on the scattering angle ϑ (with respect to the beam axis). The fact that in this process the whole range of frequencies (13.50) is emitted is a consequence of the assumption of a wide spread in the directions of the scattered wave. If we assume that the electromagnetic radiation is also anisotropic, for instance, by considering the scattering of a beam of electromagnetic waves by a beam of ultra-relativistic particles we get a unique connection between the scattering angle and the frequency:

$$\omega \approx \omega_* \frac{2(1-\cos\theta)}{(m_e c^2/\varepsilon_*)^2 + \vartheta^2}, \tag{13.51}$$

where θ is the angle between the beam axes of the electromagnetic waves and the ultra-relativistic particles.

The growth rate in the range (13.50) or at the frequency (13.51) is determined by the equation

$$\frac{\gamma^t(\vartheta)}{\omega_{pe}} \approx \frac{n_*}{n_e} \frac{m_e c^2}{\varepsilon_*} \left[\left(\frac{m_e c^2}{\varepsilon_*}\right)^2 + \vartheta^2\right] \left(\frac{\omega_{pe}}{\omega_*}\right)^3 \frac{W^t_*}{n_e m_e c^2}, \tag{13.52}$$

where W^t_* is the total "primary" electromagnetic radiation density. For frequencies above those in (13.50) one also obtains growth but with a growth rate which decreases very fast,

as $(\omega_*/\omega)^{2+\alpha}$, with increasing frequency. Moreover, there occurs here an extra numerical factor $(\gamma+1)(\gamma+2)/(2\alpha-\gamma-2)$ which for $\alpha < (\gamma+2)/2$ becomes negative, that is, the growth rate becomes a decay rate. In other words, there is practically no amplification of radiation outside the limits of the interval (13.50).

If we now compare the growth rate (13.52) with the decay rate of the process $t \to t' \pm 1$, given by formula (3.37), we obtain an estimate for the energy of the excited electromagnetic waves and at the same time a condition for amplification (if such a density did not exist already initially, no amplification occurs):

$$\frac{W^t_\omega}{\omega^2} \ll \frac{W^t_{\omega_*}}{\omega_*^2} \frac{n_*}{n_e} \frac{m_e c^2}{\varepsilon_*} \left[\left(\frac{m_e c^2}{\varepsilon_*} \right)^2 + \vartheta^2 \right], \tag{13.53}$$

where $W^t_{\omega_*}$ and W^t_ω are the spectral densities of the energies of the incident and the scattered waves, respectively.

Combining equations (13.50) to (13.53) we can estimate the possibility for amplification in each actual case. For instance, if the electromagnetic radiation spectrum is flat, $\alpha \approx 0$, we get a limitation on the angles of the scattered radiation for which amplification can take place, and hence on the angular spread of the beam:

$$\Delta\theta \lesssim \vartheta \ll \sqrt{\left(\frac{n_*}{n_e} \frac{m_e c^2}{\varepsilon_*} \right)}. \tag{13.54}$$

If we consider the Compton scattering of electromagnetic radiation by a particles beam *in vacuo* there will be no decay process $t \to t' \pm 1$, when we neglect the plasma properties of the beam itself. In that case the above-mentioned restriction does not occur and amplification is possible in a wide set of circumstances.

Turning to the case of a relativistic beam in a plasma we can from relation (13.53) obtain a rough estimate for the energy density of the electromagnetic waves which are produced through Compton scattering by a beam of relativistic electrons with a given spread $\Delta\theta$ in directions and which are stabilized by the non-linear decay process $t \to t' \pm 1$:

$$W^t \approx \frac{n_*}{n_e} \frac{m_e c^2}{\varepsilon_*} \frac{W^t_*}{(\Delta\theta)^4}; \tag{13.55}$$

we assumed here that the radiation density of the incident waves W^t_* is given. Comparing (13.54) and (13.55) we find that the case $W^t \gg W^t_*$ is well possible, although only for narrowly directed beams. The beam relaxation is determined by relation (13.48), replacing $W^l W^t$ by $W^t W^t_* \omega_{pe}^2/\omega_*^2$ in the first equality.

We emphasized already in § 3 that stimulated scattering processes involving a lowering of the frequency always lead to an amplification of the radiation if the incident particles are distributed isotropically. This is also valid in the case of Compton scattering.

Consider high-frequency radiation—whose isotropy or anisotropy is of no importance—in a region occupied by isotropically distributed ultra-relativistic particles. This high-frequency radiation will through stimulated scattering by the ultra-relativistic particles be transferred to the low-frequency end of the spectrum (Kaplan, Tsytovich and Chikhachev,

1970). The growth rate for the transfer of radiation with a maximum in the spectrum near the frequency ω_* in the frequency range

$$\frac{\omega_*}{4}\left(\frac{m_e c^2}{\varepsilon_*}\right)^2 < \omega < \omega_* \tag{13.56}$$

turns out to be equal to

$$\frac{\gamma^t(\omega)}{\omega_*} \approx \frac{n_*}{n_e}\frac{\omega_{pe}^4}{\omega_*^3\omega}\frac{W_*^t}{n_e m_e c^2}. \tag{13.57}$$

Its maximum value is reached at the lower frequency limit $\frac{1}{4}\omega_*(m_e c^2/\varepsilon_*)^2$. There

$$\frac{\gamma_{max}^t}{\omega_*} \approx \frac{n_*}{n_e}\left(\frac{\varepsilon_*}{m_e c^2}\right)^2\left(\frac{\omega_{pe}}{\omega_*}\right)^4\frac{W_*^t}{n_e m_e c^2}. \tag{13.58}$$

The energy transfer occurs also in the lower-frequency end of the spectrum, that is, when $\omega < \frac{1}{4}\omega_*(m_e c^2/\varepsilon_*)^2$, but then the growth rate decreases fast with decreasing frequency, approximately as ω^3. Comparing (13.58) with the magnitude of the non-linear transfer, we find the energy density of the low-frequency electromagnetic waves:

$$W^t \approx W_*^t\frac{n_*}{n_e}\left(\frac{\omega}{\omega_*}\right)^2 \approx W_*^t\frac{n_*}{n_e}\left(\frac{m_e c^2}{\varepsilon_*}\right)^4. \tag{13.59}$$

As one would expect, the energy density of the scattered radiation is much less than the energy of the incident radiation as in this case the energy for the excitation of radiation at low frequencies is produced by high-energy radiation. It is also important to note that the low-frequency radiation is distributed isotropically.

Equation (13.58) is important for the following reason. In a system where the optical depth for radiation at a frequency ω is large, that is, where $\gamma_{max}^t R/c > 1$ for the γ_{max}^t from (13.58), high-frequency radiation must be transferred to low-frequency radiation rather rapidly. The condition $\gamma_{max}^t R/c < 1$ is thus a necessary requirement for a quasi-stationary state of a system with a large high-frequency radiation density.

Polarization of the electromagnetic emission from a turbulent plasma

One of the basic characteristics of cosmic radiation is its polarization. On the basis of an analysis of the observed polarization one reaches often far-reaching conclusions about the nature of the mechanism of producing the radiation and about the structure of the system. The polarization properties of synchrotron radiation are well known. Of course, it is necessary to compare different mechanisms from the point of view of the polarization of the radiation produced by them.

Radiation from a system consisting of a large number of electrons can be polarized only when there is a preferred direction which may be caused in different ways. For instance, the polarization of synchrotron radiation is connected with the presence of an external, more or less uniform, magnetic field. There are also other causes for polarization such as the anisotropy of the plasma turbulence or of the particle velocity distribution when we are dealing with plasma or Compton mechanisms.

When considering the polarization of the emission from a source one must, first of all, take into account the polarization of the separate emitters, secondly, the polarization of the system of emitters, and thirdly, the change in polarization when the radiation propagates through the medium. We shall show how this can be done, using synchrotron radiation as an example.

It is well known that an electron moving along a spiral in a magnetic field produces elliptically polarized electromagnetic radiation. The direction of rotation of the electrical vector depends on whether the longitudinal velocity component is parallel or antiparallel to the vector $\boldsymbol{H}$. The degree of polarization of the emission by a single electron is different at different frequencies. In the limit of high frequencies $\omega \gg \omega_c$ it reaches 100 per cent, while at low frequencies it is 50 per cent.

The resulting polarization of the radiation from a system of electrons turns out to be linear to a very good approximation, and the electric vector lies in the plane at right angles to the magnetic field. This is connected with the fact that the number of electrons with velocity components $v_\parallel$ parallel or antiparallel to $\boldsymbol{H}$ is the same, while $v_\perp$ for the observed radiation is directed to the same side for all electrons—we remember that relativistic electrons emit in the forward direction. Moreover, the total polarization at a given frequency is composed of the polarizations of the emission of electrons with different ω_c and the degree of polarization must therefore depend on the energy distribution of the electrons. For the distribution function (12.2)—or rather for its part $f_\varepsilon \propto \varepsilon^{-\gamma}$—the degree of polarization is the same at all frequencies and equal to (Garibyan and Gol'dman, 1954; Trubnikov, 1958):

$$\Pi = \frac{\gamma+1}{\gamma+\frac{7}{3}}. \tag{13.60}$$

In this case we mean by degree of polarization the following. We determine the square of the components of the electric field strength of the wave oriented at right angles or parallel to the projection of the magnetic field on the plane perpendicular to the line of sight. The ratio of the difference of these squares of the field strengths to their sum is the degree of polarization.

If the electric field of the wave is strictly directed at right angles to the magnetic field, the linear polarization reaches 100 per cent. In actual cosmic synchrotron sources a hundred per cent polarization is impossible.

Equation (13.60) determines the maximum degree of polarization of synchrotron radiation. Inhomogeneities in the structure of the magnetic field can only decrease it. There is yet another important cause for decreasing the degree of polarization, namely, the Faraday effect. As a result of this effect the polarization vectors of the emission from different parts of the system are rotated over different angles. The plane of polarization of the radiation from a further removed part is rotated over a larger angle than from a closer-by part. If the electrons are distributed uniformly over the volume of the emitting system, the resulting degree of polarization is (Razin, 1958)

$$\Pi = \frac{\gamma+1}{\gamma+\frac{7}{3}} \frac{\sin \frac{1}{2}\psi(R)}{\frac{1}{2}\psi(R)}, \tag{13.61}$$

where the total rotation angle Ψ is determined by equation (1.12), and where R is the size of the system. In the case when $\Psi(R) \gg 1$ the polarization becomes practically unobservable.

Reabsorption also decreases the polarization. In an optically thick medium we have always $\Pi \approx 0$.

There is, however, another effect which will affect the polarization which we observe on Earth: in a magneto-active plasma there are at any point an ordinary and an extra-ordinary wave with different polarization and propagation properties. As was pointed out by Ginzburg, Zheleznyakov and Zaĭtsev (1969; see also Zheleznyakov, 1970b) it may happen that one of these two characteristic waves is preferentially absorbed so that the other one—with a very definite polarization—may emerge from the plasma. Although Zheleznyakov's analysis applies only to a very special case, recent work by Grounds (1972; see also Grounds and ter Haar, 1973) has shown that the general result will probably not be very different.

We now turn to the polarization in the case of plasma emission mechanisms. Tsytovich (1969d) has considered the general theory of the polarization of the radiation from a turbulent plasma. We give here simplified concrete expressions for the degree of polarization, applicable to those mechanisms which we have discussed earlier. It is clear that the radiation produced in the scattering of the waves of isotropic turbulence by isotropically distributed particles will be unpolarized. We must thus consider the possibility of turbulence anisotropy or anisotropy in the particle velocity distribution.

We start with anisotropic turbulence. Developed plasma turbulence is, as a rule, isotropic, but there is an important exception which has already been discussed in § 8. We remember that if non-linear processes have already transferred longitudinal plasmons to a region of the spectrum with wavenumbers $k < \omega_{He}/\sqrt{3}v_{Te}$ (see equation (8.41)) further transfer leads to a decrease in the angle ϑ between the wavevector and the magnetic field and the plasma turbulence becomes one-dimensional with the wavevectors of the longitudinal plasmons oriented parallel or antiparallel to the magnetic field. When the longitudinal plasmons are oriented in such a way, the electric field is also directed along the magnetic field and hence the electrons which scatter the radiation are also accelerated along the magnetic field.

Hence follows an important conclusion: by being scattered by isotropically distributed electrons such a one-dimensional plasma turbulence will produce linearly polarized radiation but with the electric vector along the magnetic field, and not at right angles to it, as in the case of synchrotron radiation.

To determine the degree of polarization we need find the emission probability for waves with the electric vector along and perpendicular to the magnetic field. The total scattering probability for $v_p \gg c$ is given by equation (13.20); we shall now give the probabilities for the separate components, averaging only over the direction of the motion of the particles. The probability for the scattering of a longitudinal plasmon into an electromagnetic wave with the electric field vector in a plane through the vectors $\boldsymbol{k}$ and $\boldsymbol{H}$ is:

$$u_k^{(1)} = \frac{\pi}{4}\frac{\omega_{pe}}{n_e^2}\left(\frac{\omega_{pe}}{\omega}\right)^2\left(\frac{m_e c^2}{\varepsilon}\right)^2\left\{-1+\frac{\omega}{\omega_{pe}}\left(\frac{m_e c^2}{\varepsilon}\right)^2+\frac{3}{8}\left[\frac{\omega}{\omega_{pe}}\left(\frac{m_e c^2}{\varepsilon}\right)^2-2\right]^2\right\}\sin^2\vartheta; \tag{13.62}$$

for the scattering into a wave with the electric vector perpendicular to that plane we have

$$u_k^{(2)} = \frac{\pi}{32}\frac{\omega_{pe}}{n_e^2}\left(\frac{\omega_{pe}}{\omega}\right)^2\left(\frac{m_e c^2}{\varepsilon}\right)^2\left[\frac{\omega}{\omega_{pe}}\left(\frac{m_e c^2}{\varepsilon}\right)^2 - 2\right]^2 \sin^2\vartheta, \tag{13.63}$$

where ϑ is the angle between the wavevector of the scattered wave and the magnetic field. We can then in the usual way evaluate the emission coefficients and after that the degree of polarization. After simple, though lengthy, calculations we find:

$$\Pi = \frac{(\gamma+1)(\gamma+3)}{\gamma^2+4\gamma+11}. \tag{13.64}$$

This equation for the degree of polarization, as also the earlier given expressions (13.60) and (13.61), refer to spontaneous emission when reabsorption and amplification are small. As in the case of synchrotron emission (see (13.60)) the degree of polarization is independent of the angle ϑ and for $\gamma = 3$ equations (13.60) and (13.64) give the same result: $\Pi = \frac{3}{4} = 75$ per cent. Of course, what we said earlier about the effect of the Faraday rotation on the polarization remains valid.

We shall now consider a second cause leading to polarization of scattered radiation—the anisotropy in the velocity distribution of the ultra-relativistic particles. We assume that a narrowly directed beam of relativistic electrons passes through a region with isotropic turbulence. Moreover, we shall assume for the time being that the optical depth for the amplification of electromagnetic waves is small so that only spontaneous emission is important.

Electromagnetic waves scattered by this beam will also turn out to be polarized with the electric field vector directed predominantly perpendicular to the beam axis. The most probable situation is one where the beam of relativistic electrons moves along the magnetic field so that also here, as in the case of sycnhrotron radiation, the electric radiation field vector is at right angles to the external magnetic field, in contrast to the case of anisotropic turbulence.

We shall not give the cumbersome expressions for the emission probabilities with different directions of the electric field. We write down at once the ratio of the difference between the probabilities with the electric field in the plane of the vectors $\boldsymbol{k}$ and $\boldsymbol{v}$ and the field at right angles to that plane, to their sum:

$$\frac{\bar{u}_k^{(1)} - \bar{u}_k^{(2)}}{\bar{u}_k^{(1)} + \bar{u}_k^{(2)}} = \frac{\dfrac{\omega}{\omega_{pe}}\left(\dfrac{m_e c^2}{\varepsilon}\right)^2\left[1 - \dfrac{\omega}{2\omega_{pe}}\left(\dfrac{m_e c^2}{\varepsilon}\right)^2\right]}{1 - \dfrac{\omega}{\omega_{pe}}\left(\dfrac{m_e c^2}{\varepsilon}\right)^2\left[1 - \dfrac{\omega}{2\omega_{pe}}\left(\dfrac{m_e c^2}{\varepsilon}\right)^2\right]}. \tag{13.65}$$

Equation (13.65) is written in terms of the particle energy. It can also be written in terms of the angle ϑ between the directions of the wave and of the beam which is uniquely determined by ε through equation (13.5).

It follows from (13.65) that at the maximum possible frequency for a given energy—or ϑ—the degree of polarization vanishes, and that it increases after that steeply to a maximum,

equal to approximately 100 per cent, when $\omega \approx \omega_{pe}(\varepsilon/m_e c^2)^2$ and then again decreases to zero when the frequency further decreases.

Averaging, using the distribution function (12.2), gives:

$$\Pi = \frac{4(\gamma+1)}{\gamma^2+4\gamma+11}. \tag{13.66}$$

The polarization picture changes, however, abruptly in the case when the optical depth for the scattered radiation becomes larger than unity. The scattering by an anisotropic beam then leads to amplification of electromagnetic waves and the polarization of the emission increases abruptly (Tsytovich, 1969d).

It is clear that the growth rates of the polarized and the unpolarized radiations are different. If γ^t is the total growth rate, the growth rate for the polarized part is equal to

$$\gamma^t_{12} \approx \frac{\bar{u}^{(1)}_k - \bar{u}^{(2)}_k}{\bar{u}^{(1)}_k + \bar{u}^{(2)}_k}\gamma^t. \tag{13.67}$$

Let I_ω be the radiation intensity and U_ω be the intensity of its polarized part—more precisely, I_ω and U_ω are the appropriate Stokes parameters so that $\Pi = U_\omega/I_\omega$. We then have the following equations for the change in time of the quantities I_ω and U_ω:

$$\frac{dI_\omega}{dt} = \gamma^t I_\omega + \gamma^t_{12} U_\omega, \quad \frac{dU_\omega}{dt} = \gamma^t_{12} I_\omega + \gamma^t U_\omega. \tag{13.68}$$

The solution of these equations determines the change in time of the degree of polarization:

$$\Pi = \left| \frac{1+\Pi_0-(1-\Pi_0)\exp(2\gamma^t_{12}t)}{1+\Pi_0+(1-\Pi_0)\exp(2\gamma^t_{12}t)} \right|, \tag{13.69}$$

where Π_0 is the initial degree of polarization. It follows from this that even if $\Pi_0 = 0$, for $\gamma^t_{12}t \gg 1$ the degree of polarization of the electromagnetic radiation produced as a result of amplification can turn out to be 100 per cent. This refers also to the case of transfer from high-frequency to low-frequency radiation.

Bremsstrahlung mechanisms for relativistic electrons and ions in a turbulent plasma

In concluding this section we shall briefly consider yet another emission mechanism at high frequencies in a turbulent plasma (Tsytovich, 1971c). Ultra-relativistic particles absorbing and emitting longitudinal plasmons through the Cherenkov mechanism are scattered, that is, they change the direction of their motion. It is clear that this process can be compared to collisions between particles and waves. The "collision" frequency is determined by the diffusion along the energy axis, that is,

$$\nu_{\text{eff}} = \frac{c^2 D^1_{\parallel}}{\varepsilon^2} = \frac{2\pi^2 e^2 \omega^2_{pe}}{\varepsilon^2 c} \int_{\omega_{pe}/c}^{k_{max}} W^1_{k'} \frac{dk'}{k'^3} \approx \frac{\pi}{2}\frac{m_e c^2}{\varepsilon}\frac{W^1}{n_e \varepsilon}\omega_{pe}, \tag{13.70}$$

where W^l is the energy density of the resonance plasmons, that is, plasmons with $v_p < c$. In turn this scattering must be accompanied by electromagnetic radiation according to the general properties of dipole radiation. The power of that emission is equal to

$$P_\omega^t = \frac{2e^2}{\pi c} v_{\text{eff}} \left(\ln \frac{2\varepsilon}{m_e c^2} - 1 \right) \approx \frac{e^2 \omega_{pe}}{c} \frac{m_e c^2}{\varepsilon} \frac{W^l}{n_e \varepsilon} \left(\ln \frac{2\varepsilon}{m_e c^2} - 1 \right), \tag{13.71}$$

and the emission coefficient for the case of the distribution function (12.2) is

$$J_\omega^t \approx \frac{\gamma(\gamma-1)}{4\pi} \frac{n_*}{n_e} \frac{\omega_{pe}^3}{n_e c^3} \left(\frac{m_e c^2}{\varepsilon_*} \right)^2 \left[\ln \frac{2\varepsilon_*}{m_e c^2} - 1 \right] W^l. \tag{13.72}$$

The emission power decreases with increasing energy but it also has an "advantage": as in the case of the ordinary bremsstrahlung, J_ω is independent of frequency up to very high frequencies $\sim \varepsilon_*/\hbar$. The mechanism described here can thus turn out to be important in the region of very high frequencies. Moreover, ions and electrons emit with the same intensity.

§ 14. Plasma turbulence reactors for relativistic electrons

A plasma region with a developed turbulence, "trapped" radiation and a large number of also "trapped" relativistic particles we shall call reactor.† In such a reactor part of the fast particles acquire even higher energies by being stochastically accelerated. The problem of the energetics of a reactor and that of containing such a plasma we shall not discuss here. The plasma may be contained by gravitational forces or by a magnetic field and this—or something else—may also give the energy to maintain the level of turbulence.

One thing is now important for us: the energy dissipation in a plasma turbulence reactor is accompanied by acceleration of the particles which form the relativistic electron spectrum. Later these particles may leave the reactor either through diffusion or by very possible "explosions" of the reactor connected with its "overheating".

In the next section we consider the problem in how far a plasma turbulence reactor represents actual astrophysical objects.

The evaluation of a plasma turbulence reactor is the solution of a complex self-consistent problem as here the particle distribution function, the spectral density of the electromagnetic radiation, and the turbulence energy density are all interconnected. The solution of the complete problem must thus include the writing down and the analysis of a set of kinetic equations for W_k^l, I_ω, f_ε, and so on.

It is not too difficult to write down such a system in principle, but when one starts this investigation it is advisable to try and simplify the problem so that its physical meaning is not obscured by complex and cumbersome mathematics.

Let us list the simplifications introduced here. Firstly, we shall assume that the distribution function of the relativistic electrons is described by equation (12.2) where the parameters

† Tsytovich and Chikhachev (1969, 1971) and Kadomtsev and Tsytovich (1970) were the first to consider the theory of such a reactor. We shall consider the problem here from a somewhat more general point of view and in the next chapter we shall give a generalized theory of turbulent reactors for the case of an ultra-relativistic plasma in a very strong magnetic field.

n_*, ε_*, and γ must be determined from the properties of the reactor. One can show that the function (12.2) is, indeed, a solution of the self-consistent problem of the turbulent reactor, at least in the limiting cases $\varepsilon \gg \varepsilon_*$ and $\varepsilon \ll \varepsilon_*$ (Tsytovich and Chikhachev, 1969).

Secondly, we shall not take into account the detailed spectrum of Langmuir turbulence. This can always be done in the region of phase velocities much larger than the velocity of light, as then the scattering probability is independent of k. The region of phase velocities less than or equal to the velocity of light contains a relatively small part of the plasma turbulence energy and this approximation is thus justified with good accuracy.

Thirdly, we shall assume that apart from Langmuir turbulence there is in the reactor also turbulence of other modes, such as that of the transverse plasmons. According to Kaplan and Tsytovich (1968b) there is in a turbulent plasma a continuous transfer of energy from transverse to longitudinal plasmons and vice versa in such a way that neither kind of turbulence is stationary. However, the sum of their energies $W^{\mathrm{l}}+W^{\mathrm{p}} = \text{constant}$, and this sum alone enters in what follows in the equations, in accordance with (13.21) and (13.25). In that sense taking the transverse plasmons into account enables us to assume the total turbulence energy to be constant.

Electromagnetic radiation spectrum in a turbulent reactor

By definition a plasma reactor is described by the fact that in it there is "trapped" electromagnetic radiation at frequencies appreciably higher than ω_{pe} or ω_{He}. This means that the optical depth of a reactor for such radiation is much larger than unity.

We shall consider the interior regions of the reactor where the particle distribution function and the intensity of the electromagnetic radiation are certainly isotropic so that there is no amplification of the radiation. However, there are here several damping mechanisms and for comparison we shall therefore write down all main damping rates, dropping numerical factors of order unity and restricting ourselves to the high-frequency case:

1. Collisional damping rate

$$\gamma_1^{\mathrm{t}} \approx -\left(\frac{\omega_{\mathrm{pe}}}{\omega}\right)^2 \frac{\omega_{\mathrm{pe}}}{N_{\mathrm{D}}}; \tag{14.1}$$

2. Synchrotron damping rate which for $\omega > \omega_{He}(\varepsilon_*/m_{\mathrm{e}}c^2)^2$ has the form

$$\gamma_2^{\mathrm{t}} \approx -\left(\frac{\omega_{He}}{\omega}\right)^{\frac{1}{2}(\gamma+4)} \frac{n_*}{n_{\mathrm{e}}} \frac{\omega_{\mathrm{pe}}^2}{\omega_{He}} \left(\frac{\varepsilon_*}{m_{\mathrm{e}}c^2}\right)^{\gamma-1}; \tag{14.2}$$

3. Compton damping rate, due to scattering by longitudinal plasmons, $\omega > \omega_{\mathrm{pe}}(\varepsilon_*/m_{\mathrm{e}}c^2)^2$

$$\gamma_3^{\mathrm{t}} \approx -\left(\frac{\omega_{\mathrm{pe}}}{\omega}\right)^{\frac{1}{2}(\gamma+4)} \frac{n_*}{n_{\mathrm{e}}} \left(\frac{\varepsilon_*}{m_{\mathrm{e}}c^2}\right)^{\gamma-1} \frac{\omega_{\mathrm{pe}} W^{\mathrm{l}}}{n_{\mathrm{e}} m_{\mathrm{e}} c^2}; \tag{14.3}$$

4. Bremsstrahlung damping rate, connected with the scattering of particles by plasma oscillations (see (13.70))

$$\gamma_4^{\mathrm{t}} \approx -\left(\frac{\omega_{\mathrm{pe}}}{\omega}\right)^2 \frac{n_*}{n_{\mathrm{e}}} \frac{\omega_{\mathrm{pe}} \tilde{W}^{\mathrm{l}}}{n_{\mathrm{e}} m_{\mathrm{e}} c^2} \left(\frac{m_{\mathrm{e}}c^2}{\varepsilon_*}\right)^3, \tag{14.4}$$

where $\tilde{W}^{\mathrm{l}}$ is the energy density of plasmons with $v_{\mathrm{p}} \leqslant c$; $\tilde{W}^{\mathrm{l}} \ll W^{\mathrm{l}}$.

Apart from these "pure" damping rates one must also take into account the "slipping" in frequency due to non-linear scattering and decay processes (equation (3.37)) and the Compton scattering of electromagnetic waves:

5. Decay rate corresponding to the interaction $t \to t' \pm 1$

$$\gamma_5^t \approx -\left(\frac{\omega_{pe}}{\omega}\right)^2 \frac{\omega_{pe}^2 4\pi I_\omega}{n_e m_e c^3} \approx -\left(\frac{\omega_{pe}}{\omega}\right)^3 \frac{\omega_{pe} W^t}{n_e m_e c^2}; \tag{14.5}$$

6. Compton interaction of electromagnetic waves

$$\gamma_6^t \approx -\left(\frac{\omega_*}{\omega}\right)^{\frac{1}{2}(\gamma+4)} \frac{n_*}{n_e} \frac{\omega_{pe}^4}{\omega_*^2} \left(\frac{\varepsilon_*}{m_e c^2}\right)^{\gamma-1} \frac{4\pi I_\omega^*}{n_e m_e c^3}$$

$$\approx -\left(\frac{\omega_*}{\omega}\right)^{\frac{1}{2}(\gamma+4)} \frac{n_*}{n_e} \frac{\omega_{pe}^4}{\omega_*^3} \left(\frac{\varepsilon_*}{m_e c^2}\right)^{\gamma-1} \frac{W_*^t}{n_e m_e c^2}. \tag{14.6}$$

We have thus six different damping rates. Any of these can turn out to be the most important one under different circumstances. Which one and when must be found for each actual case considered.

We proceed as follows. We shall consider two cases. In the first case we assume that the magnetic field is so strong that the damping rate (14.2) is larger than all others and in the second case we assume that the damping rate (14.3) is the largest one. The other damping rates will only restrict the applicability of the solutions.

The optical radius of the reactor is determined both by the damping and by the scattering by plasma waves. According to (5.42) we have for the optical radius: $\tau_R \approx (|\gamma^t|\sigma_\perp)^{1/2} \times (R/c)$, where R is the geometric radius of the reactor.

As in the reactors considered the main part of the longitudinal plasmon energy occurs in the region of phase velocities which are larger than the velocity of light we can take for the effective elastic scattering coefficient the relation which follows from (5.27) with $k' \approx \omega_{pe}/c$:

$$\frac{\sigma_\perp}{\omega_{pe}} \approx \left(\frac{\omega_{pe}}{\omega}\right)^4 \frac{W^l}{n_e m_e c^2}. \tag{14.7}$$

The condition $\tau_R \approx 1$ determines the frequency ω_R below which the radiation is trapped and above which the reactor is transparent. For a reactor in which the main role is played by the Compton scattering—we shall call it a Compton reactor—we have

$$\frac{\omega_R}{\omega_{pe}} \approx \left[\frac{n_*}{n_e}\left(\frac{\varepsilon_*}{m_e c^2}\right)^{\gamma-1}\left(\frac{W^l}{n_e m_e c^2}\right)^2\left(\frac{\omega_{pe} R}{c}\right)^2\right]^{2/(\gamma+12)}. \tag{14.8}$$

Similarly we have for a synchrotron reactor, with damping rate (14.2):

$$\frac{\omega_R}{\omega_{He}} \approx \left[\frac{n_*}{n_e}\left(\frac{\varepsilon_*}{m_e c^2}\right)^{\gamma-1}\left(\frac{\omega_{pe}}{\omega_{He}}\right)^5 \frac{W^l}{n_e m_e c^2}\left(\frac{\omega_{pe} R}{c}\right)^2\right]^{2/(\gamma+12)}. \tag{14.9}$$

We must note the weak dependence of the frequency ω_R on such poorly determined reactor parameters as n_*/n_e or $W^l/n_e m_e c^2$.

At frequencies below ω_R the spectral intensity or the radiation density are determined by the Rayleigh–Jeans law. However, we have here a peculiarity connected with the fact that particles with different energies emit at different frequencies.

In the low-frequency end of the spectrum where the emission comes from electrons from that part of the distribution function which is described by the relation $f_\varepsilon < \varepsilon^2$—that is, at frequencies $\omega < \omega_{He}(\varepsilon_*/m_e c^2)^2$ in the synchrotron reactor and at frequencies $\omega < \omega_{pe}(\varepsilon_*/m_e c^2)^2$ in the Compton reactor—the spectral energy density is determined by the normal Rayleigh–Jeans law:

$$W_\omega^t = \frac{k^2 \varepsilon_*}{\pi^2 c} = \frac{\omega^2 \varepsilon_*}{\pi^2 c^3}. \tag{14.10}$$

In the region of higher frequencies one must take into account the dependence of the frequency on the energy of the emitting particles and on their distribution function.

It is well known that in an optically thick medium the spectral energy density of the electromagnetic waves is determined by the relation $W_\omega^t = J_\omega^t/|\gamma^t(\omega)|$. The quantities J_ω^t and γ_ω^t were given in the preceding two sections. Some of them were written down with all numerical factors and others with the omission of factors of order unity. We need in what follows exact expressions for the spectral density of the electromagnetic waves. We gave in §13 the corresponding numerical coefficients for the case of Compton scattering and those coefficients are well known for the case of the synchrotron mechanism (see, for instance, Ginzburg and Syrovatskiĭ, 1964a; Kaplan and Pikel'ner, 1970). As a result we obtain the spectral density of the radiation in a Compton reactor:

$$W_\omega^t = \frac{(\gamma+4)(\gamma+6)(\gamma^2+4\gamma+11)}{\sqrt{2}(\gamma+1)(\gamma+3)(\gamma+5)(\gamma^2+6\gamma+16)} \frac{m_e \omega_{pe}^2}{\pi^2 c} \left(\frac{\omega}{\omega_{pe}}\right)^{5/2}, \tag{14.11}$$

and in a synchrotron reactor (we have corrected here some of the coefficients which are given in the above-mentioned literature, such as dropping a factor $(\gamma-2)/\gamma$ in the absorption coefficient):

$$W_\omega^t = \frac{\Gamma\left(\frac{3\gamma-1}{12}\right)\Gamma\left(\frac{3\gamma+19}{12}\right)\Gamma\left(\frac{\gamma+5}{12}\right)\Gamma\left(\frac{\gamma+8}{4}\right)}{\sqrt{3}(\gamma+1)\Gamma\left(\frac{3\gamma+2}{12}\right)\Gamma\left(\frac{3\gamma+22}{12}\right)\Gamma\left(\frac{\gamma+7}{4}\right)\Gamma\left(\frac{\gamma+6}{4}\right)} \frac{m_e \omega_{He}^2}{2\pi^2 c} \left(\frac{\omega}{\omega_{He}}\right)^{5/2}. \tag{14.12}$$

It is clear that the frequency dependence is the same in the two cases ($W_\omega^t \propto \omega^{5/2}$) but that the numerical coefficients are different.

At frequencies higher than ω_R the spectra of the synchrotron and Compton reactors are also the same:

$$W_\omega^t \approx J_\omega^t R \propto \left(\frac{\omega_R}{\omega}\right)^{(\gamma-1)/2}, \tag{14.13}$$

but, of course, they have different factors which determine their intensities.

In the general case, considered by Tsytovich and Chikhachev (1969, 1971), one can obtain the parameters of a reactor in which the damping rate is determined by the sum of expres-

sions (14.2) and (14.3) while the emission coefficient consists of the sum of the emission coefficients for the Compton and synchrotron mechanisms. As their frequency-dependences are the same the parameters of such a reactor differ from those considered above only in the numerical factors, the magnitudes of which, however, as will be seen in what follows, are unimportant for the determination of the spectrum of the relativistic electrons.

The relations obtained here for the reactor parameters are valid as long as the corresponding damping rates which determine the emission intensity are larger than the other damping rates from the list given at the beginning of this section. When evaluating the reactor parameters we must thus check that this is, in fact, the case. Taking the example of the Compton reactor we shall show below this can be done.

One can, apparently, always neglect the collisional damping rate. The rate γ_4^t is small compared to γ_3^t provided the energy density of the resonant plasmons is very small compared to the energy density of plasmons with $v_p \gtrsim c$. The rate γ_5^t is small compared to γ_3^t provided the following condition is satisfied:

$$\frac{n_*}{n_e}\left(\frac{\varepsilon_*}{m_e c^2}\right)^{\gamma-1} W^l > \left(\frac{\omega_R}{\omega_{pe}}\right)^{\frac{1}{2}(\gamma-2)} W^t. \tag{14.14}$$

Finally one gets from the inequality $\gamma_6^t < \gamma_3^t$:

$$W^l > \left(\frac{\omega_{pe}}{\omega_R}\right)^3 W^t. \tag{14.15}$$

Once one has obtained either from calculations or from observational data the reactor parameters one can easily verify that they are mutually consistent. One must similarly analyse the synchrotron reactor.

Acceleration of electrons through absorption of electromagnetic radiation

It is clear that if a particle in the medium absorbs more energy from the electromagnetic waves than it emits, its energy increases. Absorption will dominate emission in the case of a spectral energy density increasing with frequency so that, for instance, electrons in the electromagnetic radiation field (14.11) or (14.12) must accelerate on average (Tsytovich, 1963). This acceleration has a stochastic nature and is completely analogous to the stochastic acceleration through the absorption of plasma waves, considered in § 5. We have here again the diffusion equation (5.4) with the appropriate diffusion coefficient (5.5) where we must now take for u_k the probability for electromagnetic emission and for N_k the number of electromagnetic waves, N_k^t. As a result we have for the diffusion coefficient when acceleration occurs through absorption of electromagnetic waves:

$$D_{\parallel}^t = \pi^2 e^2 \int_0^\infty F(\omega, \varepsilon)\, W_\omega^t \frac{d\omega}{\omega}. \tag{14.16}$$

We have assumed here also that the electromagnetic radiation is isotropic.

The function $F(\omega, \varepsilon)$ is determined by the emission and absorption mechanisms for the electromagnetic waves. When evaluating it we must average the emission coefficients over

all angles. As a result we get, for instance, for the case of isotropically distributed relativistic electrons, accelerated by synchrotron radiation (Tsytovich and Chikhachev, 1969; Chikhachev, 1971) (from here on we shall again use ω_{He} for eH/m_ec, and not for $eH \sin \vartheta/m_ec$)

$$F(\omega, \varepsilon) = \frac{\sqrt{3}}{\pi} \frac{\omega_{He}}{\omega} \left(\frac{q}{3}\right)^2 \left\{ K_{4/3}\left(\frac{1}{3}q\right) K_{1/3}\left(\frac{1}{3}q\right) + \frac{q}{5}\left[K_{1/3}^2\left(\frac{1}{3}q\right) - K_{4/3}^2\left(\frac{1}{3}q\right)\right]\right\}, \tag{14.17}$$

where, as before, $q = (\omega/\omega_{He})(m_ec^2/\varepsilon)^2$. In the case of absorption in Compton scattering (an electromagnetic wave turns into a plasmon, giving its energy to an electron) we have

$$F(\omega, \varepsilon) = \frac{\omega^2(W^1 + W^p)}{\pi^2 \omega_{pe} e^2 c^2} \bar{u}_k, \tag{14.18}$$

where the probability $\bar{u}_k$ is given by equation (13.20).

We shall evaluate the coefficients for diffusion acceleration for the synchrotron mechanism, using the spectral radiation density (14.12) and for the Compton process the spectral density (14.11). The calculations are cumbersome but are done completely.

For the synchrotron mechanism we have

$$D_{\parallel}^{t} = \frac{3}{2\sqrt{3}} \frac{\Gamma\left(\frac{31}{12}\right)\Gamma\left(\frac{11}{12}\right)\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{3\gamma-1}{12}\right)\Gamma\left(\frac{3\gamma+19}{12}\right)\Gamma\left(\frac{\gamma+5}{4}\right)\Gamma\left(\frac{\gamma+8}{4}\right)}{(\gamma+1)\Gamma\left(\frac{11}{4}\right)\Gamma\left(\frac{3\gamma+2}{12}\right)\Gamma\left(\frac{3\gamma+22}{12}\right)\Gamma\left(\frac{\gamma+7}{4}\right)\Gamma\left(\frac{\gamma+6}{4}\right)}$$
$$\times \frac{m_e e^2 \omega_{He}^2}{c} \left(\frac{\varepsilon}{m_ec^2}\right)^3 \approx \frac{m_e e^2}{c} \omega_{He}^2 \left(\frac{\varepsilon}{m_ec^2}\right)^3. \tag{14.19}$$

For the Compton acceleration involving a change of electromagnetic waves into plasmons we have

$$D_{\parallel}^{t} = \frac{344}{945} \frac{(\gamma+4)(\gamma+6)(\gamma^2+4\gamma+11)}{(\gamma+1)(\gamma+3)(\gamma+5)(\gamma^2+6\gamma+16)} \frac{e^2\omega_{pe}^2(W^1+W^p)}{n_ec^3} \left(\frac{\varepsilon}{m_ec^2}\right)^3$$
$$\approx \frac{m_ee^2}{c} \omega_{pe}^2 \left(\frac{\varepsilon}{m_ec^2}\right)^3 \frac{W^1+W^p}{n_em_ec^2}. \tag{14.20}$$

The exact, γ-dependent numerical coefficients are required for the following analysis of the properties of plasma turbulence reactors. However, for many estimates it is fully sufficient to use the final approximate expressions.

In particular, we can use these formulae to obtain the rate of acceleration of electrons when they absorb electromagnetic radiation of the kind considered in the synchrotron mechanism:

$$\frac{d\varepsilon}{dt} \approx \frac{c^2 D_{\parallel}^{t}}{\varepsilon} \approx \frac{e^2}{c} \omega_{He}^2 \left(\frac{\varepsilon}{m_ec^2}\right)^2, \tag{14.21}$$

or in the Compton mechanism:

$$\frac{d\varepsilon}{dt} \approx \frac{c^2 D_{\parallel}^{t}}{\varepsilon} \approx \frac{e^2}{c} \omega_{pe}^2 \frac{W^1+W^p}{n_em_ec^2} \left(\frac{\varepsilon}{m_ec^2}\right)^2.$$

One must draw attention to the fact that the rate of energy gain depends on energy in the same way as that of the energy loss by electrons through radiation (see below).

It is very important that the obtained cubic dependence of the acceleration diffusion coefficient on the energy is a direct consequence of the spectrum, $W_\omega^t \propto \omega^{5/2}$, of the electromagnetic radiation. If the spectrum had been different, for instance $W_\omega^t \propto \omega^\nu$, we would have had for the acceleration diffusion coefficient $D_{\parallel}^t \propto \varepsilon^{2(\nu-1)}$. In particular, for the absorption of the spectrum (14.10) or a broadly thermal spectrum the acceleration diffusion coefficient would be $D_{\parallel}^t \propto \varepsilon^2$.

From this it also follows that the diffusion acceleration mechanism is effective only for those particles which absorb the electromagnetic radiation at frequencies less than ω_R, that is, for electrons with energies in the intervals:

$$\frac{\varepsilon_*}{m_e c^2} \lesssim \frac{\varepsilon}{m_e c^2} \lesssim \left(\frac{\omega_R}{\omega_{pe}}\right)^{1/2}, \quad \frac{\varepsilon_*}{m_e c^2} \lesssim \frac{\varepsilon}{m_e c^2} \lesssim \left(\frac{\omega_R}{\omega_{He}}\right)^{1/2}, \tag{14.22}$$

for the two reactors, respectively.

Cherenkov absorption of plasmons with $v_p \lesssim c$ leads, as is well known, also to the acceleration of relativistic electrons. The corresponding acceleration diffusion coefficient, determined by equation (5.6), is equal to $D_{\parallel}^l \approx (m_e \omega_{pe}/n_e)\tilde{W}^l$, where $\tilde{W}^l$ is the energy density of the resonance plasmons. At low energies the resonance acceleration is larger than the Compton one. The Compton acceleration starts to dominate at energies

$$\frac{\varepsilon}{m_e c^2} \gtrsim \left[\frac{\tilde{W}^l}{W^l} \frac{n_e c^3}{\omega_{pe}^3}\right]^{1/3}. \tag{14.23}$$

The ratio $\tilde{W}^l/W^l$ is usually very small so that (14.23) does not constitute a serious limitation for the electron energy. One can in a similar manner compare resonance and synchrotron accelerations.

Formation of a power-law energy distribution function in a plasma reactor

From the previous subsection it follows that in a plasma reactor with "trapped" electromagnetic radiation a strong acceleration of relativistic electrons takes place. In fact, the electrons also lose energy strongly.

The energy emitted by an electron in the synchrotron mechanism is determined by (12.11). By averaging this expression over angles bearing in mind that in (12.11) ω_{He} stands for $\omega_{He} \sin \vartheta$, we get:

$$P_2 = \frac{4}{9} \frac{e^2}{c} \omega_{He}^2 \left(\frac{\varepsilon}{m_e c^2}\right)^2, \quad \omega_{He} = \frac{eH}{m_e c}, \tag{14.24}$$

while when the electron is decelerated through Compton scattering, we get after integrating (13.21) over the frequencies:

$$P_3 = \frac{4}{9} \frac{e^2}{c} \omega_{pe}^2 \left(\frac{\varepsilon}{m_e c^2}\right)^2 \frac{W^l + W^p}{n_e m_e c^2}. \tag{14.25}$$

There are also other forms of loss but for electrons with high energies the above losses are the main ones. At lower energies ionization losses begin to dominate:

$$P_1 = \frac{e^2}{c}\omega_{pe}^2 \Lambda, \tag{14.26}$$

where Λ is the Coulomb logarithm—in a cosmic plasma $\Lambda \approx 40$. We can neglect ionization losses if the electron energy satisfies the relations

$$\frac{\varepsilon}{m_e c^2} \gtrsim \sqrt{\frac{9\Lambda}{4}}\left(\frac{n_e m_e c^2}{W^1}\right)^{1/2} \approx 10\left(\frac{n_e m_e c^2}{W^1}\right)^{1/2}, \quad \frac{\varepsilon}{m_e c^2} \gtrsim \sqrt{\frac{3\Lambda}{2}}\frac{\omega_{pe}}{\omega_{He}} \approx 8\frac{\omega_{pe}}{\omega_{He}}, \tag{14.27}$$

for a weak or a strong magnetic field, respectively.

The energy distribution function of the relativistic electrons is established by balancing the acceleration and the energy loss. This is again described by a diffusion equation with an additional term describing the losses:

$$\frac{\partial f_\varepsilon}{\partial t} = c^2 \frac{\partial}{\partial \varepsilon}\left[\varepsilon^2 D_{\parallel} \frac{\partial}{\partial \varepsilon}\left(\frac{f_\varepsilon}{\varepsilon^2}\right)\right] + \frac{\partial}{\partial \varepsilon}(P f_\varepsilon). \tag{14.28}$$

A quasi-stationary particle energy distribution is established rather fast in a reactor. Putting here $\partial f_\varepsilon/\partial t = 0$, integrating, and bearing in mind that f_ε vanishes both when $\varepsilon = 0$ and as $\varepsilon \to \infty$, we have

$$c^2 D_{\parallel} \frac{\partial}{\partial \varepsilon}\left(\frac{f_\varepsilon}{\varepsilon^2}\right) + P\frac{f_\varepsilon}{\varepsilon^2} = 0. \tag{14.29}$$

We can now easily verify that the solution of (14.29) produces a power-law distribution $f_\varepsilon \propto \varepsilon^{-\gamma}$ only, if *

$$\frac{c^2 D_{\parallel}}{\varepsilon P} = \text{constant} = \frac{1}{\gamma+2}. \tag{14.30}$$

However, acceleration by synchrotron absorption and synchrotron losses as well as acceleration by Compton scattering and Compton losses at once satisfy the condition (14.30) as here $D_{\parallel}^{t} \propto \varepsilon^3$ and $P \propto \varepsilon^2$. Of course, a combination of these mechanisms also satisfies this condition.

Substituting equations (14.19) and (14.24) into (14.30), we have

$$\frac{9}{32\sqrt{3\pi}} \frac{\Gamma\left(\frac{31}{12}\right)\Gamma\left(\frac{11}{12}\right)\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{3\gamma-1}{12}\right)\Gamma\left(\frac{3\gamma+19}{12}\right)\Gamma\left(\frac{\gamma+5}{4}\right)\Gamma\left(\frac{\gamma+8}{4}\right)}{(\gamma+1)\Gamma\left(\frac{11}{4}\right)\Gamma\left(\frac{3\gamma+12}{12}\right)\Gamma\left(\frac{3\gamma+22}{4}\right)\Gamma\left(\frac{\gamma+7}{4}\right)\Gamma\left(\frac{\gamma+6}{4}\right)} = \frac{1}{\gamma+2}. \tag{14.31}$$

* Chikhachev (1971) and Norman (1972) have shown that equation (14.29) always has as a possible solution the relativistic Maxwell-Boltzmann distribution, but the power-law distribution is formed more quickly and becomes stable, if the turbulence energy exceeds some not very large critical value (Chikhachev, 1971).

Similarly, substituting formulae (14.20) and (14.25) into (14.30) we get

$$\frac{86}{105}\,\frac{(\gamma+4)(\gamma+6)(\gamma^2+4\gamma+11)}{(\gamma+1)(\gamma+3)(\gamma+5)(\gamma^2+6\gamma+16)} = \frac{1}{\gamma+2}\,. \tag{14.32}$$

Equations (14.31) and (14.32) are equations to determine the index γ which gives a solution of equation (14.29). From (14.31) and (14.32) it follows that

$$\gamma = 3. \tag{14.33}$$

We thus reach the main conclusion: the acceleration of particles through synchrotron and Compton absorption with the corresponding losses gives power-law distribution functions with an index γ close to the observational data (Tsytovich and Chikhachev, 1971). For the sake of simplicity we have considered isotropic particle and wave distributions. One can, however, also prove some general theorems about the value of γ for the case of anisotropic distributions (Nikolaev, Tsytovich, and Chikhachev, 1973; Tsytovich, 1973). For the case when we have a distribution function of relativistic particles which is anisotropic but which for particle energies for which $mc^2/\varepsilon \ll 1$ is slowly varying (approximately constant) in a narrow angular range of particle velocities, and if the turbulent waves are also anisotropically distributed, but with a distribution which is likewise slowly varying (almost constant) in the same angular range $\Delta\theta = m_e c^2/\varepsilon$ for the wavevectors, all processes by which particles are scattered by turbulent waves (low- or high-velocity waves, Langmuir, Alfvén, or any other type of waves) together with synchrotron emission processes, will lead to a particle distribution function inside the turbulent reactor of the form $\propto \varepsilon^{-\gamma}$ with $\gamma = 3$.

One might ask whether other mechanisms could change the value of γ, without changing the power-type nature of the distribution function. This might occur because of inhomogeneities of the magnetic field or because of inhomogeneities in the relativistic particle density. This would lead to a diffusion of relativistic particle caused by the excitation of Alfvén waves. If one takes this diffusion into account, the distribution function will also be a power law, if the Alfvén wave spectrum is proportional to $1/\omega$. This is, in fact, the case according to the theoretical predictions for the Alfvén wave turbulence spectrum (see (4.50)). In this way we can explain the spread in the observed γ-values. The average observed γ is close to the value of γ found theoretically. If we take into account both the regular and the random component of the external magnetic field, the synchrotron mechanism corresponds to a lower value of γ and the Compton mechanism to a larger value. If both mechanisms are occurring at the same time, one finds intermediate values of γ. Thus, for plasma Compton and synchrotron turbulence reactors the formation of power-law distribution functions is not exceptional, but the rule, while the index of these functions lies close to the observed ones.

We now turn to the determination of another parameter of the distribution function of the fast electrons, the quantity ε_*. One sees easily that the drop in the distribution function at low energies which is equivalent to a decrease in the index γ appears, to begin with, when the right-hand side of equation (14.30) ceases to be constant and starts to decrease with decreasing energy. In particular, this occurs for an electron energy for which the ionization losses are larger than the losses through synchrotron emission or Compton scattering. Moreover, the electron energy distribution function becomes steeper with decreasing energy,

if the resonance acceleration through Cherenkov absorption begins to dominate while as before the losses are determined by the Compton scattering.

The maximum of the distribution function of the relativistic electrons is thus determined by the right-hand sides of the equations (14.27) under the condition that they are larger than the right-hand side of (14.23). The latter can be made very small when one assumes that the energy density of resonance plasmons is much less than the energy density of non-resonance plasmons. This condition will be assumed to be satisfied in what follows (see below).

It is clear that the actual maximum is determined by the larger of the quantities (14.27). We can write

$$\frac{\varepsilon_*}{m_e c^2} = \mathrm{Max}\left\{8\frac{\omega_{pe}}{\omega_{He}},\quad 10\left(\frac{n_e m_e c^2}{W^1}\right)^{1/2}\right\}. \tag{14.34}$$

In the region of energies less than ε_* the distribution function is, generally speaking, determined by several parameters and it is difficult to get here a general relation. The position simplifies again in the region $\varepsilon \ll \varepsilon_*$ where the main acceleration mechanism is the resonance Cherenkov absorption of longitudinal plasmons while the energy losses are the ionization losses. The solution of equation (14.29) for that case is

$$f_\varepsilon = \text{constant} \times \varepsilon^2 \exp\left[-\frac{\Lambda}{4\pi}\frac{\omega_{pe}^3 \varepsilon}{c^3 \tilde{W}^1}\right]. \tag{14.35}$$

It follows from this, firstly, that we have, indeed, for small energies the dependence $f_\varepsilon \propto \varepsilon^2$ as we assumed in (12.2) and, secondly, that the acceleration of resonance plasmons does not lead to a distortion of the power-law spectrum which we obtained, provided the energy density of the resonance plasmons satisfies the inequality (we use (14.23))

$$\frac{\omega_{pe}^3}{c^3 n_e}\left(\frac{\varepsilon_*}{m_e c^2}\right)^3 W^1 \gtrsim \tilde{W}^1 \gtrsim \frac{\omega_{pe}^2}{c^3}\varepsilon_*, \tag{14.36}$$

which is, apparently, very well probable in the conditions of a cosmic plasma. If the energy is rather low—but higher than ε_*—sometimes both the bremsstrahlung emission mechanism and the ionization losses could be important. If the system is optically thin for the turbulent bremsstrahlung, we need take into account only the spontaneous losses (13.71) which lead in the distribution function to an additional factor $\exp\{(\varepsilon_{**}/\varepsilon)^3\}$ which describes the steepening of the distribution function.

We must still determine two parameters: n_*/n_e and $W^1/n_e m_e c^2$. They are both in an essential manner connected with the mechanism of turbulent heating inside a plasma reactor which also depends on the mechanism of restraining the reactor from decaying. The solution of this problem requires an analysis of the large-scale magneto-gravitational structure of the cosmic plasma. These problems are still very far from being solved and they lie outside the framework of the present book.

It is, however, at once clear that the heating of the plasma in any such configuration where a strong liberation of gravitational, magnetic, or nuclear energy is possible must be very intense. Moreover, one may assume that in an intense heating the plasma in such reactors will change to a state where an appreciable part of the electrons becomes nearly relativistic

or even ultra-relativistic. The temperature of the main gas mass in such a reactor will also not be very small compared to $m_e c^2$. We note that collisional damping of such a plasma will certainly be unimportant and that just the collective plasma processes (turbulent heating) make it possible that any kind of energy can be transformed into thermal energy.

It follows from what we have said that in such turbulent plasma reactors the parameters n_*/n_e and $W^1/n_e m_e c^2$ will not be small. Also bearing in mind that the quantities (14.8) and (14.9) depend relatively weakly on the numerical values of these parameters, one can restrict oneself to the following rough estimates:

$$\frac{n_*}{n_e} \approx 10^{-2}, \quad \frac{W^1}{n_e m_e c^2} \approx 10^{-1}. \tag{14.37}$$

With these values of the parameters and also taking $\varepsilon_* \approx 10^2 m_e c^2$ we can write (14.8) and (14.9) in the following form:

$$\frac{\omega_R}{\omega_{He}} \approx \left(\frac{\omega_{pe} R}{c}\right)^{4/(\gamma+12)}, \tag{14.38}$$

$$\frac{\omega_R}{\omega_{He}} \approx \left(\frac{\omega_{pe}}{\omega_{He}}\right)^{10/(\gamma+12)} \left(\frac{\omega_{pe} R}{c}\right)^{4/(\gamma+12)}. \tag{14.39}$$

One can use these relations immediately for a comparison with the observational data (see § 15). We need only always bear in mind that the ratios ω_R/ω_{pe} and ω_R/ω_{He} must be so large that the energies of the particles accelerated by absorbing radiation with a frequency ω_R (that is, $\varepsilon \approx m_e c^2 (\omega_R/\omega_{pe})^{1/2}$ and $\varepsilon \approx m_e c^2 (\omega_R/\omega_{He})^{1/2}$ would be much larger

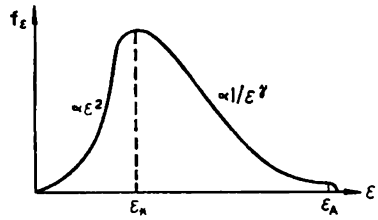

FIG. 63. Distribution function of relativistic electrons accelerated in a plasma turbulence reactor.

than ε_* because otherwise a power-law distribution function for the particle energy will not be formed in the reactor. We illustrate in Fig. 63 the nature of the electron energy distribution function.

In conclusion we remind ourselves of the important problem of the escape of the accelerated particles from the reactor. Both diffusion of the particles through its surface layer and their ejection in "eruptions" are possible. The escape through diffusion is probably accompanied by a change in the particle distribution function while an eruptive process may maintain it, even though it is accompanied by adiabatic cooling. Apparently, such "eruptions" of reactors accompanied by the ejection of a significant part of the relativistic particles are very well possible in connection with various large-scale motions of the cosmic plasma.

§ 15. The interpretation of the electromagnetic radiation of galactic nuclei, radio-galaxies and quasars

The problem of the interpretation of the radiation of powerful cosmic sources such as quasars or radio-galaxies differs from the problem of explaining the solar radio-bursts. We know more or less the conditions in which the plasma turbulence on the Sun occurs: we know the electron density and temperature and can estimate independently the velocities and densities in the beams. Observations enable us to follow the change in time of the process and to obtain its more detailed characteristics.

The position is more complicated when we try to interpret the emission of cosmic radio-sources. The particle densities, their temperatures, the beam velocities, and other parameters are badly known. We know, for instance, the magnitudes of n_e and T_e in the regions where an emission spectrum originates, but they may not be the same in the region of the radio-emission or the region of plasma turbulence.

Essentially only the spectral index α (see Fig. 53 on p. 158) and the total radiation flux are determined with any certainty, and it is not simple to deduce from the latter the spectral intensity as the size of the objects and their distances from us can be estimated only with great uncertainties. Some conclusions can also be derived from a study of the polarization of the radiation. Briefly, the observational data do not give sufficient information to make it possible at the present time to give a more or less unambiguous, generally qualitative, or even less, a quantitative interpretation of the observed powerful sources of cosmic radiation.

It is true that initially it appeared that it would be easy to interpret the radio-emission of cosmic sources. This was when astrophysicists knew essentially only one emission mechanism, the synchrotron mechanism. There was then no other choice and one could then attempt to select on the basis of the observational data a unique set for the corresponding parameters. This could often be done but sometimes there were insurmountable difficulties. Let us consider this in some detail.

Synchrotron interpretation of the radio-emission of cosmic sources

Let us assume that we can determine from the observations the spectral index $\alpha = \frac{1}{2}(\gamma - 1)$ the spectral intensity I_ω, and the size of the system R. We then also know the index γ of the electron distribution function.

We shall assume further that for some frequency $\omega = \omega_*$ the emission spectrum has a maximum. This frequency may lie outside the observed radio-band. There may be several reasons for the fall in the spectrum in the low-energy region, but to begin with we assume that this effect is connected with reabsorption (Slysh, 1963). We then get from equation (14.12) for the increasing part of the spectrum ($\omega < \omega_* = \omega_R$):

$$I_\omega \approx \frac{m_e \omega^2}{(2\pi)^3} \left(\frac{\omega}{\omega_{He}}\right)^{1/2} \tag{15.1}$$

(here and henceforth we omit numerical factors of order unity). Equation (12.16) gives for

the descending part of the spectrum

$$I_\omega \approx \frac{J_\omega R}{4\pi} \approx \frac{e^2 \omega_{He} n_*}{4\pi c} \left(\frac{\varepsilon_*}{m_e c^2}\right)^{\gamma-1} \left(\frac{\omega_{He}}{\omega}\right)^{\frac{1}{2}(\gamma-1)} R. \tag{15.2}$$

As we have here assumed that the frequency at the maximum of the spectrum is a characteristic frequency at which the optical depth turns out to be of order unity, we get by analogy with (14.9), but neglecting the "complications" due to scattering (the plasma is not turbulent), yet another condition:

$$\frac{\omega_*}{\omega_{He}} = \left[\frac{n_*}{n_e}\left(\frac{\varepsilon_*}{m_e c^2}\right)^{\gamma-1} \frac{\omega_{pe}}{\omega_{He}} \frac{\omega_{pe} R}{c}\right]^{2/(\gamma+4)}. \tag{15.3}$$

To these relations we add the obvious inequality

$$\omega_* > \omega_{He}\left(\frac{\varepsilon_*}{m_e c^2}\right), \tag{15.4}$$

and the often used assumption that the total energy of the relativistic particles and of the magnetic field must be equal, $H^2 = 4\pi n_* \varepsilon_* \delta$, or

$$\left(\frac{\omega_{He}}{\omega_{pe}}\right)^2 \approx \frac{n_*}{n_e} \frac{\varepsilon_*}{m_e c^2} \delta. \tag{15.5}$$

The coefficient δ takes the contribution from the relativistic ions into account. One usually assumes that $\delta \approx 1$ to 100. The set of equations (15.1) to (15.5) determines the parameters of the emitting synchrotron region. An analysis of this system shows that in certain cases one can obtain a physically reasonable solution but in other cases not. We shall explain what the problem is.

Assume that only the decreasing spectrum has been observed, $I_\omega \propto \omega^{-(\gamma-1)/2}$. Equation (15.2) then gives us for given I_ω and R a connection between ω_{He}, ε_*, and n_*. One of these three parameters can be eliminated using equation (15.5). The remaining parameters are greatly restricted by inequality (15.4) so that in final reckoning these parameters even if they are not determined uniquely are severely restricted in their possible values.

We now proceed as follows. Using the values obtained for ω_{He}, n_* and ε_* we must determine from (15.3) the frequency of the maximum in the spectrum, ω_*. It must, by definition, lie in the low-frequency part below the limits of the observed spectrum. If the ω_* found in this way lies in the observed part of the spectrum (but its maximum is not observed) or in the shorter-wavelength part of the spectrum, the solution obtained is inconsistent and bears no relation to reality. One can also try to choose the parameters ω_{He}, n_* and ε_* in a different way using the remaining free choice, as (15.4) is only an inequality, but usually one can not gain much.

If the maximum is also observed the condition (15.1) can be applied to the increasing part of the spectrum. The frequency ω_{He} is here determined uniquely and if it is in agreement with the remaining relations (15.2) to (15.5) this solution can be assumed to be satisfactory, but as a rule this does not happen. It is important that equation (15.1) is very tight.

The observed maximum in the spectrum can also be ascribed to the influence of the plasma on the synchrotron radiation, that is, condition (12.18). There appears then another parameter, n_e.

The method described here has been applied many times to the analysis of cosmic radio-sources; the results can be found in many papers and books (Shklovskiĭ, 1960a; Ginzburg and Syrovatskiĭ, 1964a; Kaplan and Pikel'ner, 1970) where the reader can also find the corresponding numerical estimates. The general conclusions are the following. When evaluating extended objects such as clusters of radio-galaxies the results obtained are satisfactory, that is, the computed ω_* indeed fall in the unobserved low-frequency radio-band or are actually observed. One can show (Kellermann and Paolini-Toth, 1968; Kellermann, Jauncey, Cohen, Shaffer, Clark, Broderick, Rönnäng, Rydbeck, Matveyenko, Moiseyev, Vitkevitch, Cooper, and Batchelor, 1971) that if the effective temperature in the centimetre band is less than 10^{12} °K this radiation can be explained by the synchrotron hypothesis.

However, when evaluating compact and powerful radio-sources such as quasars it has not been possible to tie everything up tidily. The large intensity requires a large density of ultra-relativistic electrons and the frequency ω_* obtained from (15.3) turns out to be too high. The observed intensity of the radio-emission turns out to be much higher than (15.1).

However, even in those cases when the solution of the set (15.1) to (15.5) turns out to be physically admissible there appear new difficulties. The main one is connected with the following considerations. Let us assume that a cloud of relativistic electrons is expanding isotropically. Its magnetic field will then decrease according to $H \propto R^{-2} \propto t^{-2}$ (we assume conservation of magnetic flux). In accordance with the condition of adiabatic invariance the particle energy decreases as $\varepsilon \propto \sqrt{H} \propto t^{-1}$. The density decreases as $n_* \propto R^{-3} \propto t^{-3}$. Hence we get the way the radiation flux should decrease with time, $F_\omega \propto R^2 I_\omega$ (Shklovskiĭ, 1960b, 1965b, 1971a):

$$F_\omega \propto t^{-2\gamma} \tag{15.6}$$

for the decreasing part of the spectrum. In the increasing part of the spectrum, (15.1), the flux of the radio-emission increases as t^3. It follows from (15.3) that the maximum in the spectrum drifts to the low-frequency end of the spectrum as $\omega_* \propto t^{-2(2\gamma+3)/(\gamma+4)}$. Finally, we also find from this and from (15.6) the change in flux when the frequency of the maximum changes:

$$F_{\omega_*} \propto \omega_*^{(7\gamma+3)/(4\gamma+6)}. \tag{15.7}$$

Kellermann and Paolini-Toth have succeeded in observing a systematic change in the flux and in the position of the maximum. It can be made to agree with equations (15.6) and (15.7) by putting $\gamma \approx 1$.

However, putting $\gamma \approx 3$ we get such a rapid decrease in the radio-emission flux with time that it is incomprehensible why such expanding clouds have not yet been observed. We can, however, assume that the index γ also changes during the expansion (Shklovskiĭ, 1971b); however, in these cases γ only increases (Kardashev, 1964). We mention also the work of Ozernoĭ and Sazonov (1968, 1969) who assumed for an explanation of the change in the spectra that there is an ultra-relativistic dispersion of the macroscopic clouds and of the plasma.

Summarizing we can say the following. The synchrotron mechanism can explain the radio-emission of extended and of more or less stationary formations. The application of this mechanism to rapidly changing and compact objects meets with serious difficulties which sometimes can be removed by suitable assumptions, but often these difficulties turn out to be insurmountable.

There are usually no such difficulties when one interprets powerful cosmic sources of optical or X-ray emission.

Plasma interpretation of the emission from powerful radio-sources

The emission of cosmic radio-sources can be either spontaneous or stimulated. We shall consider the advantages and deficiencies of plasma emission mechanisms for both cases.

Spontaneous plasma emission mechanisms have certain advantages as compared to the synchrotron mechanism but when interpreting the observed objects one must always assume the presence of plasma turbulence with a relatively high energy density.

The advantages of spontaneous plasma mechanisms consist in the following. Firstly, there are here more parameters which can be chosen in such a way that different peculiarities of the spectra can be explained, for instance, the presence of several maxima and minima in the spectrum of a single object. Secondly, there is also greater scope for the interpretation of the observations of the polarization. In particular, it is important that in plasma mechanisms the electric vector of the polarized radiation can also be directed along the magnetic field strength vector. Thirdly, plasma emission mechanisms connect the acceleration of relativistic electrons and their emission. We can here obtain a great variety of spectra (see Getmantsev, 1970, 1971 for an interpretation of the observations by Braude, Lebedeva, Megn, Ryabov and Zhouck, 1969) and explain the different time variations of the spectra (Kaplan, 1967), and in particular one can obtain also a decrease in time of the index γ (Gaĭlitis and Tsytovich, 1964b). It is important that the small "inertia" of the plasma turbulence makes it possible to explain fast changes both in the spectrum and in the intensity of the radiation.

However, the greatest advantage of plasma mechanisms occurs when stimulated processes are taken into account. We discussed earlier in § 12 that although in the synchrotron mechanism it is very well possible to get stimulated amplification of the electromagnetic radiation, it requires rather rigid conditions which are often unlikely to be satisfied in astrophysical objects.

The stimulated amplification of the electromagnetic radiation in plasma mechanisms is subject to fewer restrictions and it is thus more probable that these effects will occur under cosmic conditions. Stimulated processes "amplify" all advantages of the plasma emission mechanisms. Firstly, one can now also obtain complex spectra. Secondly, in this case it is possible to produce radiation with 100 per cent polarization. Thirdly, one can certainly expect here a change in flux and in spectra on a time scale equal to the light dimensions of the object. Fourthly, stimulated processes in a turbulent plasma lead to a rapid acceleration of an appreciable part of the electrons in the plasma.

In the framework of the first book on plasma astrophysics we cannot discuss here the various actual objects. One may hope that such a study will be made in the near future.

We restrict ourselves here to a brief mention of studies which have already been made in this direction and of some preliminary estimates.

Authors such as Shklovskiĭ, Ginzburg and Kardashev started to talk about the possibility of a plasma interpretation of the radio-emission of quasars or of other compact objects such as the small source in the Crab Nebula in 1964 or 1965 when it became clear that the application of the synchrotron mechanism was running into insurmountable difficulties.

Ginzburg and Ozernoĭ (1966b) were the first to attempt to give a quantitative estimate. In their paper they assumed that the emission of quasars can be explained by the passage of shock waves through their atmosphere which excite plasma turbulence in the same way as happens in the solar corona for the excitation of type II bursts. One must note that almost always the first attempt to apply plasma mechanisms to a study of any object reduces to using results obtained in the study of the sporadic solar radio-emission (see Ginzburg, Zheleznyakov and Zaĭtsev, 1969 for such a study of pulsars). All deficiencies of the corresponding theories of the solar radio-emission are even magnified in such a transfer. In particular, in the paper by Ginzburg and Ozernoĭ which we just quoted they obtained an unacceptably large value for the plasma turbulence energy density in the shock waves: $W^{l} \approx 20 n_e T_e$. Moreover, to obtain the production of emission at high frequencies they required large values of ω_{pe} or ω_{He} which are also improbable. Colgate, Lee and Rosenbluth (1970) took into account the increase in photon frequency in scattering (see § 5) but neglected the important processes for producing relativistic electrons in a turbulent plasma and their emission (§§ 5 and 13).

Zheleznyakov (1967c) applied ideas about the synchrotron instability to interpret the radio-emission of quasars and other point sources. He gave estimates for the growth rates and the conditions for the excitation (necessary density and temperature of the "cold plasma). It turned out that the density of such a plasma must be high and its temperature must in some cases be comparable to $m_e c^2$. The non-linear effects were not estimated. This can, incidentally, easily be done by using equation (12.73). For instance, to obtain an intensity of the order of 10^{-8} erg/s at frequencies of the order of $\omega \approx 10^9\ s^{-1}$ and assuming that $\omega_{pe} \approx 10\omega_{He}$ we need a density of relativistic particles of $n_* \approx 1\ cm^{-3}$.

Moreover, it follows from the analysis given in §§ 12 and 13 that the synchrotron instability apparently leads in turn to the development of longitudinal plasmon turbulence, the conversion of which into electromagnetic waves is not subject to strict restrictions on the frequency and particle energy such as are imposed through the requirement of a direct excitation of the electromagnetic radiation field.

The present authors (Kaplan and Tsytovich, 1970a) gave estimates for the growth rates and showed that even for the smallest sizes of these sources the growth rates are sufficiently large to guarantee the excitation of electromagnetic radiation through the Compton scattering by a beam of relativistic electrons. We can now use equation (13.44) to estimate the longitudinal plasmon energy density and use formula (13.47) to determine also the density of the electromagnetic radiation and hence to estimate the spectral intensity.

Unfortunately, the difficulty mentioned at the beginning of this section, of insufficient information about the parameters of cosmic sources, which one meets with when interpreting the radio-emission of these sources, occurs here in full measure. For instance, using equation (13.47) we cannot give an unambiguous estimate of W^{t} as the quantities n_*/n_e

and ε_* are unknown, even if one might unambiguously determine W^1, for instance, from relation (13.44). However, one can easily check that by choosing the unknown parameters arbitrarily but within reasonable limits, one can obtain the intensity of the radiation which corresponds to the observational data. It is, however, possible to approach the problem somewhat differently, using the observational data (Krivorutskiĭ and Tsytovich, 1970). The observed emission refers to the surface regions of the objects where the escape of anisotropic particles and electromagnetic radiation leads to plasma instabilities. The energy density of the escaping emission is approximately known. It must be the source of the turbulence independent of what is the mechanism for the emission of electromagnetic waves. By virtue of (4.57) the power of the excitation of Langmuir oscillations by electromagnetic radiation is

$$Q \approx \frac{\omega_{pe}^4}{\omega_*^3} \frac{(W^t)^2}{n_e m_e c^2}. \tag{15.8}$$

This enables us to estimate the energy density W^1 which in the above-quoted paper was found to be comparable to $n_e T_e$ for quasars which indicates that the quasars are, indeed, objects with a strong plasma turbulence.

The steepening of the observed spectrum of some cosmic radio sources (see, for instance Braude, Lebedeva, Megn, Ryabov, and Zhouck, 1969) could be explained either by the influence of the turbulent bremsstrahlung described by (13.71) (Tsytovich, 1973) or by ionization losses (Braude and Kaner, 1973).

We hope that future observations and a more detailed analysis of the various processes will make it possible to obtain better defined estimates of the parameters. In any case, the physics of plasma processes which probably occur in such sources is qualitatively intelligible and the equations necessary for the calculations are available.

Infra-red emission sources in galactic centres and the possibility of a direct emission from a plasma turbulence reactor

We gave in § 11 data on the infra-red sources in galactic nuclei and quasars. We can try to identify these sources with a plasma reactor in which according to the results of § 14 fast particles can be accelerated and a power-law distribution law can be formed. In such a case the observed frequency of the maximum in the radiation $\omega_{ir} \approx 2{\cdot}5 \times 10^{13}\ s^{-1}$ must be compared with the frequency ω_R which determines the maximum in the emission spectrum of the reactor.

It is true that the observed frequency-dependence of the intensity of the infra-red source agrees poorly with the theoretical ideas ($I_\omega \propto \omega^{7/2}$ instead of $\propto \omega^{5/2}$ for $\omega < \omega_{ir}$). The observed spectrum decreases also very steeply at the high-frequency side. One can, however, mention a number of reasons which would enable us to explain such a discrepancy, in particular, the conditions for emission in inhomogeneous surface layers of such objects.

We shall see what estimates one obtains when applying (14.40) and (14.41) to such objects. We assume that the size of the source is $R \approx 10^{19}$ cm. We also take $\gamma = 3$ in both cases. We then have from equation (14.40): $\omega_{pe} \approx 3 \times 10^8\ s^{-1}$. However, the energy of the particles accelerated in the plasma reactor, $\varepsilon \approx m_e c^2\ (10^5)^{1/2} \approx 300\ m_e c^2$ is only an order of magnitude larger than the energy at the maximum of the electron distribution function $\varepsilon_* \approx$

$10\,m_e c^2$. The plasma reactor can therefore not produce the power-law particle distribution in a sufficiently wide range of energies.

In the synchrotron reactor there are two parameters: ω_{pe} and ω_{He}. To fix the ideas, we assume that $\omega_{pe} \approx 10\omega_{He}$. We then get for the same values of the other parameters: $\omega_{pe} \approx 3\times10^8\ s^{-1}$ and $\omega_{He} \approx 3\times10^7\ s^{-1}$. The power-law spectrum occupies here the energy range from $\varepsilon_* \approx 60\,m_e c^2$ to $\varepsilon = 10^3\,m_e c^2$. According to (15.1) we get for the radiation intensity at the maximum of the spectrum $I_\omega \approx 10^{-2}$ erg/cm^2, that is, with a certain margin over the observed data. This margin, incidentally, is necessary as the spread in luminosity of infra-red sources reaches five orders of magnitude.

A plasma reactor with $n_e \approx 10^8\ cm^{-3}$ and $H \approx 2$ Oe might thus correspond to the observed data of the infra-red source in the centre of galaxies.

Norman and ter Haar (1972) have shown that conditions are favourable for the existence of a plasma turbulence reactor in the accreted disc in the black-hole model of galactic nuclei developed by Lynden-Bell and Rees (1971) and that in that case one may expect electromagnetic emission with a peak in the infra-red.

Relativistic electron beams in the intergalactic and the galactic space and the interpretation of outburst emission

When interpreting the emission from bursts from galaxies into the intergalactic space we have at the present time at least two difficulties. The first one is that quasi-linear relaxation of beams should rapidly smear out the outbursts and the effect should not be observable. This difficulty is in fact fictitious and is resolved in the same way as for type III bursts on the Sun when we take non-linear stabilization into account. The second difficulty consists in the very fast decrease in the emission flux (see (15.7)) because of the spread in the burst if we decide on the synchrotron hypothesis of its emission from the point of energy.

It is, however, possible to use theoretical ideas about plasma turbulence to solve these difficulties without special efforts (Motz and Tsytovich, 1972). Although at the moment there are several parameters which cannot be determined from the observations one can at least verify that there are no pronounced discrepancies with the observations. At the present there are strong observational arguments in favour of the assumption that the outbursts may move with relativistic velocities. Hence, we must use the theory of the interaction of relativistic beams, taking non-linear stabilization effects into account.

In principle the case is possible, which we shall consider here, where beams of fast particles bursting from the nucleus into the intergalactic space turn out to be denser than the intergalactic medium itself. One must then consider the intergalactic medium to be "beams" in the cloud of relativistic particles and change to the coordinate system in which the cloud of relativistic particles is at rest.

Let u be the average translational velocity of the cloud of relativistic particles, $\varepsilon_* = m_e c^2 (1-u^2/c^2)^{-1/2}$ be the electron energy corresponding to that velocity, and $\Delta\varepsilon_*$ the spread in energies. If $\varepsilon_* \approx m_e c^2$ the change to the system of coordinates fixed in the cloud does not introduce any essential changes. Of more interest is the case when $\varepsilon_* \gg m_e c^2$.

It is important to take into account the change in the velocity dispersion when changing to a moving system of coordinates. In particular, the velocity v of a particle with energy

$\varepsilon = m_e c^2(1-v^2/c^2)^{-1/2}$ when we change to a system of coordinates moving with velocity u is equal to

$$v' = \frac{v-u}{1-vu/c^2} \approx c\,\frac{\varepsilon_*^2-\varepsilon^2}{\varepsilon_*^2+\varepsilon^2}, \quad \frac{|v-c|}{c} \ll 1, \quad \frac{|u-c|}{c} \ll 1. \tag{15.9}$$

This means that in the system of coordinates of the cloud the spread in the velocities of its particles can turn out to be non-relativistic,

$$\frac{\Delta v'}{c} \approx \frac{\Delta\varepsilon_*}{\varepsilon_*} \ll 1, \tag{15.10}$$

while the spread in particle velocities $\Delta v'$ in the intergalactic medium (the "beam") is very small:

$$\Delta v' \approx \Delta v\left(\frac{m_e c^2}{\varepsilon_*}\right)^2. \tag{15.11}$$

In other words, under such circumstances there must again occur a hydrodynamic instability leading to a fast increase in the velocity spread, that is, to a heating of the intergalactic medium. After that the system will change to a normal kinetic instability and the rest of the considerations can follow the methods described in the preceding sections. In particular, one can show that the characteristic time-scales for the relaxation of such "beams" can be very long. This is connected with the stabilization of such beams by non-linear effects. In particular, estimates indicate that galactic outbursts can be stabilized rapidly. Indeed, if there would simply be quasi-linear relaxation, the galactic outbursts would die literally instantaneously and it would be impossible to observe them. For instance, if we take $n_*/n_e \approx 0{\cdot}1$, $\varepsilon_*/m_e c^2 \approx 10^2$, $\Delta\theta \approx 0{\cdot}1$, $\omega_{pe} \approx 10^2\ s^{-1}$, we get from (12.62) $\gamma^1 \approx 10\ s^{-1}$, that is, corresponding to a distance of about 10^9 cm which is negligible compared to galactic distances ($\approx 10^{22}$ cm). Assuming that as a result of the development of the hydrodynamic instability the beam reaches the limits of the kinetic instability and $(\Delta\theta)^2 \gg (m_e c^2/\varepsilon_*)^2$, we get from (12.65) and (12.66)

$$(\Delta\theta)^2 \approx \left(\frac{n_*}{n_e}\right)^{1/2}\left(\frac{m_e c^2}{\varepsilon_*}\right)^{1/2} \approx \frac{\Delta k}{k}, \tag{15.12}$$

that is,

$$\frac{\gamma^1}{\omega_{pe}} \approx \left(\frac{n_*}{n_e}\,\frac{m_e c^2}{\varepsilon_*}\right)^{1/2}. \tag{15.13}$$

We noted that the "temperature" of the particles in the cloud is non-relativistic, but if $\Delta\varepsilon_*$ is not too different from ε_*, it is not too small compared to $m_e c^2$, that is, in actual fact we have $k_* \ll \Delta k$ and "absolute" stabilization, described by (12.67), does not occur. The power of the generation can be described by the formula

$$Q \approx \frac{\gamma^2(\Delta k)^2}{\alpha^1} \approx 30\omega_{pe} n_e m_i c^2\left(\frac{n_*}{n_e}\right)^2\left(\frac{m_e c^2}{\varepsilon_*}\right)^2\left(\frac{\Delta\varepsilon_*}{\varepsilon_*}\right)^2. \tag{15.14}$$

As we have already shown in the preceding chapter, complete stabilization through scattering by electrons starts after a time determined in this case by the growth rate

$$\frac{\gamma^1}{\omega_{pe}} \approx \frac{Q}{\omega_{pe} n_e m_e c^2}\,\frac{\varepsilon_*}{\Delta\varepsilon_*}\left(\frac{\varepsilon_*}{m_e c^2}\,\frac{n_e}{n_*}\right)^{1/2} \approx 3\times 10^4\,\frac{\Delta\varepsilon_*}{\varepsilon_*}\left(\frac{n_*}{n_e}\,\frac{m_e c^2}{\varepsilon_*}\right)^{3/2}. \tag{15.15}$$

Unfortunately, neither n_*/n_e nor m_ec^2/ε_* are known. However, it is clear from the considerations developed here that is should be possible to attain complete stabilization of beams at relatively small distances. For instance, for $\Delta\varepsilon_*/\varepsilon_* \approx 0{\cdot}1$, $n_*/n_e \approx 0{\cdot}1$, and $m_ec^2/\varepsilon_* \approx 10^{-2}$ the characteristic stabilization length is only somewhat greater than the quasi-linear relaxation length.

The difficulty about the quasi-linear relaxation is thus completely removed. Another difficulty is, in fact, of an energic nature. However, the intergalactic "beam" adds energy to the outburst as even in the hydrodynamic stage—up to the kinetic stage which is stabilized —it can give off an appreciable fraction of its energy (Faĭnberg, Shapiro and Shevchenko, 1970). One obtains easily the total energy flux of the intergalactic flux over the cross-section of the cloud, $F \approx 4\pi R^2 n_* \varepsilon_* c$; assuming that $R \approx 10^{20}$ to 10^{21} cm, and $n_* \approx 10^{-5}$ cm^{-3}, we get $F \approx 3\times 10^{41}\,(\varepsilon_*/m_ec^2)$ erg/s. For the given values of ε_*/m_ec^2 this agrees with the observed radio-fluxes of intergalactic outbursts. From an energic point of view there arises thus also no difficulty.

We must yet discuss the problem of the emission mechanism. In principle it could be the synchrotron mechanism in the magnetic fields of the outburst itself or in the intergalactic field, but it can also be a plasma mechanism. To compare them we estimate W^1 and $H^2/8\pi$. In the intergalactic space $H^2/8\pi \approx 10^{-15}$ to 10^{-16} erg/cm^3 (that is, $H \approx 3$ to 10×10^{-8} Oe; see, for instance, Ginzburg and Syrovatskiĭ, 1964a). We can estimate the value of W^1 as follows. An intergalactic "beam" penetrating into the outburst always passes rapidly through the hydrodynamic stage and then reaches stabilization in the kinetic stage and it produces thus in the surface region some turbulent energy density. Subsequent particles in this "beam" experiencing the hydrodynamic instability are already incident upon a turbulent plasma and thus, being stabilized, penetrate further without loss of energy, give off "their" portion of the energy, and thereby create turbulence in the next layer of the outburst. Such a wave of turbulence generation passes through the whole of the outburst. Its velocity $v \approx c(\Delta\varepsilon_*/\varepsilon_*)^2$, although somewhat lower than the light velocity, is comparable with it when $\Delta\varepsilon_* \lesssim \varepsilon_*$. We can thus estimate W^1 from the condition of non-linear stabilization by electrons:

$$\frac{W^1}{n_e m_e c^2} \approx \sqrt{\left(\frac{n_*}{n_e}\,\frac{m_ec^2}{\varepsilon_*}\right)}\,\frac{\Delta\varepsilon_*}{\varepsilon_*}\,. \tag{15.16}$$

If we take $n_* \approx 0{\cdot}1\ n_e$, $m_ec^2/\varepsilon_* \approx 10^{-2}$, $\Delta\varepsilon_*/\varepsilon_* \approx 0{\cdot}3$, $n_e \approx 10^{-6}$ cm^{-3}, we get $W^1 \approx 10^{-14}$ erg/cm^3 which is larger than the energy density of the intergalactic magnetic field. In this connection it is improbable that such a field can prevent the penetration of the intergalactic "beam" into the outburst. Moreover, it is clear from this estimate that the field of the outburst itself must be rather large in order to influence the processes for exciting turbulence by the intergalactic "beam" and also for the synchrotron mechanism to dominate over the plasma mechanism.

The problem of the outbursts from the nuclei of galaxies, the turbulence generated by the outbursts, and their stabilization are, apparently, important for the theory of the origin of galactic cosmic rays and the "dissipation" of supernova shells (Ginzburg, 1966; Tsytovich, 1966a; Parker, 1970) and also for the theory of the heating of the intergalactic gas (Ginzburg and Ozernoĭ, 1965, 1966a; Wentzel, 1971).

CHAPTER IV

Pulsar Emission

(Turbulent processes in a relativistic plasma with a strong magnetic field)

§ 16. Observational data about pulsars

THE discovery in 1967 of the pulsars by Hewish and his collaborators (Hewish, Bell, Pilkington, Scott, and Collins, 1968) not only presented astrophysicists with very interesting objects to study, but at the same time presented some new problems in physics. One usually assumes that pulsars are rotating neutron stars. The theoretical study of pulsars is developing in two directions. One is the study of the physics of neutron stars. Papers concerned with the second class are connected with studying emission mechanisms which are able to produce very powerful electromagnetic waves—which are observed in the shape of separate pulses—in wavelength bands, starting with radio-waves and ending with *X*-rays.

The study of neutron stars had begun before the discovery of pulsars. However, at that time it was assumed that the emission of a neutron star would hardly have a complex character. It was assumed that the temperature at the surface of a neutron star was of the order of 10^7 °K and that its radiation was produced by the usual thermal bremsstrahlung mechanism and thus was concentrated in the X-ray end of the spectrum. The discovery of the pulsars stimulated further developments in the study of the structure of neutrons stars, of the properties of the matter in their interior and in their surface layers, and their dynamic properties. All these problems lie outside the framework of the present book and we shall not further be concerned with them.

The most difficult problem for pulsars has turned out to find an explanation of the peculiarities of their emission which has nothing in common with the earlier proposed X-ray emission of a hot surface. A study of mechanisms for this emission is the main topic of the present chapter.

The necessary data from the physics of neutron stars are the following ones: these objects have a radius of about 10^6 cm, they have a strong magnetic field at their surface which according to some estimates can reach values of 10^9 Oe, and even 10^{12} Oe. The possibility that the magnetic field may also be considerably weaker is not excluded. Neutron stars rotate with a large angular velocity (corresponding to the period with which the pulses are repeated) and one assumes that the rotation axis and the symmetry axis of the magnetic field do not coincide and may even be mutually perpendicular. The velocities with which the pulsars move in space are large and possibly reach several hundreds of km/s.

RADIO-EMISSION OF THE PULSARS

Pulsars—of which at the beginning of 1972 65 were known (see, for instance, a list by Manchester and Taylor (1972) to which 4 where later added)—emit radio-waves in a wide band of frequencies (some spectra are given in Fig. 64) with a strict periodicity. The periods undergo, however, some changes which may be a smooth increase or sudden changes—which are most often decreases in period, although, apparently, increases have also been found.

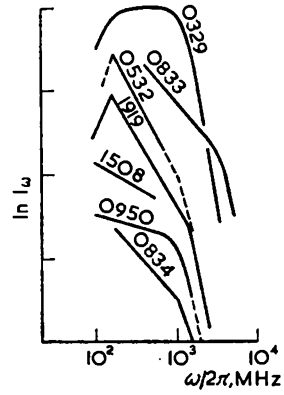

FIG. 64. Radio-spectra of several pulsars.

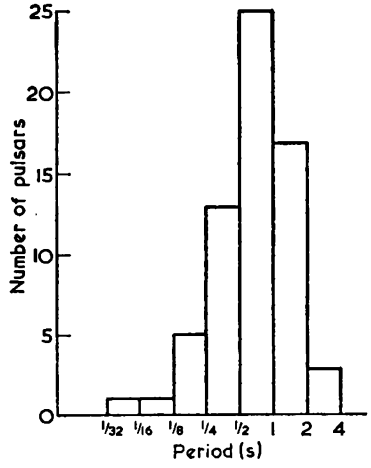

FIG. 65. Period-distribution of the pulsars.

One usually assumes that the periodicity of the pulses is determined by the rotation of the pulsar, the smooth change in its period by the slowing down of this rotation, and the sudden changes by a rearrangement of the structure of its surface layers or interior ("star-quakes"; see, for instance, Pines and Shaham, 1972). Apparently, the younger the pulsar, the smaller its period, and the larger the change in its period. The distribution of pulsars over periods is given in Fig. 65, and the statistical dependence of the period on the slowing-

down of the rotation (that is, the characteristic time of the existence of the pulsar) is given in Fig. 66.

The pulsar in the Crab Nebula (PSR 0531) has the shortest period and the fastest increase. This pulsar differs from the others through some peculiar properties, in particular, it shows optical and X-ray pulses. The pulsar PSR 0531 will therefore be considered separately.

The shape of the pulses and their length Δt do not show such a stability as the period; consecutive pulses are often completely dissimilar. The average shape of the pulses, though,

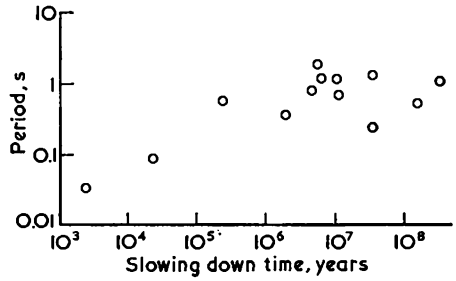

FIG. 66. Statistical connection between period and slowing-down time.

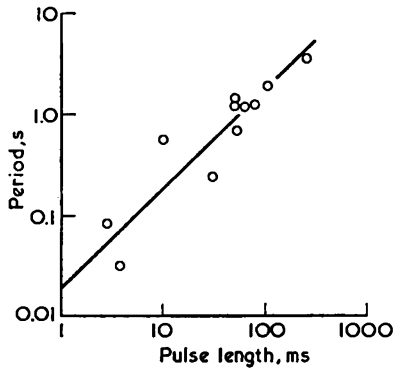

FIG. 67. Statistical connection between period and pulse length.

is conserved and the length of the "average" pulse depends statistically on the period P, as follows (Fig. 67):

$$\frac{\Delta t}{P} \approx \frac{1}{20} \quad \text{to} \quad \frac{1}{30}. \tag{16.1}$$

We show in Fig. 68 the average shapes of the pulses of a number of pulsars. In this figure, as well as in other figures and in the text, we indicate the pulsar by a number which gives the hours and minutes of the right ascension of their position, as given by Manchester and Taylor (1972). The shorter the pulse, the simpler its form. In some cases where the pulse consists of subpulses one has observed a systematic shift of their position, corresponding to the so-called secondary periodicity. According to the shape of the pulses the pulsars can be divided into classes: S with simple pulses, C with complex pulses, and D with drifting subpulses.

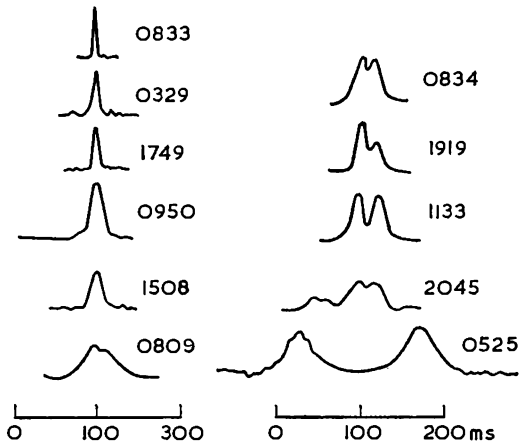

FIG. 68. Some characteristic average pulse shapes.

In some cases, for instance, for the pulsar PSR 0950 one has observed in the time between pulses a small "interpulse" the intensity of which in the case of PSR 0950 is 1·5% of the main pulse. The shape of the pulse is sometimes asymmetric with the leading side falling more steeply. It is difficult to notice a dependence of the width of the pulse on wavelength.

The radiation from some pulsars shows strong (up to 100%), and from others, on the other hand, relatively weak (from 5 to 10% up to 30%) linear polarization. Both the polarization angle and the degree of polarization change during a single pulse (Fig. 69). In a number of cases circular polarization (up to 30%) has been observed.

The intensity of the pulses shows strong variations. Often the pulses disappear and appear again after minutes, or much longer time intervals (weeks or months). The change in intensity of some pulsars (smoothed over periods of eight days) is shown in Fig. 70.

The radio-emission spectra of pulsars, shown in Fig. 64 is, as a rule, more complicated than the spectra of other radio-sources. It can be described by a power-law only along

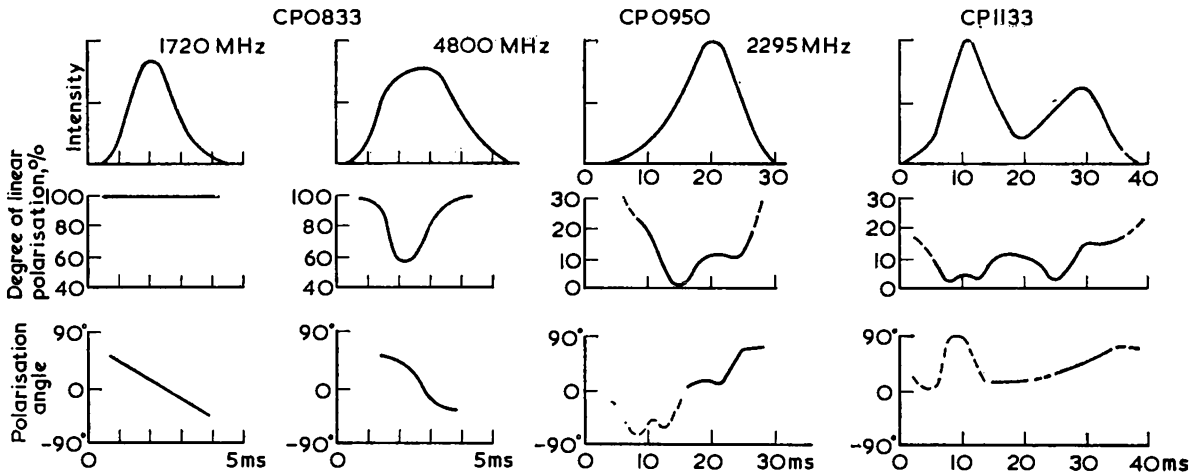

FIG. 69. Change in intensity, degree of polarization, and polartzation angle during a pulse for several pulsars.

separate sections. Apparently, for $\omega \lesssim 6\times10^8\ s^{-1}$ and $\omega \gtrsim 10^{10}\ s^{-1}$ the spectra decrease steeply. The spectral index in the frequency range $6\times10^8\ s^{-1} < \omega\ 10^{10}\ s^{-1}$ changes between the limits of 0·6 and 2. For larger frequencies the spectral index increases to 3 or 4.

The spectral intensity of the pulsar radio-emission is rather undetermined due to the uncertainty in the estimates of the distances and sizes of the objects. The size of the star is

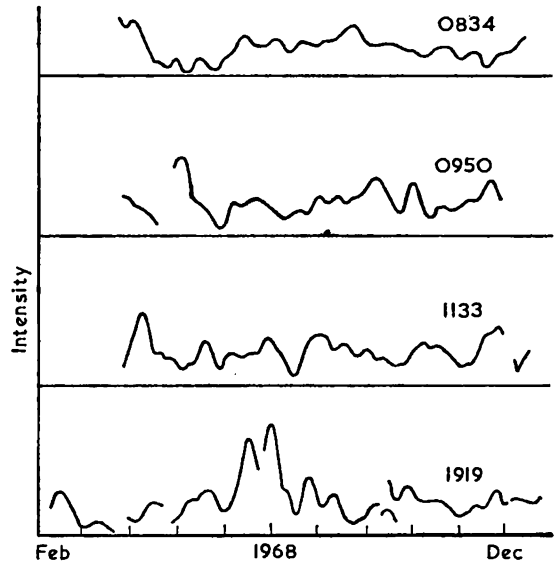

FIG. 70. Fluctuations in the radio-emission flux of some pulsars, smoothed over 8-day time intervals.

about 10^6 cm. The radio-emitting region may be large, but one can estimate an upper limit for its size from the pulse length, ($R \lesssim c\Delta t$). Apparently, the size of the radio-emitting regions and their distance from the pulsar surface are not larger than 10^7 cm (Drake and Craft, 1968). One can then obtain from the observed radio-flux around the frequency $6\times10^8\ s^{-1}$ the vast intensity $I_\omega \approx 10^2$ to 10^4 erg/cm^2 which corresponds to an effective temperature $T_{\text{eff}} \approx 10^{23}$ to 10^{25} °K. The total average radio-power of a pulsar is of the order of 10^{27} to 10^{28} erg/s.

THE CRAB PULSAR

So far only one pulsar has shown infra-red, optical, and X-ray emission. This is PSR 0531 in the Crab Nebula, which is the remnant of the supernova which flared up in 1054 AD. There is one object (Cen X3) which shows pulsar-like behaviour in its X-ray emission.

We show in Fig. 71 the average shape of the pulses from PSR 0531 at different wavelengths In general, they are similar, but there are also significant differences, in particular, the width of the fine structure at frequencies $\omega/2\pi \approx 430$ MHz and 196 MHz.

We show in Fig. 72 the spectrum of PSR 0531 together with the spectrum of the Crab Nebula. The spectral index of the radio-emission of the pulsar is $\alpha = 2{\cdot}9$, and in the X-ray band $\alpha = 1{\cdot}2$. It is important to note that in the radio-band the spectral index of the

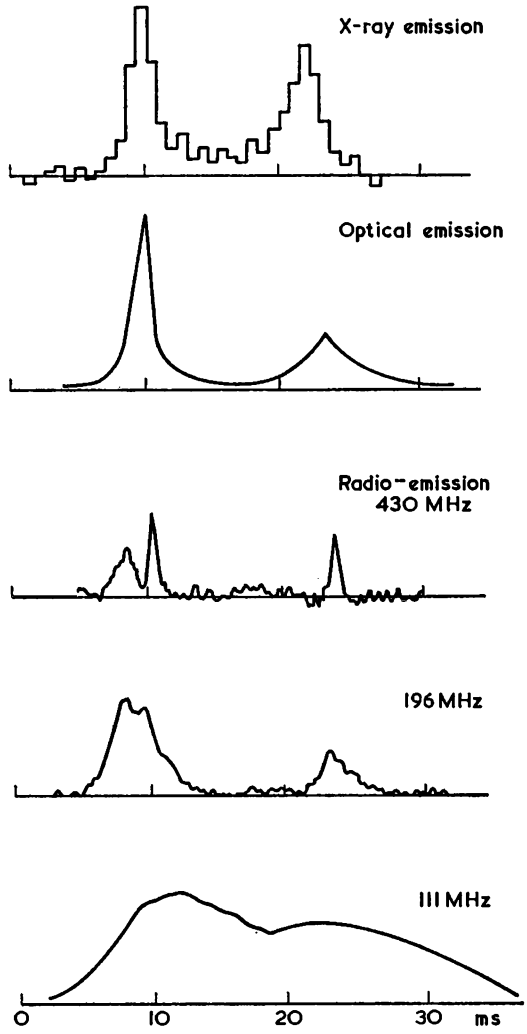

FIG. 71. Average pulse shape of the Crab pulsar (at different emission frequencies).

emission from the nebula is $\alpha = 0{\cdot}3$, while in the optical and X-ray bands $\alpha = 1$, that is, there are considerable differences between the indexes.

The main radio-pulse is 80% polarized, the interpulse has a weak, 11%, polarization. However, a small burst, a precursor of the main pulse which occurs only in the radio-band, is 100% polarized. We show in Fig. 73 the polarization of the optical emission of PSR 0531. The optical emission is on average 25% polarized and this value changes systematically through the pulse, reaching a clearly pronounced minimum close to the intensity maximum.

The spectral intensity of the emission of PSR 0531 in the radio-band is $I_\omega = 10^7$ erg/cm^2, corresponding to an effective temperature $T_{\text{eff}} \approx 10^{30}$ °K and a total luminosity $L \approx 10^{31}$ erg/s; in the optical band we have $I_\omega = 10^2$ erg/cm^2, $T_{\text{eff}} \approx 10^{11}$ °K, $L \approx 10^{33}$ erg/s; and in the X-ray band $I_\omega \approx 10$ erg/cm^2, $T_{\text{eff}} \approx 10^2$ °K, $L \approx 10^{35}$ erg/s.

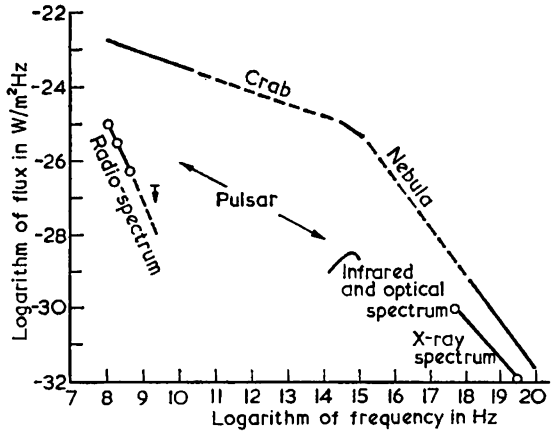

FIG. 72. Spectra of the Crab pulsar and the Crab Nebula.

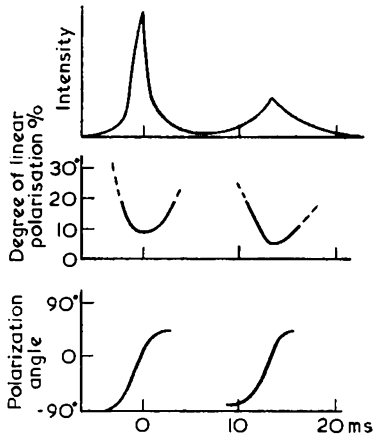

FIG. 73. Changes in intensity, degree of polarization, and polarization angle of the optical emission of the Crab pulsar.

Even before the discovery of a pulsar in the Crab Nebula a compact radio-source had been observed which emitted very strongly in the metre band; apparently, this is the same pulsar PSR 0531, the pulses of which are smeared out by interstellar scattering in the long-wavelength region of the radio-spectrum.

The properties of the Crab Nebula itself must, probably, be connected with the characteristics of the pulsar; for instance, one usually assumes that it supplies the relativistic electrons which guarantee the brightness of this nebula in the continuum. It is true, that the spectral characteristics of the radiation from the pulsar and from the nebula are different, but they can be interpreted taking into account the de-excitation of the particle energy. A more intimate connection of the magnetic fields of the pulsar and of the nebula is possible.

A characteristic feature in the structure of nebulae which are supernova shells, such as the Crab Nebula, consists in filaments consisting of gas with a relatively low temperature ($T_e \approx 10^4$ °K) and a high density ($n_e \approx 10^3$ cm^{-3}). The thickness of the filaments is of the

order of 3×10^{16} cm and is smaller than their lengths by several factors of ten. It is, however, possible that the filaments are regions of interstellar gas dragged along by the motion of the supernova shell which is left behind so that there is no direct connection with the pulsar.

One has also observed "wisps" in the Crab Nebula. These are elongated formations visible in the continuous optical spectrum. They move from the centre of the nebula, that is, probably from the pulsar, with a velocity of the order of $\frac{1}{10}$ of the velocity of light. "Wisps" appear with intervals of the order of several months. It is possible that the elongation of "wisps" is connected with the magnetic field; in that case it is important to note that they move predominantly at right angles to the field. The size of a "wisp" is of the order of 5×10^{16} cm and their volume emission coefficient is about 100 times larger than the average emission coefficient in the nebula.

One has also observed other formations on a larger scale ($\sim 10^{18}$ cm) which drift more slowly in the nebula, but also from the centre to the periphery. Apparently, the pulsar is the centre of the activity of the whole shell.

The example of the Crab Nebula shows clearly that pulsars may have been formed in supernova outbursts. It has been possible to identify a few more pulsars with nebulae which are remnants of supernova shells which are left behind. However, in other similar nebulae oo pulsars have been observed; this can possibly be explained by the narrow directional diagram of their emission. We note that there are in those cases centres of activity.

One can find more detailed data on pulsars and supernova shells in many review articles (Shklovskiĭ, 1968; Hewish, 1970; Ginzburg, 1971; Woltjer, 1971; Kardashev, 1970; Smith 1972). The most complete summary of data is contained in a review by ter Haar (1972).

§ 17. A relativistic plasma in a strong magnetic field

It follows from theoretical calculations of the formation and evolution of neutron stars that one may expect at their surface a strong dipole magnetic field with a field strength of up to 10^9 to 10^{12} Oe so that in the plasma surrounding the pulsar there may be a very strong magnetic field. Moreover, the radiation energy density of pulsars is so large that the plasma in the space near the pulsar will most probably be heated to ultra-relativistic energies—both by the radiation and by turbulent processes. It is thus impossible to study pulsar emission mechanisms without first analyzing the properties of an ultra-relativistic plasma in a very strong magnetic field. We shall now consider such a plasma, but first we shall say a few things about an ultra-relativistic plasma when there is no magnetic field present.

Dispersion relations for an ultra-relativistic plasma without a magnetic field

We shall consider very briefly the collective properties of an ultra-relativistic plasma in which the average energy of the various kinds of particles $\varepsilon_{\alpha*}$ may be different but always much larger than the corresponding values of $m_\alpha c^2$. Furthermore, collisions are very rare in a relativistic plasma so that we shall assume that the plasma is completely collisionless. The energy distribution function of the particles in such a plasma is non-Maxwellian and the distribution (12.2) is probably the best approximation. Moreover, electron-positron pairs can be formed in an ultra-relativistic plasma.

The problem of pair-formation in an ultra-relativistic plasma under cosmic conditions has often been discussed (see, for instance, Bisnovatyĭ-Kogan, Zel'dovich, and Syunyaev, 1971). However, pair-formation only appreciably affects the properties of a collisional plasma in those cases where the processes take place so slowly that particle collisions manage to produce a sufficient number of pairs. We shall consider here the opposite case of processes which take place with characteristic times which are much shorter than the time for pair production. It is true, that in a sufficiently dense plasma collisionless pair-production is also possible (Tsytovich, 1961b), but for this it is necessary that the plasma frequency, defined below, exceeds $2m_e c^2/\hbar \approx 2\times 10^{21}\,\mathrm{s}^{-1}$. In any case the presence of positrons in a plasma can easily be taken into account in the same way as we take into account the presence in the plasma of other particles with different signs of their charge.

The plasma frequency for each kind of particle is determined by the formula

$$\omega_{p\alpha}^2 = \frac{8\pi e^2 c^2}{3}\int \frac{f_\varepsilon^{(\alpha)}\, d\varepsilon}{\varepsilon} = \frac{4\pi(\gamma-1)}{3}\,\frac{e^2c^2 n_\alpha}{\varepsilon_{\alpha *}}, \tag{17.1}$$

where we have used equation (12.2) and understand by n_α the density of the particle considered.

The dispersion relation for transverse electromagnetic waves, that is, for $\omega-\omega_p \ll \omega_p$, has the following form (Silin and Rukhadze, 1961):

$$\omega^{\mathrm{p}}(k) = \sqrt{\left(\omega_p^2+\frac{6}{5}c^2k^2\right)} \approx \omega_p+\frac{3}{5}\,\frac{c^2k^2}{\omega_p}, \tag{17.2}$$

where $\omega_p^2 = \sum_\alpha \omega_{p\alpha}^2$, that is, we include electrons, positrons, and ions.

For high-frequency electromagnetic waves, when $\omega \gg \omega_p$, we have

$$\omega^{\mathrm{t}}(k) = \sqrt{(\omega_t^2+c^2k^2)} \approx ck, \tag{17.3}$$

where the frequency ω_t—which plays the role of a plasma frequency at high frequencies—does not differ significantly from the quantity ω_l to be introduced in a moment.

The dispersion relation for longitudinal plasmons can be written in the form

$$\omega^{\mathrm{l}}(k) = \omega_p \sqrt{\left(3\lambda^2\left[\frac{1}{2}\,\lambda \ln\frac{\lambda+1}{\lambda-1}-1\right]\right)}, \quad \lambda = \omega/ck. \tag{17.4}$$

Equation (17.4) can not be applied in the region where the light cone $\omega = ck$ intersects the dispersion curve. The applicability of (17.4) is limited by the condition

$$\lambda-1 \gg \left(\frac{m_\alpha c^2}{\varepsilon_{\alpha *}}\right)^2, \tag{17.5}$$

that is, to the super-light region. The intersections of the dispersion curve with the light cone—which we shall denote by ω_l—can be determined from the equation

$$\omega_l^2 = \sum_\alpha \omega_{l\alpha}^2, \tag{17.6}$$

where

$$\omega_{l\alpha} = \omega_{p\alpha} \sqrt{\left(3\left[\ln \frac{2\varepsilon_{\alpha *}}{m_\alpha c^2} - C + \frac{1}{2} - \frac{d \ln \Gamma(\gamma)}{d\gamma}\right]\right)}, \tag{17.7}$$

and where we have taken, as before, the distribution function (12.2) while $C = 0{\cdot}577\ldots$ is the Euler constant. For $\gamma = 3$ we have

$$\omega_{l\alpha} = \omega_{p\alpha} \sqrt{\left(3\left[\ln \frac{2\varepsilon_{\alpha *}}{m_\alpha c^2} - 1\right]\right)}. \tag{17.8}$$

This equation can be obtained formally from (17.4) by replacing $\lambda - 1$ by $\frac{1}{2}(m_\alpha c^2/\varepsilon_{\alpha *})^2$. If $\varepsilon_{e*} \approx 10^4 m_e c^2$, we have $\omega_{le} \approx 5\,\omega_{pe}$. If the ion energies are not less than the electron energies, while the positron density is much lower than the electron density, we have $\omega_l \approx \omega_{le}$.

As a result we get the following scheme for the dispersion curves (Fig. 74). At low wave-numbers two curves start from the point $\omega = \omega_p$, $k = 0$, one for the transverse waves (17.2)

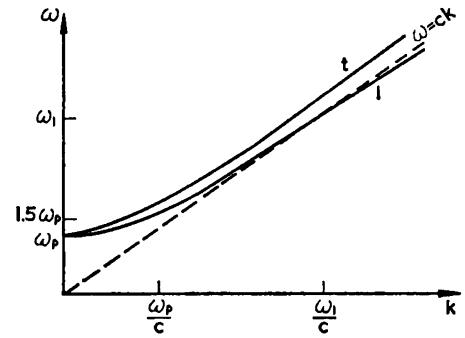

FIG. 74. Dispersion curves for transverse and longitudinal waves in a relativistic plasma without a magnetic field.

and a second one for the longitudinal waves (17.4) which for small k (large λ) has the following form:

$$\omega^l(k) = \sqrt{\left(\omega_p^2 + \frac{3}{5} c^2 k^2\right)} \approx \omega_p + \frac{3}{10} \frac{c^2 k^2}{\omega_p}. \tag{17.9}$$

The curve for the transverse waves stays all the time above the light cone, that is, their phase velocity is always larger than c. For the longitudinal waves, the phase velocity for $\omega < \omega_l$ is also larger than c, but for $\omega > \omega_l$ it becomes less than c.

It is clear that Landau damping will occur near $\omega = \omega^l$ as the particle velocities are also close to c. We shall assume that it is effective for plasma waves with a phase velocity corresponding to the velocities of electrons with energies of, say, $3\varepsilon_{e*}$. The quantity λ_d for waves with Landau damping is then

$$\lambda_d = \frac{v_p}{c} = 1 - \frac{1}{2}\left(\frac{m_e c^2}{3\varepsilon_{e*}}\right)^2. \tag{17.10}$$

Substituting (17.10) into (17.4) we get an estimate for the maximum frequency of the plasma waves:

$$\omega_d^2 \approx 3\omega_p^2\left[\ln\frac{3\varepsilon_{e*}}{m_e c^2}-1\right] \approx 1{\cdot}1\omega_l^2, \tag{17.11}$$

for $\varepsilon_{e*} \approx 10^4 m_e c^2$. It is clear that in the small frequency interval $\omega_d-\omega_l \approx 0{\cdot}04\,\omega_l \approx 0{\cdot}2\,\omega_p$ plasma waves can be excited in an ultra-relativistic plasma through a beam instability. The value $\varepsilon_{e*} \approx 10^4 m_e c^2$ is taken as an example as in a number of attempts to interpret pulsar emission one has obtained this value for ε_{e*} or one close to it.

Non-linear interactions in an ultra-relativistic plasma without a magnetic field

Transfer of energy either along the dispersion curves of one mode or from transverse to longitudinal waves and the other way round is produced by the normal Compton scattering by the ultra-relativistic electrons in the plasma itself. The formulae of § 13 derived for the case of scattering of waves with phase velocities $v_p > c$ are valid also in the present case. We must replace ω_{pe} by ω_p, defined in the previous subsection.

Two kinds of non-linear interactions are important here. First of all, energy transfer takes place at frequencies of the order of ω_p both between the l and t modes, and also within each mode, and secondly, energy transfer occurs from the frequency region near ω_p to higher frequencies of the transverse electromagnetic waves ($\omega \gg \omega_p$).

The energy transfer near the frequency ω_p is here also produced by Compton scattering in contrast to the case of a non-relativistic plasma where due to the mutual cancellation of the Compton and the non-linear scattering by electrons such a transfer was connected with the non-linear scattering by the polarization "coats" of the ions. Using equation (13.25) and the new definition (17.1) of the frequency of the plasma oscillations we obtain an estimate for the growth rate for the transfer of energy at frequencies of the order of ω_p between different modes and within one mode:

$$\frac{\gamma^{p,l}}{\omega_p} \approx \frac{W^{p,l}}{n_*\varepsilon_*}, \tag{17.12}$$

where $n_*\varepsilon_*$ is essentially the thermal energy of the ultra-relativistic plasma $\left(n_*\varepsilon_* = \sum_\alpha n_\alpha \varepsilon_{\alpha*}\right)$.

The energy transfer is here also accompanied by a heating of the particles which in this case is significantly larger as in the transfer process the frequency may change by an amount comparable with ω_p. The characteristic rate for the heating of the plasma is:

$$\frac{\gamma_*}{\omega_p} \approx \frac{c^2 D_{\parallel}^{l\,p}}{\omega_p \varepsilon_*^2} \approx \left(\frac{W^{l,p}}{n_*\varepsilon_*}\right)^2. \tag{17.13}$$

The growth and decay rates for energy transfer in the high-frequency region of the electromagnetic mode are also determined by equations (13.25) and (13.26) when we take the new definition of the plasma frequency into account. For the frequency range $\omega < \omega_p(\varepsilon_*/m_e c^2)^2$ we have

$$\frac{\gamma^t(t+e \rightleftharpoons t'+e')}{\omega_p} \approx \frac{\gamma^l(l+e \rightleftharpoons l'+e')}{\omega_p} \approx \pm\frac{\omega_p}{\omega}\frac{W^{l,t}}{n_*\varepsilon_*}, \tag{17.14}$$

and for the higher frequencies $\omega > \omega_p(\varepsilon_*/m_ec^2)^2$ we have

$$\frac{\gamma^t(t+e \rightleftharpoons t'+e')}{\omega_p} \approx \frac{\gamma^l(l+e \rightleftharpoons l'+e')}{\omega_p} \approx \pm\left(\frac{\omega_p}{\omega}\right)^{(\gamma+4)/2}\left(\frac{\varepsilon_*}{m_ec^2}\right)^{\gamma+2}\frac{W^{l,t}}{n_*\varepsilon_*}. \quad (17.15)$$

As in the case of a non-relativistic plasma, the stimulated energy transfer involving an increase in frequency corresponds to absorption if it is connected with scattering by isotropically distributed particles (equations (17.14) and (17.15) then determine the decay rates) or an amplification if the scattering occurs by anisotropically distributed particles. In that case equations (17.14) and (17.15) determine the appropriate growth rates.

We must interpret, in equations (17.14) and (17.15) $W^{l,t}$ as the total energy of all plasmons, both longitudinal and transverse, with frequencies $\omega \approx \omega_p$.

At high frequencies $\omega \gg \omega_p$ decay interactions such as $t \rightarrow t'+l$ also occur. The corresponding growth rate is

$$\frac{\gamma^t(t \rightleftharpoons t'+l)}{\omega_p} \approx \left(\frac{\omega_p}{\omega}\right)^3\frac{W^t}{n_*\varepsilon_*} \approx \left(\frac{\omega_p}{\omega}\right)^2\frac{\omega_p W^t_\omega}{n_*\varepsilon_*}. \quad (17.16)$$

A comparison of the different growth rates determines the energy level of plasma and electromagnetic waves established as a result of non-linear interactions. For instance, if the excitation of electromagnetic radiation is determined by the Compton scattering by a beam of ultra-relativistic electrons and the loss of energy by the decay interactions, we get from comparing (17. 15) and (17.16)

$$W^t_\omega \approx \frac{W^{l,t}}{\omega_p}\left(\frac{\omega_p}{\omega}\right)^{\gamma/2}\left(\frac{\varepsilon_*}{m_ec^2}\right)^{\gamma+2}. \quad (17.17)$$

The energy level $W^{l,t}$ is determined by the conditions for the excitation of plasma turbulence.

Dispersion relations for an ultra-relativistic plasma in a very strong magnetic field

This case is the most important one for the interpretation of pulsar phenomena. For the sake of simplicity we restrict the problem through several assumptions justified by the physics of pulsar phenomena, and by the conditions for the applicability of the theory to explain these phenomena.

We shall assume that the gyro-frequency which depends for relativistic particles only on the energy and not on the mass is much higher than the plasma frequency. Under those conditions the difference in mass of electrons and ions is not very important but differences in their average energies influence the dispersion relations. We note that the results of the preceding subsection refer to the case $\omega_p \gg eHc/\varepsilon_*$ while now we are interested in the case where the inequality is the other way round.

In a very strong magnetic field the motion of a particle is practically strictly parallel or antiparallel to the magnetic field lines. This feature can be explained in many ways. If an electron possesses a noticeable momentum component $p_\perp$ at right angles to the field synchrotron emission will be large and this will rapidly make that component smaller.

Going from the regions with a very strong field (near the pulsar) to a region of a less strong field any charged particles will undergo a decrease in $p_\perp$ by virtue of the conservation of the adiabatic invariants.

The directionality of the pulsar emission showing itself in its pulsed nature also indicates that the particles move in a narrow solid angle which is, apparently, determined by the geometry of the magnetic field. The independence of the pulse width of frequency shows that the distribution of the particles over the angles θ between their velocity vector and the direction of the magnetic field is, apparently, independent of their energy, at any rate for large energies. This means that the particle velocity distribution function can be written as the product of two functions, one of which, f_ε, depends only on the particle energy and the other, which we shall denote by f_θ, depending only on the angle θ. For the first one we shall take the approximation:

$$f_\varepsilon = \frac{(\gamma-1)\varepsilon_*^{\gamma-1}}{(\varepsilon+\varepsilon_*)^\gamma} n_* . \tag{17.18}$$

The angular distribution of the particle velocities we need only in the next section. If it is sufficiently narrow, that is, if all $\theta \ll 1$, we can to a first approximation assume that the particle motion is one-dimensional, directed strictly along the magnetic field lines. The function (17.18) was written down in a system of coordinates in which the plasma as a whole is at rest; in that case the motions parallel and antiparallel to the vector $\boldsymbol{H}$ are equally probable.

The one-dimensional nature of the distribution function somewhat changes the numerical factors in the definition of the characteristic plasma frequencies. The main plasma frequency is now equal to

$$\omega_p^2 = \sum_\alpha \omega_{p\alpha}^2 = \sum_\alpha 4\pi e^2 \int_{m_\alpha c^2}^\infty \frac{\partial}{\partial p_\alpha}(v_\alpha f_\varepsilon^\alpha)\, d\varepsilon = \sum_\alpha 2\pi(\gamma-1)\frac{e^2c^2 n_\alpha}{\varepsilon_{\alpha *}}, \tag{17.19}$$

where α indicates the kind of particle and $\varepsilon_{\alpha *}$ the average energy of particles of that kind. The main plasma frequency is determined by the particles with the lowest energy (in a non-relativistic plasma by the particles with the smallest mass).

We determine the gyro-frequency $\omega_{H\alpha}$ in terms of the Alfvén velocity* v_A which in an ultra-relativistic plasma with a one-dimensional distribution function is given by the equation

$$\frac{v_A^2}{c^2} = \frac{(\gamma-2)H^2}{8\pi n_* \varepsilon_*} = \frac{1}{\sum_\alpha \omega_{p\alpha}^2/\omega_{H\alpha}^2}, \tag{17.20}$$

whence we have for the gyro-frequency of a particle of the kind α

$$\omega_{H\alpha} = \sqrt{\left[\frac{1}{4}(\gamma-1)(\gamma-2)\right]}\frac{eHc}{\varepsilon_{\alpha *}}. \tag{17.21}$$

* In a weak magnetic field $v_A = H/\sqrt{(4\pi n_i m_i)}$. Usually this quantity is much less than the velocity of light. In strong fields $v_A > c$, but the true velocity of propagation of Alfvén waves is $v_A c/\sqrt{(v_A^2+c^2)}$ which is always smaller than c. Bearing this in mind, we retain for v_A the name Alfvén velocity, taking into account that it is not equal to the velocity of the Alfvén waves.

We must substitute in (17.20) and (17.21) $[(\gamma-1)\ln(\varepsilon_{max}/\varepsilon_*)]^{-1}$ instead of $\gamma-2$ in the case when $\gamma = 2$. We shall assume in what follows that $v_A \gg c$.

One can evaluate the dispersion relations in such a plasma by standard methods. There are three branches in the low-frequency region where $\omega \ll \omega_{H\alpha}$. One of them corresponds at low frequencies to longitudinal waves, that is, it is analogous to the Langmuir branch. The two others correspond to Alfvén and fast magneto-sound waves. We shall retain for all these waves the same indices as they had in the case of a non-relativistic plasma.

We shall take into account the possibility of the propagation of waves at large angle ϑ to the magnetic field. The light cone corresponds to the condition $\omega/k_z = \omega/k\cos\vartheta = c$ because, according to our assumptions, all particles move strictly along the field lines.

For propagation strictly along the magnetic field (or rather at an angle $\vartheta \ll m_e c^2/\varepsilon_*$ to the magnetic field lines) we have for the longitudinal "Langmuir" wave branch the relation:

$$\omega^l(k) = \begin{cases} \omega_p + 3k^2c^2/8\omega_p, & kc \ll \omega_p; \\ kc + \omega_p^2/kc, & kc \gg \omega_p. \end{cases} \tag{17.22}$$

It is important that in contrast to a non-relativistic plasma, it is here also possible that $\omega = ck \gg \omega_p$. Equation (17.22) is valid up to the intersection of this branch with the light cone $\omega = ck_z$ at a frequency $\omega_l^2 = \sum_\alpha \omega_{l\alpha}^2$ or, in the case when $\omega_{pe} = \omega_{li}$ as $\omega_l \approx \omega_{le}$, where

$$\omega_l^2 = \frac{8\pi e^2 c^2 n_*}{(\gamma-2)\varepsilon_*}\left(\frac{\varepsilon_*}{m_e c^2}\right)^2 \approx \frac{4\omega_p^2}{(\gamma-1)(\gamma-2)}\left(\frac{\varepsilon_*}{m_e c^2}\right)^2. \tag{17.23}$$

At frequencies above ω_l there is for longitudinal waves with $\vartheta \ll m_e c^2/\varepsilon_{e*}$ very strong Landau damping (Fig. 75). The magnitude of the damping rate depends strongly on the choice of the maximum energy $\varepsilon_{max,\alpha}$ for which equation (17.18) is still applicable, and also on the value of the parameter γ. In particular, for the case when $2 < \gamma < 4$ we have for the Landau damping rate:

$$\frac{\gamma^l}{\omega^l} \approx -\sum_\alpha \frac{\pi\gamma(4-\gamma)}{16}\left(\frac{m_\alpha c^2}{\varepsilon_{max,\alpha}}\right)^{4-\gamma}\left(1-\frac{\omega^2}{c^2k_z^2}\right)^{(\gamma-2)/2}. \tag{17.24}$$

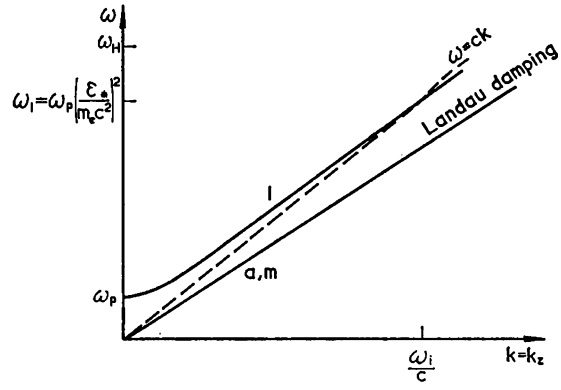

FIG. 75. Dispersion curves for low-frequency ($\omega \ll \omega_H$) waves in a very strong magnetic field. The case of propagation along the magnetic field ($\vartheta \ll m_e c^2/\varepsilon_*$).

When $\gamma > 4$ we must replace in this equation $\varepsilon_{\max,\alpha}$ by $\varepsilon_{\alpha *}$ and also change the numerical factor.

Thus, in contrast to a non-relativistic plasma where longitudinal waves occupy a narrow frequency range of width $\sim 0{\cdot}2\omega_{pe}$ or an ultra-relativistic plasma without a magnetic field where the frequency interval for the longitudinal waves is of the order of $5\omega_p$, here, in a plasma with a magnetic field, the longitudinal waves occupy a wide range of frequencies of extent $\sim \omega_p \varepsilon_*/m_e c^2$.

There is, however, something else that is important. The plasma wave branch considered is longitudinal only when the propagation occurs at an angle $\vartheta \ll m_e c^2/\varepsilon_*$. If we have $\vartheta \gg m_e c^2/\varepsilon_*$, such a wave turns out to be longitudinal only in the immediate vicinity of ω_p, and for $\omega \gg \omega_p$ it changes into the ordinary transverse electromagnetic wave with $\omega = ck$ and nowhere intersects the light cone. Apparently, in actual conditions longitudinal plasma waves are concentrated only close to the frequency ω_p. The quantity W^l, the longitudinal plasmon energy density, will refer only to those waves.

If the average energies of the different kinds of particles are very different, a new type of waves occurs here which is analogous to ion-sound. Let us consider the case when $\varepsilon_{i*} \ll \varepsilon_{e*}$ when, according to equation (17.19), $\omega_{pi} \gg \omega_{pe}$. The frequency, analogous to the ion-sound waves, is less than ω_{pe}. The dispersion relation for such waves has the simple form:

$$\frac{2\omega_{pe}^2}{c^2 k_z^2 - \omega^2} - \frac{\omega_{pi}^2}{\omega^2} = 0. \tag{17.25}$$

Hence it follows that

$$\omega \approx ck_z \left(1 - \frac{\omega_{pe}^2}{\omega_{pi}^2}\right). \tag{17.26}$$

In order that these waves are not subject to strong Landau damping, the following inequality must be satisfied:

$$\left(\frac{m_e c^2}{\varepsilon_{e*}}\right)^2 \ll \frac{\varepsilon_{i*}}{\varepsilon_{e*}} \ll \left(\frac{m_i c^2}{\varepsilon_{i*}}\right)^2. \tag{17.27}$$

In an ultra-relativistic plasma, the difference between ε_{i*} and ε_{e*} must be very large in order that longitudinal sound waves can occur.

Waves analogous to magneto-sound or Alfvén waves can propagate in a plasma in a strong magnetic field, independent of the ratio of the average energies of different particles. If their propagation is strictly longitudinal along the magnetic field ($\vartheta \ll m_e c^2/\varepsilon_{e*}$) their dispersion relations are the same and of the form:

$$\omega = \frac{kv_A}{\sqrt{(1 + v_A^2/c^2)}} \approx ck\left(1 - \frac{c^2}{2v_A^2}\right), \tag{17.28}$$

when $v_A \gg c$. The phase velocities of these waves are smaller than the velocity of light, but the electrical fields of the plasma pulsations are perpendicular to the direction of the magnetic field and to the particle motions and we have therefore also here no Landau damping nor a build-up when the particle distribution function is inverted.

Let us now turn to the case of the propagation of waves at large angles, $\vartheta \gg m_e c^2/\varepsilon_{e*}$, to the direction of the external magnetic field. We shall, as before, still assume that the motion of the particles is strictly one-dimensional along the external magnetic field.

The longitudinal "Langmuir" waves now have frequencies close to ω_p that is, wavenumbers $k \ll \omega_p/c$. Instead of the first of equations (17.22) we have

$$\omega \approx \omega_p + \frac{2+\cos^2\vartheta}{8\omega_p} c^2k^2. \tag{17.29}$$

Waves with large values of k become transverse with a rather complicated dispersion relation:

$$\omega^2 = (2+\tan 2\vartheta \ln \sin^2\vartheta)\omega_p^2 + c^2k^2 - \frac{c^2}{v_A^2} \frac{(2+\sin^2\vartheta)\cos^2\vartheta \tan 2\vartheta}{\tan 2\vartheta + [2\omega_p^2/c^2k^2 \cos^2\vartheta]} c^2k^2. \tag{17.30}$$

To a first approximation the second term is here the largest, that is, $\omega \approx ck$ – the usual dispersion relation for a transverse wave. The first term in (17.30) takes into account the effect of the plasma frequency. It is always positive and changes from $2\omega_p^2$ to ω_p^2 when the angle ϑ changes from 0 to $\pi/2$. The third term which takes into account a correction of the order of the small quantity c^2/v_A^2 is negative and can therefore, at least in principle, reduce the phase velocity of the waves considered to below that of the velocity of light, that is, it can lead to Landau damping. A simple analysis shows, however, that such a reduction of the phase velocity to values below c is possible only in narrow intervals for the angle ϑ and the frequency ω:

$$\frac{c}{v} \gg \vartheta \gg \frac{\omega_p}{\omega}; \quad \frac{\omega_p \varepsilon_{e*}}{m_e c^2} \gg \omega \gg \frac{v_A}{c}\omega_p. \tag{17.31}$$

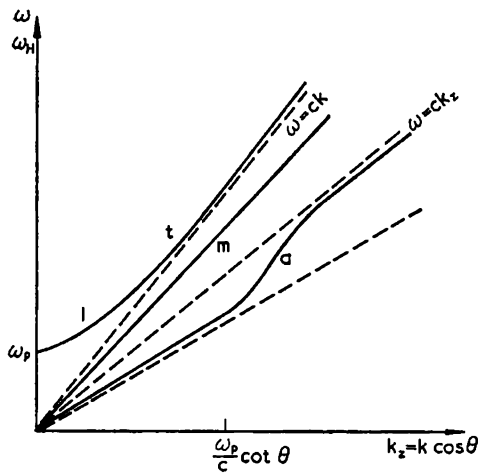

FIG. 76. The same as for Fig. 75, but for the case of propagation of the waves at an angle to the magnetic field ($\vartheta_A \gg m_e c^2/\varepsilon_*$). *l*: the part of the dispersion curve where the oscillations are longitudinal; *t*: where they are transverse; *m* and *a* where they are analogous to magneto-sound and to Alfvén waves.

Magneto-sound waves with $\vartheta > m_e c^2/\varepsilon_{e*}$ do not suffer Landau damping, as before, and their dispersion relation is

$$\omega^{m}(\boldsymbol{k}) \approx ck\left[1-\frac{c^2}{v_A^2}\,\frac{1+\cos^2\vartheta}{4}\right]. \tag{17.32}$$

The fact is that the particles do not here have velocity components at right angles to the magnetic field—in contrast to the case of a non-relativistic plasma with an isotropic particle distribution function, where there is therefore Landau damping for the magneto-sound waves. It is true that in the case considered there is a small component of the electric field of the wave in the direction of the magnetic field which therefore leads to Landau damping, but its relative magnitude is of the order ω^2/ω_H^2 and the damping is therefore very small at low frequencies.

The dispersion relation for the analogue of the Alfvén waves, propagating at an arbitrary angle ϑ is:

$$\begin{aligned}\omega^{a}(\boldsymbol{k}) &\approx c|k_z|\left[1-\frac{c^2}{v_A^2}\,\frac{\omega_p^2}{2\omega_p^2+c^2k_\perp^2}\right]\\ &\approx ck|\cos\vartheta|\left[1-\frac{c^2}{v_A^2}\,\frac{\omega_p^2}{2\omega_p^2+c^2k^2\sin^2\vartheta}\right].\end{aligned} \tag{17.33}$$

In contrast to the magneto-sound waves we have here a much larger component of the electric field of the wave along the external magnetic field. This leads to the appearance here of the plasma frequency ω_p and to an appreciable Landau damping for the Alfvén waves in the case considered.

In the range of small wave numbers the dispersion curve for the Alfvén waves has the usual form: $\omega = c\,|\,k_z\,|\,(1-c^2/2v_A^2)$, and for large wavenumbers $\omega = c\,|\,k_z\,|$ (Fig. 76). However, near the frequency

$$\omega_A \approx \omega_p \cot\vartheta \tag{17.34}$$

there is a bend in the dispersion curve and there the Landau damping has a maximum. The evaluation of the Landau damping rate in a general form (Tsytovich and Kaplan, 1972b) leads to the expression

$$\frac{\gamma_k^a}{\omega^a(\boldsymbol{k})} = \sum_\alpha \frac{16\pi^2e^2m_\alpha\omega_p^2c^6k_\perp^2}{v_A^4(2\omega_p^2+c^2k_\perp^2)(1-\omega^2/c^2k_z^2)^{3/2}}\left[\frac{\partial}{\partial\varepsilon}(v_\alpha f_\varepsilon^\alpha)\right]_{\varepsilon=\varepsilon_{res}}, \tag{17.35}$$

where we must substitute for the particle energy the energy ε_{res} given by the relation

$$\varepsilon_{res} = m_\alpha c^2\left[1-\frac{\omega^2(\boldsymbol{k})}{c^2k_z^2}\right]^{-1/2} \approx m_\alpha c v_\alpha\left[\frac{2\omega_p^2+c^2k_\perp^2}{4\omega_p^2}\right]^{1/2}. \tag{17.36}$$

At the bend frequency (17.34), where the Landau damping is at its largest, we have for the case of the distribution function (17.18)

$$\frac{\gamma_k^a}{\omega_A} \approx -\pi\gamma\,\frac{c}{v_A}\,\frac{\omega_p c^2k_\perp^2}{(2\omega_p^2+c^2k_\perp^2)^{3/2}}\sum_\alpha\frac{m_\alpha c^2}{\varepsilon_{\alpha*}}. \tag{17.37}$$

When $\omega \ll \omega_A$ the damping rate decreases as ω^2 when the frequency is lowered. At high frequencies ($\omega \gg \omega_A$) equation (17.35) leads to a constant value of the damping, but, in fact, the applicability of (17.35) is restricted by the condition

$$\frac{\omega}{\omega_A} \leqslant \frac{c^2}{v_A^2}\left(\frac{\varepsilon_{e*}}{m_e c^2}\right)^2. \tag{17.38}$$

At high frequencies the damping rate γ^a again decreases.

One can similarly consider ion-sound waves propagating at an angle in an ultra-relativistic plasma with a strong magnetic field (see Tsytovich and Kaplan, 1972b). There appear here two altered magneto-sound waves, a "fast" and a "slow" one.

Instability of Alfvén waves in an ultra-relativistic plasma in a strong magnetic field

The expression we gave a moment ago for the Landau damping is correct when the particle energy distribution function has no inversion, that is, when everywhere the condition $\partial f_\varepsilon^\alpha/\partial\varepsilon < 0$ is satisfied. In the opposite case there occurs now a build-up of Alfvén waves through Cherenkov emission. The growth rate of such a build-up can be evaluated using the following expression for the probability of the Cherenkov emission:

$$u_k^a = 4\pi^2 \frac{c^4}{v_A^4} \frac{8\omega_p^2 c^4 k_\perp^2 k_z^2}{(2\omega_p^2 + k_\perp^2 c^2)^3} \frac{e^2 v^2}{\omega^a} \delta(\omega^a - k_z v_z), \tag{17.39}$$

where v_z is the component of the velocity of the emitting particles in the direction of $\boldsymbol{H}$ (to be general, we have treated the case when the particle may move also at an angle to the magnetic field). We obtained expression (17.39) through the usual method of calculating the emission coefficient, taking into account that the unit polarization vector of an Alfvén wave in an ultra-relativistic plasma in a strong magnetic field is given by the relation

$$\boldsymbol{e}_k \approx \left\{1,\ 0,\ -\frac{c^2}{v_A^2} \frac{c^2 k_\perp k_z}{2\omega_p^2 + c^2 k_\perp^2}\right\}. \tag{17.40}$$

In particular, the Landau damping rate (17.35) for Alfvén waves can be evaluated using equation (17.39) together with its direct definition from the dielectric tensor components.

In the complex non-stationary conditions of the circumpulsar plasma there may occur various particle distribution functions, among which possibly also inverted ones, that is, ones for which $\partial f_\varepsilon^\alpha/\partial\varepsilon > 0$. For a given distribution function of this kind we can always evaluate the growth rate:

$$\gamma_k^a = \frac{1}{2} \sum_\alpha \int u_k^a \frac{\partial}{\partial\varepsilon} (v_A f_\varepsilon^\alpha)\, dp_\alpha. \tag{17.41}$$

To estimate the instability when there is an inverted distribution function we can use the same equation (17.37), changing its sign and dropping numerical factors of order unity:

$$\frac{\gamma^a}{\omega_A} \approx \frac{c}{v_A}\left(\frac{ck_\perp}{\omega_p}\right)^2 \sum_\alpha \frac{m_\alpha c^2}{\varepsilon_{\alpha *}}. \tag{17.42}$$

The Alfvén waves, excited by this instability, are directed at a finite angle ϑ to the magnetic field and have frequencies in the range $\omega_A \lesssim \omega \lesssim \omega_A \sum_\alpha \varepsilon_{\alpha *}/m_\alpha c v_A$. In other words, they occupy a wide angular spectrum and occur, in particular, with frequencies of the order ω_p. The growth rate (17.42) is rather large and the inversion of the distribution function will therefore under actual conditions disappear rapidly due to quasi-linear relaxation and the formation of a plateau. We shall below estimate the role of the quasi-linear relaxation.

The inversion of particle distribution functions leads also to the build-up of the analogue of ion-sound waves, with a growth rate

$$\frac{\gamma_k^s}{\omega_p} \approx \left(\frac{m_i c^2}{\varepsilon_{i*}}\right)^2 \left(\frac{\varepsilon_{i*}}{\varepsilon_{e*}}\right)^{(\gamma-2)/2}. \tag{17.43}$$

The instability of the longitudinal "Langmuir" waves is probably insignificant, as in that case only waves moving strictly along the magnetic field are excited, that is, waves moving within a very narrow solid angle and corresponding to a very narrow frequency range.

Of most importance is therefore the Cherenkov instability for Alfvén waves, and this will just determine the quasi-linear relaxation of the population inversion.

The instability of an ultra-relativistic plasma in a strong magnetic field can also be connected with cyclotron emission of Alfvén and other plasma waves. As now the condition

$$\omega - k_z v_z = \pm \frac{eHc}{\varepsilon} \tag{17.44}$$

must be satisfied, it is clear that such an instability occurs when there are in the plasma particles with energies $\varepsilon \gg \varepsilon_*$, as always $\omega - k_z v_z < \omega < \omega_H = eHc/\varepsilon_*$.

For instance, cyclotron generation of longitudinal waves with $k \ll \omega_p/c$ is possible only for particles with energies

$$\varepsilon > \frac{eHc}{\omega_p} \approx \varepsilon_* \frac{v_A}{c}. \tag{17.45}$$

For cyclotron generation of Alfvén waves one needs particles with energies

$$\varepsilon > \frac{eHc}{\omega(1-\omega/ck_z)} > \varepsilon_* \frac{v_A^2}{c^2}. \tag{17.46}$$

The growth rate of the cyclotron instability is determined by both the longitudinal and the transverse derivatives of the particle distribution function, that is, it is proportional to the

quantity

$$k_z \frac{\partial f}{\partial p_z} \pm \frac{\omega_H}{v_\perp} \frac{\partial f}{\partial p_\perp} = \omega \frac{\partial f}{\partial \varepsilon} \pm \frac{\omega_H}{\varepsilon} \frac{\partial f}{\partial \theta^2} - \frac{2\omega}{\varepsilon} \theta^2 \frac{\partial f}{\partial \theta^2}, \tag{17.47}$$

where $f = f_\varepsilon f_\theta$ is the complete particle momentum distribution function. The signs in (17.47) correspond to the signs in equation (17.44).

Waves with a phase velocity larger than the velocity of light can be built up when we have the plus-sign in (17.47) when simultaneously the two conditions $\theta^2 \ll 1$, $\partial f/\partial\theta^2 > 0$ (inverted population with respect to the angular distribution) are satisfied. This case is, apparently, not very probable.

Alfvén and magneto-sound waves can be built up also in the case of a normal, say, Gaussian distribution with respect to the angular variables, that is, when $\partial f/\partial\theta^2 < 0$. It is true that the energy of such particles must be large:

$$\varepsilon > \varepsilon_* \left(\frac{v_A}{c}\right)^2 \frac{\omega_H}{\omega}. \tag{17.48}$$

The cyclotron emission probability for an Alfvén wave for the case when $\omega - k_z v_z = -eHc/\varepsilon$ is:

$$u_k^a = \frac{\pi e^2 v_\perp^2}{\omega^a(k)(1 + c^2 k_\perp^2/2\omega_p^2)} \delta\left(-\omega \frac{c^2}{v_A^2} \frac{\omega_p^2}{2\omega_p^2 + c^2 k_\perp^2} + \frac{eHc}{\varepsilon}\right), \tag{17.49}$$

while the cyclotron emission probability for a magneto-sound wave is

$$u_k^m = \frac{\pi^2 e^2 v_\perp^2}{\omega^m(k)} \delta\left(-\omega_k^m \frac{c^2}{v_A^2} \frac{1 + \cos^2\vartheta}{4} + \frac{eHc}{\varepsilon}\right). \tag{17.50}$$

To evaluate the growth rate we can use for f_ε equation (17.18), while we assume that the function f_θ is Gaussian with a dispersion $\overline{\theta^2}$ which drops out from the final expression for the growth rate.

As a result of straightforward calculations we obtain the following expression for the growth rate of Alfvén waves when there is cyclotron emission from the "tail" of the power-law particle distribution function:

$$\frac{\gamma^\alpha}{\omega^a(k)} = \frac{2\pi}{\gamma - 1} \left(\frac{\omega_p}{\omega_H}\right)^2 \frac{\omega_p^2}{\omega_p^2 + c^2 k_\perp^2} \left(\frac{\omega_p^2}{2\omega_p^2 + c^2 k_\perp^2}\right)^\gamma \left(\frac{\omega^a(k)}{\omega_H}\right)^{\gamma - 2} \left(\frac{c}{v_A}\right)^{2\gamma}. \tag{17.51}$$

The build up of Alfvén waves with frequencies of the order of ω_A have the largest growth rate. In that case

$$\frac{\gamma_{max}^a}{\omega_A} \approx \left(\frac{c}{v_A}\right)^{3\gamma}. \tag{17.52}$$

We can similarly determine the growth rate for the cyclotron excitation of magneto-sound waves by the "tail" of the particle distribution:

$$\frac{\gamma_k^m}{\omega^m(k)} = \frac{\pi}{\gamma - 1} \left(\frac{\omega_p}{\omega_H}\right)^2 \left(\frac{\omega^m(k)}{\omega_H}\right)^{\gamma - 2} \frac{(1 + \cos^2\vartheta)^{2\gamma}}{4^\gamma} \left(\frac{c}{v_A}\right)^{2\gamma}. \tag{17.53}$$

The maximum growth rate is reached for waves with a very high frequency, $\omega^{m}(\boldsymbol{k}) \sim \omega_H$, that is, close to the upper limit of the frequency range. The corresponding maximum magnitude of the growth rate is

$$\frac{\gamma^{m}_{max}}{\omega_H} \approx \left(\frac{c}{v_A}\right)^{2(\gamma-1)}. \tag{17.54}$$

It is here very important that although in the cyclotron instability considered only particles with a high energy take part and that here only a restricted range of frequencies of the waves is excited, a similar instability occurs when the particle distribution function decreases when the energy and angles increase, that is, for the most probable form of this function for the case of an ultra-relativistic plasma in a strong magnetic field. Quasi-linear relaxation tends here to smooth out the distribution function and, in particular, tends to make it isotropic. However, radiative compression and the motion of particles in a field with decreasing intensity again establish anisotropy and thereby stabilize the instability.

The cyclotron instability, especially for Alfvén waves has thus in an ultra-relativistic plasma in a strong magnetic field a universal character (in contrast to the Cherenkov instability due to the inversion of the function f_ε), and is practically always stabilized and must thus lead to the appearance of strong Alfvén turbulence in such a plasma.

Non-linear wave interactions in an ultra-relativistic plasma in a strong magnetic field

As in the case of an ultra-relativistic plasma without a magnetic field, the non-linear interactions are essentially Compton scattering processes by the electrons in the plasma. The magnetic field, however, considerably decreases the efficiency of energy transfer in these interactions. Moreover, we must here take into account the magnetization and also the one-dimensionality of the particle motion.

The magnetization of the motion leads to the fact that only electrons with the smallest energies will play an essential part in the non-linear transfer. This means that the plasma wave energy goes from one branch to another with a relatively small change in frequency. The most important of these processes is the energy transfer from the Alfvén branch, where it is stored because of the instability, to longitudinal plasmons with frequencies $\omega \gtrsim \omega_p$, as in the given case ω_A and ω_p are comparable. The growth rate for such a transfer through Compton scattering is, however, strongly diminished as compared to (17.12) because the small components of the electric field of the wave at right angles to H are responsible for this transfer. We have

$$\frac{\gamma^{l}(a+e \rightleftharpoons l+e')}{\omega_p} \approx \left(\frac{c}{v_A}\right)^{4} \frac{W^{l}}{n_* \varepsilon_*}. \tag{17.55}$$

Comparing the growth rates (17.52) and (17.55) we can estimate the energy density of Langmuir and Alfvén waves which is established in an ultra-relativistic plasma as a result of its being made turbulent through the cyclotron excitation by an anisotropic distribution. We start then from the assumption that in a state of a well-developed turbulence the energy

densities of longitudinal and of Alfvén waves are comparable. We have

$$W^{\mathrm{l}} \approx W^{\mathrm{a}} \approx n_* \varepsilon_* \left(\frac{c}{v_{\mathrm{A}}}\right)^{3\gamma-4}. \tag{17.56}$$

We must use for γ the value of that index in the high energy region, that is, estimate it, for instance, from the X-ray emission spectrum. The energy density of the plasma turbulence will be comparable to the energy density of the ultra-relativistic particles when $\gamma \approx \frac{4}{3}$. If $\gamma > \frac{4}{3}$, we have $W^{\mathrm{l}} \approx W^{\mathrm{a}} \ll n_* \varepsilon_*$, as in the case of normal weak turbulence.

If Alfvén waves are excited by a stabilized inverted population (with a growth rate (17.42) with $\omega_{\mathrm{p}} \approx ck_{\perp}$), the energy density of the plasma waves turns out to be very large:

$$W^{\mathrm{l}} \approx W^{\mathrm{a}} \approx n_* m_{\mathrm{i}} v_{\mathrm{A}}^3 / c, \tag{17.57}$$

which is larger than the energy density of the resonance particles, that is, of the particles in the inversion region. This means that under the conditions of an ultra-relativistic plasma in a strong magnetic field the quasi-linear relaxation as a rule always succeeds in adjusting the population inversion.

It is true that in the inhomogeneous plasma in the vicinity of a pulsar the inversion of the distribution function may be established rapidly due to the escape of particles, but for this it is necessary that there exist large particle concentration gradients with characteristic inhomogeneity scale-lengths of the order of (Tsytovich and Kaplan, 1972b)

$$L \leqslant \frac{c}{\omega_{\mathrm{p}}} \frac{v_{\mathrm{A}}^2}{c^2}. \tag{17.58}$$

The problem of how the circumpulsar plasma becomes turbulent remains as yet an unclear one.

Apparently, the cyclotron instability and partly the instability arising from the repeated occurrence of inverted populations will nevertheless continuously introduce turbulence in the circumpulsar plasma, creating in it turbulence of Alfvén and longitudinal waves in the range of frequencies of the order ω_{p}. The energy density of this turbulence is probably not much lower than, and might even be comparable with the energy density of the ultra-relativistic particles. There are here many problems such as, for instance, the determination of the spectra of such turbulence, which remain completely unexplored.

§ 18. Emission by an ultrarelativistic plasma in a strong magnetic field

It is very important for an interpretation of the pulsars to analyze the features of the emission which arises in an ultra-relativistic plasma in a strong magnetic field. First of all it is here essential that the particles move almost strictly along the magnetic field, that is, with $p_{\perp} \ll m_{\mathrm{e}} c$ when $\varepsilon \gg m_{\mathrm{e}} c^2$. A second peculiarity is connected with the suppression of non-linear transfers in a strong magnetic field. The conditions for the propagation of electromagnetic waves are also changed.

The emission by an ultra-relativistic plasma without a magnetic field, considered at the beginning of the preceding section, differs little from the case of a non-relativistic plasma with an admixture of relativistic particles. We can therefore in that case use the results of

§§ 12 and 13 taking into account the appropriate redefinition of the plasma frequency. The conditions for the propagation of high-frequency electromagnetic waves in an ultrarelativistic plasma hardly differs from the conditions of their propagation in vacuo.

If a strong magnetic field is present many emission characteristics are radically altered. We need a special discussion which will be given in the present section.

Emission mechanism in a very strong magnetic field

In a strong magnetic field there are three basic emission mechanisms: magneto-bremsstrahlung (magneto-brems radiation), Compton scattering, and cyclotron scattering.

We shall assume that the particle moves at such a small angle to the magnetic field $\theta \ll m_e c^2/\varepsilon$ that practically the whole of its magneto-bremsstrahlung is concentrated in the first harmonic. The frequency of this emission is, however, not equal to $\omega_H = eHc/\varepsilon$, the angular rotational frequency of the particle in the magnetic field. Taking the Doppler effect into account gives for the observed emission frequency which is generated in that case,

$$\omega \approx \frac{\omega_H}{1-(v/c)\cos\vartheta} \approx \frac{2\omega_{He}(m_e c^2/\varepsilon)}{(m_e c^2/\varepsilon)^2+\vartheta^2} \leqslant 2\omega_{He}\frac{\varepsilon}{m_e c^2}, \tag{18.1}$$

where in contrast to the preceding section we have retained for ω_{He} the non-relativistic gyro-frequency $\omega_{He} = eH/m_e c$.

To fix the ideas we shall assume in what follows that the emitting particles are electrons, although all we say equally refers also to other particles, such as protons or positrons.

One could call the mechanisms of such a magneto-bremsstrahlung a synchrotron mechanism as the emission is by a relativistic particle, or a cyclotron mechanism as the main part of the emission occurs at the first harmonic. We retain here the general term, magneto-bremsstrahlung. The linear connection between the maximum frequency of the emission and the energy of the emitting particle, which distinguishes this mechanism from the synchrotron mechanism is important for what follows.

Solving equation (18.1) we find the connection between the particle energy, the emission frequency, and the direction of the wavevector of the electromagnetic wave, ϑ:

$$\frac{\varepsilon}{m_e c^2} = \frac{1}{(\omega_{He}/\omega) \pm \sqrt{[(\omega_{He}/\omega)^2 - \vartheta^2]}}. \tag{18.2}$$

Strictly speaking the angle ϑ is reckoned here from the velocity vector, but if we assume that $\vartheta \gg \theta$ we can consider ϑ to be the angle between the wavevector of the electromagnetic wave and the field direction.

The directional diagram of the radiation considered is restricted to a solid angle $\pi\vartheta_{\max}^2 = \pi(\omega_{He}/\omega)^2$. Two particles, the one with the largest, and the one with the smallest energy emit at the same frequency and in the same direction.

The total emission power of one electron is determined by the well-known formula of the magneto-brems mechanism:

$$-\frac{d\varepsilon}{dt} = P_1(\varepsilon) = \frac{2}{3}\frac{e^4H^2}{m_e^2c^3}\left(\frac{p_\perp}{m_e c}\right)^2 = \frac{2}{3}\frac{e^2}{c}\omega_{He}^2\left(\frac{p_\perp}{m_e c}\right)^2 = \frac{2}{3}\frac{e^2}{c}\omega_{He}^2\left(\frac{\varepsilon}{m_e c^2}\right)^2\theta^2, \tag{18.3}$$

where $p_\perp$ is the particle momentum component at right angles to the magnetic field. In the approximation considered $p_\perp \ll m_e c$, $p_\perp \approx \varepsilon\vartheta/c$.

As in all magneto-brems mechanisms polarized radiation is produced here. In the general case the radiation is elliptically polarized and the ratio of the lengths of the semi-axes of the polarization ellipse is equal to

$$1-\frac{m_e c^2}{\varepsilon}\frac{\omega}{\omega_{He}} = \pm\sqrt{\left[1-\left(\frac{\omega}{\omega_{He}}\vartheta\right)^2\right]}, \tag{18.4}$$

where we have used equation (18.2). When $\vartheta = \omega_{He}/\omega$, the radiation is linearly polarized, that is, at the edges of the directional diagram. The degree of polarization is here also a maximum. We remember that the polarization of the radiation from several pulsars, in particular, the Crab pulsar, has just these properties.

The magnetic field in the vicinity of the pulsar has probably a rather large curvature. The motion of electrons along curved magnetic field lines leads to the production of radiation which, in general, can even be called synchrotron radiation as it is generated at a high harmonic of the "frequency" of the motion along a curved orbit $2\pi c/R$, where R is the radius of curvature of the magnetic field lines. For the frequency of this emission we have

$$\omega \approx \frac{2\pi c}{R}\left(\frac{\varepsilon}{m_e c^2}\right)^2, \tag{18.5}$$

and for its total power

$$P_2 = \frac{e^2}{c}\left(\frac{2\pi c}{R}\right)^2\left(\frac{\varepsilon}{m_e c^2}\right)^2. \tag{18.6}$$

For small values of R which one may expect in the neighbourhood of pulsars this radiation may also turn out to be considerable.

The next emission mechanism is the Compton scattering of plasma turbulence waves into high-frequency radiation. We shall assume that the whole energy of this turbulence lies in plasmons with frequencies of the order of ω_p. The frequency of the electromagnetic emission generated in the scattering is then

$$\omega = \frac{\omega_p}{1-(v/c)\cos\vartheta} \leqslant 2\omega_p\left(\frac{\varepsilon}{m_e c^2}\right)^2. \tag{18.7}$$

The connection between the radiation frequency, the angle ϑ, and the particle energy is then determined by the equation

$$\frac{\varepsilon}{m_e c^2} = \frac{1}{\sqrt{[2(\omega_p/\omega)-\vartheta^2]}}. \tag{18.8}$$

The opening of the directional diagram is $\vartheta_{max} \leqslant \sqrt{(2\omega_p/\omega)}$. There is here, however, an important difference with the case of the magneto-brems mechanism. In that case the emission more or less uniformly filled the whole of the directional diagram within the limits of the solid angle $\pi(\omega_{He}/\omega)^2$. Now, however, in the scattering mechanisms the radiation is concentrated at the edges of the directional diagram which is now a hollow cone.

This effect can be explained as follows. A longitudinal wave is in the scattering converted into a transverse one only when the projection of the electric field on the direction of motion of the particle is different from zero, that is, when $\vartheta \neq 0$. Such a scattering can be considered to be the result of the absorption of a longitudinal wave and such an emission of a transverse wave that the transverse momentum components of the particle are not changed. This is the essential difference between this mechanism and the magneto-brems one. Connected with this effect is the fact that the emission power no longer increases with increasing particle energy as was the case for Compton scattering without a strong magnetic field.

The emission power of a single relativistic electron in the Compton scattering of longitudinal plasmons in a plasma in a strong magnetic field is equal to

$$-\frac{d\varepsilon}{dt} = P_3 = \frac{8\pi e^4 W^1}{3m_e^2c^3} = \frac{4}{3(\gamma-1)}\frac{e^2}{c}\omega_p^2\left(\frac{\varepsilon_*}{m_ec^2}\right)^2\frac{W^1}{n_*\varepsilon_*}. \tag{18.9}$$

The radiation appearing in Compton scattering turns out to be linearly polarized due to the one-dimensional motion of the particle, as was shown in § 13. In this case the polarization is 100% and the electric vector of the electromagnetic wave lies in a plane through its wavevector and the magnetic field strength vector.

The same relativistic particle emits at two different frequencies both in the magneto-brems mechanism and in the Compton scattering mechanism. The maximum frequency of the electromagnetic waves emitted by a particle with a given energy ε is determined by equation (18.1) if $\varepsilon < m_ec^2\omega_{He}/\omega_p$ and by equation (18.7) for electrons with a higher energy.

The third emission mechanism which we call cyclotron scattering differs from the Compton scattering only in that in it the transverse component of the particle momentum also changes. The properties of the cyclotron mechanism are a combination of the two preceding mechanisms.

The frequency of the electromagnetic radiation which is generated in the cyclotron scattering of longitudinal waves is determined by the relation

$$\omega = \frac{\omega_p \pm \omega_H}{1-(v/c)\cos\vartheta} = 2\frac{\omega_p \pm \omega_{He}(m_ec^2/\varepsilon)}{(m_ec^2/\varepsilon)^2+\vartheta^2}. \tag{18.10}$$

Hence it follows that particles with a relatively low energy, when $\varepsilon \ll m_ec^2\omega_{He}/\omega_p$, emit at a frequency close to the magneto-brems emission and the frequency of the emission of more energetic particles is of the same order as in the case of the normal Compton scattering. We shall write down the expressions for the total emission power separately for the two limiting cases.

For particles with energies $\varepsilon \ll m_ec^2\omega_{He}/\omega_p$ the total power of the spontaneous emission in cyclotron scattering is

$$-\frac{d\varepsilon}{dt} = P_4 = \frac{8\pi}{15}\frac{e^4p_\perp^2}{m_e^4c^5}\left(\frac{\omega_H}{\omega_p}\right)^4 W^1 = \frac{1}{5}\left(\frac{p_\perp}{m_ec}\right)^2\left(\frac{\omega_{He}}{\omega_p}\frac{m_ec^2}{\varepsilon}\right)^4 P_3 = \frac{4\pi W^1}{5H^2}\left(\frac{\omega_{He}}{\omega_p}\frac{m_ec^2}{\varepsilon}\right)^4 P_1. \tag{18.11}$$

For particles with higher energies, $\varepsilon \gg m_e c^2 \omega_{He}/\omega_p$, the total power of the spontaneous emission in cyclotron scattering is

$$-\frac{d\varepsilon}{dt} = P_5 = \frac{8\pi}{15}\frac{e^4 p_\perp^2}{m_e^4 c^5}\left(\frac{\omega_p}{\omega_H}\right)^2 W^1 = \frac{1}{5}\left(\frac{p_\perp}{m_e c}\right)^2 \left(\frac{\omega_p}{\omega_{He}}\frac{\varepsilon}{m_e c^2}\right)^2 P_3 = \frac{4\pi W^1}{5H^2}\left(\frac{\omega_p}{\omega_{He}}\frac{\varepsilon}{m_e c^2}\right)^2 P_1. \tag{18.12}$$

The two formulae, (18.11) and (18.12) are written in a form which makes it possible to see clearly the role of the cyclotron scattering. As long as the energy ε does not differ too much from the limiting value $m_e c^2 \omega_{He}/\omega_p$ the cyclotron scattering is less important than the magneto-brems emission or the Compton scattering as here, on the one hand, $p_\perp \ll m_e c$ and, on the other hand, as a rule, $W^1 \lesssim n_* \varepsilon_* \ll H^2/4\pi$ (this follows from the condition $\varepsilon_* < m_e c^2 \omega_{He}/\omega_p$). However, at low energies the role of the cyclotron scattering can become the determining one.

Cyclotron scattering at low energies gives the maximum effect. It is true that equation (18.11) is derived for the case $\varepsilon \gg m_e c^2$ but there are no other lower limits for the value of ε, for (18.11) to be valid. For a fixed angle θ the quantity P_4 decreases with increasing energy, as ε^{-2}. For energies $\varepsilon > m_e c^2 \omega_{He}/\omega_p$ the emission power again increases with increasing ε, butt his increase is limited by the requirement $p_\perp \ll m_e^2 c^3 \omega_{He}/\omega_p \varepsilon$ which was assumed when deriving equation (18.12) so that P_5 is not larger than the normal power of the Compton losses in a plasma.

The relative role of the last two mechanisms is determined by the ratio of the plasma wave energy density and the magnetic field energy density. We find from equations (18.11) and (18.12) that as long as the particle energy lies within the limits

$$\left(\frac{4\pi W^1}{5H^2}\right)^{1/4} \ll \frac{\varepsilon}{m_e c^2}\frac{\omega_p}{\omega_{He}} \ll \left(\frac{5H^2}{4\pi W^1}\right)^{1/2}, \tag{18.13}$$

the magneto-brems emission dominates the cyclotron scattering. Outside this energy interval the emission is larger for the cyclotron scattering.

The polarization properties of the radiation in cyclotron scattering are different in different energy ranges. When $\varepsilon \ll m_e c^2 \omega_{He}/\omega_p$ the polarization of the radiation is the same as for the magneto-brems radiation. The directional diagram of the radiation has the same form as for the magneto-brems radiation.

Moreover, for energies $\varepsilon \gg m_e c^2 \omega_{He}/\omega_p$ the degree of polarization in cyclotron scattering is somewhat lowered as compared to the case of Compton scattering as there occurs also a component of the electric field at right angles to the plane through the vectors $\boldsymbol{k}$ and $\boldsymbol{H}$. The degree of polarization is here approximately $1-(\omega_p/\omega_H)^2$. The directional diagram of the radiation is a hollow cone.

Cyclotron scattering and magneto-brems radiation depend in the same way on the angle θ or $p_\perp$ and their comparison is thus independent of the choice for these quantities.

The relative role of the Compton scattering (equation (18.9)) and of the magneto-brems emission depends already significantly on the choice of the values of $p_\perp$ or θ. Comparing (18.3) and (18.9) we can see that when $p_\perp < m_e c(4\pi W^1/H^2)^{1/2}$ the losses through magneto-brems emission are less than the Compton losses.

As we consider an ultra-relativistic plasma, the state of which is appreciably controlled by the magnetic field, we may assume that the condition $4\pi W^1/H^2 \ll 1$ will be satisfied.

If we assume that the plasma turbulence energy density is of the order of $W^1 \approx n_* \varepsilon_*$ (see § 17) Compton scattering dominates magneto-brems emission only for energies $\varepsilon < < m_e c v_A/\sqrt{(\overline{\theta^2})}$. In the very-high electron energy region we can neglect Compton and cyclotron scattering at frequencies $\omega \approx \omega_p(\varepsilon/m_e c^2)$ provided the angle θ is not too small.

Spectral emission coefficients

The powers of the particle energy losses given in the preceding sub-section were obtained by integrating the emission probabilities—given in the tables of the Appendix—over the radiation frequency and over the angles for particles with a given energy. We shall now determine the spectral dependence of the emission coefficients averaging the appropriate probabilities by means of a distribution function chosen in a well-defined way and integrating them over the angle ϑ.

It is impossible to use here a strictly one-dimensional particle velocity distribution function as the emission probabilities also depend on the component $p_\perp$. In fact, the distribution function now depends on two arguments, ε and θ. As a first approximation we can make the following assumption: we assume that the distribution over the angle θ is independent of the distribution over the energy ε and that the latter is determined by the already well-known power-law dependence. We can then assume that $f_{\varepsilon,\theta} = f_\varepsilon^{(1)} f_\theta^{(2)}$, where $f_\varepsilon^{(1)}$ is, say, the function (17.18) while we can write for $f_\theta^{(2)}$ a Gaussian function with dispersion $\overline{\theta^2}$.

It is difficult to say in how far these assumptions agree with real conditions in pulsars. Apparently, the angular distribution is independent of the energy distribution as the pulse width of a pulsar does not show an appreciable change with frequency. The emission spectra, at least in some frequency ranges, have a power-law character so that the use of (17.18) is also justified. Something else is important for us: a function of the form described is an exact solution of the self-consistent problem of the distribution of turbulence and particles in a relativistic plasma (see § 19).

The evaluation of J_ω^t for all mechanisms can be done by standard methods and we shall thus at once give the results.

At relatively low frequencies the largest role is played by cyclotron scattering for $\varepsilon \ll \ll m_e c^2 \omega_{He}/\omega_p$. The emission coefficient—apart from a factor of order unity—is equal to

$$J_\omega^t \approx \frac{\omega_{He}^3}{n_* c^3} \left(\frac{\varepsilon_*}{m_e c^2}\right)^{\gamma+1} \left(\frac{\omega_{He}}{\omega}\right)^{\gamma+2} W^1 \overline{\theta^2}. \tag{18.14}$$

Equation (18.14) is valid for frequencies $\omega \gg \omega_{He} \varepsilon_*/m_e c^2$. At lower frequencies the emission is approximately proportional to ω^{-2}. One can estimate its magnitude roughly as

$$J_\omega^t \approx \frac{\omega_{He}^3}{n_* c^3} \frac{\omega_{He}^2 \varepsilon_*}{\omega^2 m_e c^2} W^1 \overline{\theta^2}. \tag{18.15}$$

The coefficient for magneto-brems emission is for $\omega > \omega_{He} \varepsilon_*/m_e c^2$ equal to

$$J_\omega^t \approx \frac{\omega_{He}^3}{n_* c^3} \left(\frac{\omega_p}{\omega_{He}}\right)^4 \left(\frac{\varepsilon_*}{m_e c^2}\right)^{\gamma+1} \left(\frac{\omega_{He}}{\omega}\right)^{\gamma-2} \frac{H^2}{4\pi} \overline{\theta^2}. \tag{18.16}$$

For $\omega < \omega_{He}\varepsilon_*/m_ec^2$ we have for this coefficient $J^t_\omega \propto \omega^2$. Comparing (18.16) and (18.14) we see that the cyclotron scattering is more important than the magneto-brems emission at frequencies

$$\omega < \frac{\omega^2_{He}}{\omega_p}\left(\frac{4\pi W^1}{H^2}\right)^{1/4}. \tag{18.17}$$

When $\omega < \omega_{He}\varepsilon_*/m_ec^2$ the role of the cyclotron scattering is even more important.

Cyclotron scattering at high energies ($\varepsilon > m_ec^2\omega_{He}/\omega_p$) leads to the following emission coefficient:

$$J^t_\omega \approx \frac{\omega^3_{He}}{n_*c^3}\left(\frac{\omega_p}{\omega_{He}}\right)^{\frac{1}{2}(\gamma+7)}\left(\frac{\varepsilon_*}{m_ec^2}\right)^{\gamma+1}\left(\frac{\omega_{He}}{\omega}\right)^{\frac{1}{2}(\gamma-3)} W^1\overline{\theta^2}. \tag{18.18}$$

This formula is valid when $\omega > \omega_p(\varepsilon_*/m_ec^2)^2$. At lower frequencies $J^t_\omega \propto \omega^{3/2}$.

Finally, Compton scattering leads to the following emission coefficient:

$$J^t_\omega \approx \frac{\omega^3_p}{n_*c^3}\left(\frac{\omega_p}{\omega}\right)^{\frac{1}{2}(\gamma+1)}\left(\frac{\varepsilon_*}{m_ec^2}\right)^{\gamma+1} W^1. \tag{18.19}$$

Here also $\omega > \omega_p(\varepsilon_*/m_ec^2)^2$. At lower frequencies $J^t_\omega \propto \omega^{-1/2}$.

The formulae which we have just given determine the frequency-dependence of the emission spectrum under the assumption that $\overline{\theta^2}$ is independent of the particle energy. It is, of course, possible that $\overline{\theta^2}$ is different for particles with different energies. In those cases the emission spectrum will change as an energy dependence of $\overline{\theta^2}$ is equivalent to a change in the particle distribution function. In particular, if we assume that the mean square of the transverse momentum component is independent of energy—that is, $\overline{p^2_\perp}$ = constant—we have $\overline{\theta^2} \propto \varepsilon^{-2}$ which leads, for instance, to an extra factor $(\omega_{He}/\omega)^2$ in (18.14), and so on.

If we apply these formulae to obtain estimates, we must bear in mind that (18.14), (18.16) and (18.18), (18.19) determine the total emission coefficient within the limits of the whole directional diagram. To determine the emission coefficient per unit solid angle, we must divide these expressions by the magnitude of the solid angle, that is, (18.14) and (18.16) by $(\omega_{He}/\omega)^2$ and (18.18) and (18.19) by ω_p/ω.

Reabsorption coefficients

We can also use standards methods to evaluate the reabsorption coefficients. We need only take into account some features of a system which are connected with the directional features of the particle motion along the magnetic field. In particular, we get in the damping rate a larger contribution from the derivative $\partial f/\partial\theta^2$ than from $\partial f/\partial p_\parallel$. Moreover, the damping rate depends very strongly on ϑ.

One can show that there is, in general, no amplification of the electromagnetic waves for the mechanisms considered. Amplification arises for mechanisms connected with cyclotron transitions when there is a change in population of states with higher harmonics in ω_H. In magneto-brems emission mechanisms and cyclotron scattering for $\varepsilon < m_ec^2\omega_{He}/\omega_p$ transitions are possible only corresponding to the first harmonic of ω_H and there occurs

no change in population. In cyclotron scattering for $\omega_p > \omega_H$—that is, for energies $\varepsilon > m_e c^2 \omega_{He}/\omega_p$—changes in population may occur, but here the optical depths are small.

At frequencies $\omega > \omega_{He}\varepsilon_*/m_e c^2$ the expression for the cyclotron damping has the form ($\gamma^t = \gamma^t_+ + \gamma^t_-$):

$$\frac{\gamma^t_\pm(\omega, \vartheta)}{\omega_{He}} \approx -\left(\frac{\varepsilon_*}{m_e c^2}\right)^{\gamma+2} \frac{W^1}{n_* \varepsilon_*} \left(\frac{\omega_{He}}{\omega}\right)^{\gamma+5} \left\{1 \pm \sqrt{\left[1-\left(\frac{\omega}{\omega_{He}}\vartheta\right)^2\right]}\right\}^{\gamma+4} \times \sqrt{\left[1-\left(\frac{\omega\vartheta}{\omega_{He}}\right)^2\right]} \left[2-\left(\frac{\omega\vartheta}{\omega_{He}}\right)^2\right]. \tag{18.20}$$

At lower frequencies, $\omega < \omega_{He}\varepsilon_*/m_e c^2$, we get the estimate:

$$\frac{\gamma^t}{\omega_{He}} \approx -\left(\frac{\omega_{He}}{\omega}\right)^5 \frac{W^1}{n_* \varepsilon_*} \left(\frac{\varepsilon_*}{m_e c^2}\right)^2. \tag{18.21}$$

The reabsorption coefficients (18.20) and (18.21) are of comparable large magnitude which favours the formation of turbulent plasma reactors in circumpulsar space, as we shall see in the next section.

The magneto-brems damping rate for $\omega > \omega_{He}\varepsilon_*/m_e c^2$ is determined by the equation

$$\frac{\gamma^t_\pm(\omega, \vartheta)}{\omega_{He}} \approx -\left(\frac{\omega_p}{\omega_{He}}\right)^4 \left(\frac{\varepsilon_*}{m_e c^2}\right)^{\gamma+2} \frac{H^2}{4\pi n_* \varepsilon_*} \left(\frac{\omega_{He}}{\omega}\right)^{\gamma+1} \frac{2-(\omega\vartheta/\omega_{He})^2}{\sqrt{[1-(\omega\vartheta/\omega_{He})^2]}} \times \left\{1 \pm \sqrt{\left[1-\left(\frac{\omega\vartheta}{\omega_{He}}\right)^2\right]}\right\}^\gamma. \tag{18.22}$$

In the low-frequency region $\gamma^t \propto \omega^{-1}$.

In the highest-frequency region, $\varepsilon > m_e c^2 \omega_{He}/\omega_p$, $\omega > \omega_p(\varepsilon_*/m_e c^2)^2$ we have for the damping rate for cyclotron scattering:

$$\frac{\gamma^t(\omega, \vartheta)}{\omega_p} \approx -(\gamma+4)\left(\frac{\varepsilon_*}{m_e c^2}\right)^{\gamma+2} \frac{W^1}{n_* \varepsilon_*} \left(\frac{\omega_p}{\omega}\right)^{\gamma/2} \vartheta^4 \left(1-\frac{\omega\vartheta^2}{2\omega_p}\right)^{\frac{1}{2}(\gamma+2)}. \tag{18.23}$$

Finally, for the Compton scattering when $\omega > \omega_p(\varepsilon_*/m_e c^2)^2$ the reabsorption coefficient is equal to

$$\frac{\gamma^t(\omega, \vartheta)}{\omega_p} = \frac{4\pi\gamma}{\gamma-1}\left(\frac{\varepsilon_*}{m_e c^2}\right)^{\gamma+2} \left(\frac{2\omega_p}{\omega}\right)^{\frac{1}{2}(\gamma+2)} \vartheta^2 \left(1-\frac{\omega\vartheta^2}{2\omega_p}\right)^{\frac{1}{2}(\gamma+4)} \frac{W^1}{n_* \varepsilon_*}, \tag{18.23'}$$

and at lower frequencies $\gamma^t \propto \omega^{-3/2}$.

We show in Fig. 77 the ϑ-dependence of the reabsorption coefficients. In the general case for an arbitrary distribution function there is a factor $4-(d \ln f_\varepsilon/d \ln \varepsilon)$ (for $f_\varepsilon \propto \varepsilon^{-\gamma}$ it is equal to $\gamma+4$) which in principle can lead to amplification if in the particle energy region considered f_ε increases very steeply (faster than ε^4) with increasing ε. However as equation (18.23) itself corresponds to the higher-energy case it is difficult to envisage the appearance of such a behaviour of f_ε.

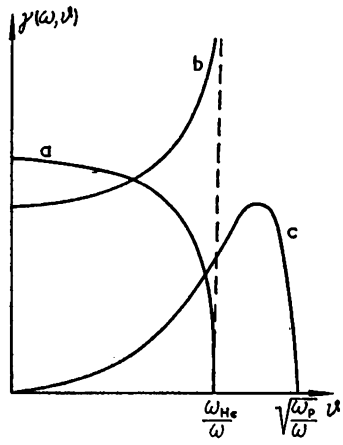

FIG. 77. Dependence of the reabsorption coefficients on the angle between the wavevector and the magnetic field: a) cyclotron scattering at low frequencies; b) magneto-brems emission; c) scattering at high frequencies.

RADIATIVE COMPRESSION OF THE PARTICLE MOTION TOWARDS THE MAGNETIC FIELD LINES

The emission of electromagnetic waves in the magneto-brems mechanism and in cyclotron scattering inevitably leads to a loss of transverse momentum component of the particles, that is, to a stretching of their motions along the magnetic field lines. The transverse momentum component decreases relatively faster for small θ than the total energy of the particle so that the angle θ decreases still faster.

Using the emission probabilities we can easily obtain formulae describing the changes in $p_\perp$ or θ. We shall give them for the case considered here of the motion of particles at small angles to the magnetic field.

Magneto-brems emission gives the following value for the loss in $p_\perp^2$ or the decrease in θ:

$$\frac{1}{2p_\perp^2}\frac{dp_\perp^2}{dt} = \frac{1}{\theta}\frac{d\theta}{dt} = -\frac{2}{3}\frac{e^4H^2}{m_e^3c^5}\frac{m_ec^2}{\varepsilon}. \tag{18.24}$$

Comparing (18.24) with (18.3) we see that the relative losses in the transverse momentum component are, indeed, larger than the relative losses $P_1(\varepsilon)/\varepsilon$ in energy, as there is in (18.3) an additional small factor $(p_\perp/m_ec)^2$.

Similarly one finds for the losses in transverse momentum component and for the decrease in the angle θ when plasma waves are cyclotron scattered by particles with energies

$\varepsilon < m_e c^2 \omega_{He}/\omega_p$:

$$\frac{1}{2p_\perp^2}\frac{dp_\perp^2}{dt} = \frac{1}{\theta}\frac{d\theta}{dt} = -\frac{8\pi}{15}\frac{e^4 W^1}{m_e^3 c^5}\frac{m_e c^2}{\varepsilon}\left(\frac{\omega_{He}}{\omega_p}\frac{m_e c^2}{\varepsilon}\right)^4. \tag{18.25}$$

Here also the relative decrease in $p_\perp$ is faster than in ε, that is, (18.25) is larger than P_4/ε, because of the presence of the same small factor $(p_\perp/m_e c)^2$ in (18.11).

Cyclotron scattering by particles with larger energies, $\varepsilon > m_e c^2 \omega_{He}/\omega_p$, also leads to a decrease in $p_\perp$ but in a relatively slowed down tempo because of the mutual cancellation of transitions with $\omega_p + \omega_H$ and $\omega_p - \omega_H$. Now we have

$$\frac{1}{2p_\perp^2}\frac{dp_\perp^2}{dt} = -\frac{16\pi}{3}\frac{e^4 W^1}{m_e^3 c^5}\frac{m_e c^2}{\varepsilon}. \tag{18.26}$$

Compton scattering without a change in $p_\perp$ (emission power P_3) does, of course, not lead to a compression of the particle motion towards the magnetic field lines.

At low energies, and not too small ratios of the quantities W^1 and $H^2/4\pi$, that is, provided $\varepsilon < m_e c^2 (\omega_{He}/\omega_p)(4\pi W^1/H^2)^{1/4}$, the losses in transverse momentum and the stretching of the motions along the magnetic field lines are thus connected with cyclotron scattering and at high energies with magneto-brems emission.

The absorption of electromagnetic waves in the same mechanisms increases $p_\perp^2$ and θ. As a result of the balance between absorption and emission a particle distribution in $p_\perp^2$ and θ is formed. In other words, in a plasma in a strong magnetic field a turbulent reactor produces both a particle energy distribution function and a particle distribution function in directions of motion. We shall discuss the theory of turbulent reactors in a relativistic plasma in the next section.

§ 19. Turbulent reactors in a relativistic plasma

We developed in § 14 the theory of plasma turbulent reactors in a non-relativistic plasma. In such reactors a distribution function is formed for the relatively small number of relativistic particles while the main part of the plasma stays non-relativistic. We now turn to the analysis of relativistic turbulent reactors in which all particles are relativistic. However, here also we must separate from the total distribution function the part describing the particles with high energies and consider the formation of just that part of the distribution function. Indeed, the most important role of turbulence is just the acceleration of fast particles. Moreover, for the interpretation of observational data it is also important to have some idea about the distribution function of the fast particles. One could, of course, pose in principle the problem of determining the complete particle distribution function in a relativistic turbulent plasma.

In the present section we shall first give a generalization of the theory of a turbulent reactor developed in § 14 to the case of a completely relativistic plasma in a relatively weak magnetic field, and then consider the case of a relativistic plasma in a strong magnetic field.

TURBULENT REACTORS IN A RELATIVISTIC PLASMA IN A WEAK MAGNETIC FIELD

We can here immediately apply many results from § 14, bearing in mind the special features of a relativistic plasma (Kaplan and Tsytovich, 1970b, 1971). In particular, one can neglect in a relativistic plasma ionization losses which decrease rapidly with increasing particle energy. The plasma frequency is also diminished which in turn weakens the relative role of such effects as the scattering of electromagnetic waves by Langmuir turbulence waves. The theory of such relativistic reactors is therefore in general even simpler than that of reactors in a non-relativistic plasma.

We start our considerations with an analysis of the particle acceleration. Particles with a low energy acquire energy when Langmuir turbulence waves are scattered by them. The characteristic growth rate for the scattering process $1+e \rightleftarrows 1'+e'$ is determined by equation (17.12) whence we can immediately estimate the tempo of the acceleration:

$$\frac{d\varepsilon}{dt} \approx \frac{c^2 D_{\parallel}^{1}}{\varepsilon} \approx \omega_p \left(\frac{W^1}{n_* \varepsilon_*}\right)^2 \frac{\varepsilon_*^2}{\varepsilon}. \tag{19.1}$$

This equation describes the acceleration of a particle with an energy less than that at the maximum of the distribution function, $\varepsilon < \varepsilon^*$. Particles with higher energies are accelerated in the reactor through the synchrotron or Compton absorption of the "trapped" electromagnetic radiation. The tempo of acceleration is then determined by equation (14.21). We rewrite it in the following form:

$$\frac{d\varepsilon}{dt} \approx \frac{e^2}{c} \omega_{He}^2 \left(\frac{\varepsilon}{m_e c^2}\right)^2, \tag{19.2}$$

$$\frac{d\varepsilon}{dt} \approx \frac{e^2}{c} \omega_p^2 \left(\frac{\varepsilon_*}{m_e c^2}\right)^2 \left(\frac{\varepsilon}{m_e c^2}\right)^2 \frac{W^1}{n_* \varepsilon_*}. \tag{19.3}$$

The energy at the maximum of the distribution function is determined by the intersection of the curves describing the acceleration for $\varepsilon < \varepsilon_*$ and for $\varepsilon > \varepsilon_*$. In the case of a very weak magnetic field when $H^2 \ll 4\pi W^1$ we get the following estimate from (19.1) and (19.3):

$$\frac{\varepsilon_*}{m_e c^2} \approx 0{\cdot}3 \left(\frac{cW^1}{m_e \omega_{pe}^3}\right)^{2/7} \approx 0{\cdot}3 \left(\frac{c^3 n_*}{\omega_{pe}^3}\right)^{2/5} \approx \frac{3 \times 10^6}{n_*^{1/5}}. \tag{19.4}$$

In the last two equalities we have taken $W^1 \approx n_* \varepsilon_*$, and n_* is in cm^{-3}. We remember that here ω_{pe} is the non-relativistic definition of the plasma frequency which differs from ω_p, the relativistic plasma frequency, by a factor $(\varepsilon_*/m_e c^2)^{1/2}$.

In the case of a stronger magnetic field, when $H^2 \gg 4\pi W^1$, the energy ε_* is determined byy comparing (19.1) and (19.2); we get

$$\frac{\varepsilon_*}{m_e c^2} \approx 0{\cdot}3 \left(\frac{cW^1}{m_e \omega_{pe}^3}\right)^{2/7} \left(\frac{4\pi W^1}{H^2}\right)^{2/7} \approx 10^7 \frac{n_*^{1/3}}{H^{4/3}}. \tag{19.5}$$

Using equation (18.23′) we determine the frequency ω_R for which the plasma reactor becomes transparent with respect to absorption through Compton scattering. By analogy with the method of § 14 but omitting the scattering of electromagnetic waves by longitudinal

plasmons, we have:

$$\frac{\omega_R}{\omega_*} \approx \frac{\omega_R}{\omega_p}\left(\frac{m_e c^2}{\varepsilon_*}\right)^2 \approx \left[\frac{\omega_p R}{c}\left(\frac{m_e c^2}{\varepsilon_*}\right)^2 \frac{W^1}{n_* \varepsilon_*}\right]^{2/(\gamma+4)}. \tag{19.6}$$

In the case of a weak magnetic field, $4\pi W^1 \gg H^2$, substituting (19.4) and $W^1 \approx n_* \varepsilon_*$ into (19.6) we find

$$\frac{\omega_R}{\omega_*} \approx \left(3^{5/2}\frac{\omega_{pe}^4 R}{c^4 n_*}\right)^{2/(\gamma+4)}. \tag{19.7}$$

If $\omega_R \gg \omega_*$, a power-law particle energy distribution will be formed in the energy range

$$\varepsilon_* < \varepsilon < \varepsilon_* \sqrt{\frac{\omega_R}{\omega_*}}, \tag{19.8}$$

with approximately the same index γ as in the case of a turbulent reactor in a non-relativistic plasma, for instance, $\gamma = 3$. In the case of a synchrotron reactor the results are very similar.

Comparing the non-relativistic plasma turbulence reactor with the analogous relativistic one, we can reach the following conclusions. In both cases a power-law particle distribution is formed with $\gamma \approx 3$. The energy of the relativistic particles at the maximum in the distribution function is in the relativistic reactor appreciably higher and the power-law distribution spans a larger energy range. A relativistic reactor is *ceteris paribus* non-transparent up to higher frequencies.

Spectral radiation intensity in an optically dense relativistic plasma in a strong magnetic field

One can evaluate this quantity easily using the expressions given in § 18 for the emission and reabsorption coefficients. We shall give here more exact expressions taking into account the dependence of I_ω on the angle ϑ and the presence of two signs in equation (18.2), that is, the possibility of emission at the same frequency by particles with different energy in the same direction.

The spectral radiation density in a given direction in an optically dense medium at those frequencies, at which under the condition $\varepsilon < m_e c^2 \omega_{He}/\omega_p$ the cyclotron scattering is the determining factor, is described by the formula:

$$I_\omega = \frac{m_e \omega^5 \vartheta^2}{2(2\pi)^3 \omega_{He}^3} \frac{\left\{1+\sqrt{\left[1-\left(\frac{\omega\vartheta}{\omega_{He}}\right)^2\right]}\right\}^{\gamma+2} + \left\{1-\sqrt{\left[1-\left(\frac{\omega\vartheta}{\omega_{He}}\right)^2\right]}\right\}^{\gamma+2}}{\left\{1+\sqrt{\left[1-\left(\frac{\omega\vartheta}{\omega_{He}}\right)^2\right]}\right\}^{\gamma+4} + \left\{1-\sqrt{\left[1-\left(\frac{\omega\vartheta}{\omega_{He}}\right)^2\right]}\right\}^{\gamma+4}}. \tag{19.9}$$

We obtain the same formula (19.9) for the spectral radiation intensity for the case of the magneto-brems mechanism, but with the index $\gamma+2$ in the numerator replaced by $\gamma-2$,

and $\gamma+4$ in the denominator by γ. At frequencies $\omega \ll \omega_{He}/\vartheta$ we have in both cases

$$I_\omega = \frac{m_e \omega^5 \overline{\theta^2}}{8(2\pi)^3 \omega_{He}^3}. \tag{19.10}$$

The emission intensity in a given direction is cut off at the frequency ω_{He}/ϑ and in that point the magnitude of the intensity I_ω is also determined by (9.10) (Fig. 78).

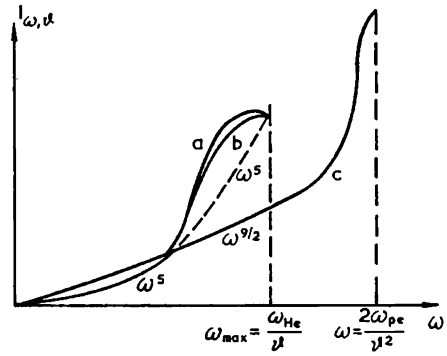

FIG. 78. The spectral intensity of the "trapped" electromagnetic radiation in a turbulent relativistic plasma in a strong magnetic field: a) cyclotron scattering at low frequencies; b) magnetobrems emission; c) scattering at high frequencies.

In the case of cyclotron scattering by particles with a large energy we have:

$$I_\omega = \frac{m_e \omega^{9/2} \overline{\theta^2}}{4(2\pi)^3(\gamma+4)\omega_{He}^2 \sqrt{(2\omega_p)}} \frac{1}{\sqrt{[1-(\omega\vartheta^2/2\omega_p)]}}. \tag{19.11}$$

The maximum value of I_ω is here restricted by the condition for cyclotron transitions $k_\perp v_\perp \leqslant \omega_H$. Hence we find

$$I_\omega^{\max} = \frac{m_e \omega^{7/2} [\overline{\theta^2}]^{1/2}}{4(2\pi)^3(\gamma+4)\omega_{He}\sqrt{(2\omega_p)}}. \tag{19.12}$$

This maximum is reached for ω close to ω_p/ϑ^2.

For Compton scattering we find

$$I_\omega = \frac{m_e \omega^{5/2}}{(2\pi)^3 \gamma \sqrt{(2\omega_p)}} \frac{1}{\sqrt{[1-(\omega\vartheta^2/2\omega_p)]}}. \tag{19.12'}$$

PARTICLE ACCELERATION THROUGH ABSORPTION OF ELECTROMAGNETIC RADIATION

The coefficients for diffusion acceleration of particles when radiation is absorbed are also determined in a well-known way. In the case of cyclotron scattering, $\varepsilon \ll m_e c^2 \omega_{He}/\omega_p$, we have for the longitudinal diffusion coefficient:

$$D_{\parallel}^{t} = \frac{\overline{\theta^2}\theta^2}{2\pi(\gamma-1)^2} \frac{\omega_{He}^4}{n_* c^3} \frac{W}{n_* \varepsilon_*} \frac{\varepsilon \varepsilon_*^3}{m_e^2 c^6} \Psi_1(\gamma), \tag{19.13}$$

where $\Psi_1(\gamma)$ is a function of γ:

$$\Psi_1(\gamma) = \int_0^1 (1-2q)^2[1+(1-2q)^2]\frac{q^{\gamma+2}+(1-q)^{\gamma+2}}{q^{\gamma+4}+(1-q)^{\gamma+4}}q^4\,dq. \tag{19.14}$$

The numerical values of this and of some other functions are given in the table.

TABLE OF THE FUNCTIONS $\Psi_n(\gamma)$

γ	$\Psi_1(\gamma)$	$\Psi_2(\gamma)$	$\Psi_3(\gamma)$	$\Psi_4(\gamma)$
−2·2	0·419		0·716	
−2·0	0·236		0·533	
−1·0	0·219		0·310	
0·0	0·216		0·285	
+1·0	0·215		0·279	
+2·0	0·215	0·800	0·278	1·333
+3·0	0·215	0·510	0·277	0·935
+4·0	0·215	0·467	0·277	0·856
+5·0	0·215	0·554	0·277	0·827

For scattering in the magneto-brems mechanism we have

$$D^t_{\parallel} = \frac{\overline{\theta^2}\theta^2}{\pi(\gamma-1)^2}\frac{\omega_p^4}{n_* c^3}\frac{H^2}{4\pi n_*\varepsilon_*}\frac{\varepsilon^5\varepsilon_*^3}{m_e^6 c^{14}}\Psi_2(\gamma), \tag{19.15}$$

where

$$\Psi_2(\gamma) = \int_0^1 [1+(1-2q)^2]\frac{q^{\gamma-2}+(1-q)^{\gamma-2}}{q^{\gamma}+(1-q)^{\gamma}}q^4\,dq. \tag{19.16}$$

In the case of acceleration through cyclotron scattering by particles with a high energy, for the case of the spectral intensity (19.11), we have

$$D^t_{\parallel} = \frac{2\overline{\theta^2}\theta^2}{21\pi(\gamma-1)^2}\left(\frac{\omega_p}{\omega_{He}}\right)^4\frac{\omega_p^4}{n_* c^3}\frac{W^1}{n_*\varepsilon_*}\frac{\varepsilon^9\varepsilon_*^3}{m_e^{10}c^{22}}\frac{1}{\gamma+4}. \tag{19.17}$$

For the Compton scattering we get

$$D^t_{\parallel} = \frac{2}{3\pi(\gamma-1)^2}\frac{\omega_p^4}{n_* c^3}\frac{W^1}{n_*\varepsilon_*}\frac{\varepsilon\varepsilon_*^3}{m_e^2 c^6}. \tag{19.17'}$$

As we should have expected, (19.17) is larger than (19.13) only when $\varepsilon > m_e c^2\omega_{He}/\omega_p$, even though it depends very steeply on energy.

The transverse diffusion coefficients are determined similarly; they describe the increase in angle θ when electromagnetic waves are absorbed. It is more convenient to introduce here another definition of $D^t_{\perp}$, namely, one such that the change with $p_{\perp}$ in the kinetic equation

would be described by a term of the form

$$\frac{\partial}{\partial p_\perp^2}\left((m_e c)^2 D_\perp^t \frac{\partial f}{\partial p_\perp^2}\right). \tag{19.18}$$

We have here taken into account also the change of $p_\perp$ due to the change in the total momentum. In the case when $\theta \ll m_e c^2/\varepsilon$ the main term in (19.18) has the form

$$\frac{\partial}{\partial \theta^2}\left(\frac{D_\theta^t}{(m_e c)^2}\frac{\partial f}{\partial \theta^2}\right), \quad D_\theta^t = D_\perp^t \left(\frac{m_e c^2}{\varepsilon}\right)^4. \tag{19.19}$$

In the case where the angular spread of the directions of motion increases through cyclotron scattering we have

$$D_\theta^t = \frac{\overline{\theta^2}\theta^2}{2\pi(\gamma-1)^2}\frac{\omega_{He}^4}{n_* c^3}\frac{\varepsilon_*^3 m_e^4 c^6}{\varepsilon^5}\frac{W^1}{n_*\varepsilon_*}\Psi_3(\gamma), \tag{19.20}$$

where

$$\Psi_3(\gamma) = \int_0^1 (1-2q)^2[1+(1-2q)^2]\frac{q^{\gamma+2}+(1-q)^{\gamma+2}}{q^{\gamma+4}+(1-q)^{\gamma+4}} q^2\, dq. \tag{19.21}$$

Similarly we have for the increase in the spread in directions of motion through the magnetobrems mechanism of absorption of electromagnetic waves:

$$D_\theta^t = \frac{\overline{\theta^2}\theta^2}{\pi(\gamma-1)^2}\frac{\omega_p^4}{n_* c^3}\frac{H^2}{4\pi n_*\varepsilon_*}\frac{\varepsilon_*^3}{\varepsilon c^2}\Psi_4(\gamma), \tag{19.22}$$

where

$$\Psi_4(\gamma) = \int_0^1 [1+(1-2q)^2]\frac{q^{\gamma-2}+(1-q)^{\gamma-2}}{q^\gamma+(1-q)^\gamma} q^2\, dq. \tag{19.23}$$

Finally, for cyclotron scattering by particles with a high energy:

$$D_\theta^t = \frac{2\overline{\theta^2}\theta^2}{21\pi(\gamma-1)^2}\left(\frac{\omega_p}{\omega_{He}}\right)^2\frac{\omega_p^4}{n_* c^3}\frac{W^1}{n_*\varepsilon_*}\frac{\varepsilon\varepsilon_*^3}{m_e^2 c^6}\frac{1}{\gamma+4}. \tag{19.24}$$

From these formulae one can get estimates for the characteristic times for the increase in particle energies and in the spread of their motions over angles θ. We remember that these powerful acceleration mechanisms operate only for those values of the particle energies for which the "trapped" electromagnetic radiation is absorbed.

TURBULENT REACTOR IN A RELATIVISTIC PLASMA IN A STRONG MAGNETIC FIELD

When writing down the kinetic equation for the particle distribution function in energies ε and angles θ, we need retain in it only the largest terms. A formal criterion for the choice of the largest terms is the lowest power in the expansion in the small parameter $k_\perp v_\perp/\omega_H$. Application of this criterion shows that in the problem considered the largest term describ-

ing the diffusion acceleration is expression (19.19) and the largest losses are connected with decreases in $p_\perp^2$ or in the angle θ. Retaining these two terms we can write the kinetic equation in the form

$$\frac{\partial f_{\varepsilon,\theta}}{\partial t} = \frac{\partial}{\partial \theta^2}\left(\frac{D_\theta^t}{(m_e c)^2}\frac{\partial f_{\varepsilon,\theta}}{\partial \theta^2}\right) + \frac{\partial}{\partial \theta^2}\left(\frac{d\theta^2}{dt} f_{\varepsilon,\theta}\right). \tag{19.25}$$

The quantity $d\theta^2/dt$ is determined from equations (18.24) to (18.26).

Integration of (19.25) gives for a quasi-stationary distribution:

$$\frac{D_\theta^t}{(m_e c)^2}\frac{\partial f_{\varepsilon,\theta}}{\partial \theta^2} + \frac{d\theta^2}{dt} f_{\varepsilon,\theta} = 0. \tag{19.26}$$

Substituting equations (18.25) and (19.20) into (19.26) we get the equation

$$\Psi_3(\gamma)\frac{\partial f_{\varepsilon,\theta}}{\partial \theta^2} = -\frac{8}{15}\frac{f_{\varepsilon,\theta}}{\overline{\theta^2}}. \tag{19.27}$$

Similarly we find by substituting equations (18.24) and (19.22) into (19.26)

$$\Psi_4(\gamma)\frac{\partial f_{\varepsilon,\theta}}{\partial \theta^2} = -\frac{4}{3}\frac{f_{\varepsilon,\theta}}{\overline{\theta^2}}. \tag{19.28}$$

Solving equation (19.28) enables us to find the parameters of the particle distribution in that energy region where the magneto-brems losses are important, and equation (19.27) determines $f_{\varepsilon,\theta}$ in the energy region where cyclotron scattering dominates. In other words, equation (19.28) is valid for the energy range (18.13), and equation (19.27) for lower energies.

It follows at once, both from (19.27) and from (19.28) that the particle distribution function in the angle θ has a Gaussian nature and that the angular dispersion $\overline{\theta^2}$ is independent of energy. This dispersion cannot be determined from the first approximation in the kinetic equation for the particle distribution function. In the actual circumstances of the circumpulsar space it is most probable that the parameter $\overline{\theta^2}$ is determined by the magnetic field geometry and the conditions for the ejection of particles. Substituting the distribution

$$f_{\varepsilon,\theta} \propto \varepsilon^{-\gamma} e^{-\theta^2/\overline{\theta^2}} \tag{19.29}$$

into (19.27) and (19.28) we get an equation to determine the index γ. In the low-energy (cyclotron scattering) region

$$\Psi_3(\gamma) = \tfrac{8}{15}, \tag{19.30}$$

and in the high-energy (magneto-brems emission) region

$$\Psi_4(\gamma) = \tfrac{4}{3}. \tag{19.31}$$

Comparing equations (19.30) and (19.31) with the data in the table for the functions $\Psi_3(\gamma)$ and $\Psi_4(\gamma)$ we verify that in the low-energy region $\gamma = -2$ and in the high-energy region $\gamma = 2$.

A self-consistent solution of the equation for the reactor in an ultra-relativistic plasma in a strong magnetic field leads thus to a distribution function of the more energetic electrons of the form $f_\varepsilon \propto \varepsilon^{-2}$ in fair agreement with the observational date for the pulsar PSR 0531. In the low-energy region we obtain here a distribution function of the form $f_\varepsilon \propto \varepsilon^2$.

Such a solution violates the condition under which the diffusion acceleration given above were obtained, that is, it does not correspond to the energy region $\varepsilon > \varepsilon_*$. Moreover, the distribution function $f_\varepsilon \propto \varepsilon^2$ turns out to be strongly unstable in a plasma in a magnetic field (see § 17), which can lead to a fast spreading-out of it, that is, to the formation of a plateau. Nevertheless, this does not mean that the solution of the equation for a turbulent reactor leads to the occurrence of an instability. To determine f_ε in the low-energy region one must use the diffusion coefficients calculated for energies $\varepsilon < \varepsilon_*$. Using these expressions one can show that one gets then a distribution function which is almost constant, or, more exactly, which decreases rather slowly with energy. The approximation (17.18) is then fully satisfactory.

We now make some remarks about the influence of the Compton scattering on the distribution function which is formed in a turbulent reactor in a strong magnetic field. As in such a process the longitudinal energy changes, only diffusion along the energy axis, described by equation (19.17′), can be important.

Let us formulate the equations which describe both the increase in energy through Compton scattering and the losses in energy in the same mechanism through spontaneous emission (equation (18.9)). We get

$$\frac{\partial f_{\varepsilon,\theta}}{\partial \varepsilon} = -\frac{P_3}{D_{\parallel}^{t}} f_{\varepsilon,\theta} = -\frac{\gamma}{\varepsilon} f_{\varepsilon,\theta}. \tag{19.31′}$$

From this it is clear that Compton scattering forms a power-law spectrum, without, however, determining the index γ which can be found, say, by solving equation (19.25).

Compton scattering therefore does not change the numerical value of γ in that region of the particle distribution function where its power-law character is produced, even if their losses are larger than other ones. Moreover, this scattering can extend the power-law function also to the region of higher energies. The quantity ε_* in the distribution function (17.18) is thus determined from (18.13) that, is, from the boundary energy for which the magnetobrems emission is larger than the cyclotron scattering:

$$\frac{\varepsilon_*}{m_e c^2} \approx \frac{\omega_{He}}{\omega_p} \left(\frac{4\pi W^1}{5H^2}\right)^{1/4}. \tag{19.32}$$

The cyclotron scattering at energies $\varepsilon > m_e c^2 \omega_{He}/\omega_p$ can also lead to diffusion in the angle θ. However, the losses in $p_\perp^2$ are here relatively small and the second term in (19.26) therefore does not cancel the first one for energies corresponding to the expression on the extreme right-hand side of inequality (18.13). This means that the absorption of electromagnetic radiation increases the spread in the particle velocities more efficiently than the spread is reduced by radiative compression.

The solution of equation (19.26) gives a more or less isotropic distribution. One must say that probably at such high frequencies the optical depth of the plasma is small and there

is no reactor at all. If, however, the optical depth of the plasma is large, an isotropic reactor will be formed here of the type considered at the start of this section. We must bear in mind that in this case all peculiar features of an ultra-relativistic plasma in a strong magnetic field, which are connected with the one-dimensional motion of the particles, such as the Alfvén instability, disappear.

We see thus that in a relativistic plasma in a strong magnetic field turbulent reactors are formed, in which power-law energy distribution functions of fast particles are produced. For not too high energies the distribution function of the angle θ is always Gaussian in nature but it is not possible to find the magnitude of the dispersion of this distribution in the framework of the first approximation of the theory. The magnitude of $\overline{\theta^2}$ can be given either from other considerations such as the condition of the conservation of the adiabatic invariant when the particle moves in a non-uniform magnetic field or from an analysis of the next approximation in the kinetic equation obtained by retaining higher-order terms in the expansion in $k_\perp v_\perp/\omega_H$. To do this one needs take into account the higher terms in the expansion in terms of $k_\perp v_\perp/\omega_H$ also in the emission probabilities.

§ 20. The interpretation of pulsar emission

Immediately after the discovery of pulsars there appeared the first attempts to interpret the pulsed nature of their emission. Two hypotheses were advanced. According to one of them pulsars were pulsating white dwarfs in the atmospheres of which shock waves formed which generated radio-emission bursts approximately in the same way as occurs in the solar corona for type II bursts. However, after pulsars were observed with very short periods which could not be explained even by harmonics of the white dwarf pulsations this hypothesis was dropped. According to the other hypothesis, first suggested by Gold (1968, 1969) the pulsars are rotating neutron stars. If one assumes that the emission has a directional character the rotation of the emitting region leads to the observation of the emission in a given direction as short pulses the relative length of which is of the order of the width of the directional diagram (Fig. 79). At the moment this hypothesis is assumed to be the most probable one.

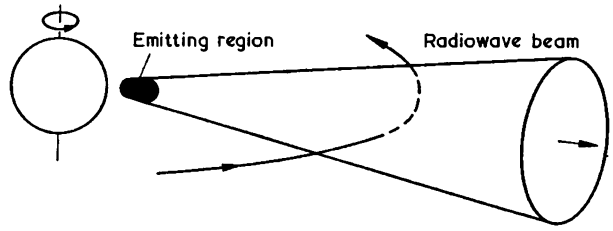

FIG. 79. Rotating neutron star model with "light-house" type emission. The active region rotates with an angular velocity corresponding to the interval between pulses.

The directional nature of the electromagnetic radiation of pulsars and the restrictions of the emitting regions is usually ascribed to the influence of a magnetic field. Present-day theoretical studies of pulsars start thus from the model of a rotating neutron star with a strong magnetic field, the axis of which does not coincide with the rotational axis (Hewish, 1970; Ginzburg, 1971; ter Haar, 1972)

In the total pulsar problem one can, somewhat arbitrarily, distinguish the following divisions: a) the physics of neutron stars and the behaviour of matter at the surface of pulsars; b) the mechanisms of transferring rotational energy to the space immediately touching the pulsar and the properties of this space (structure of the magnetic field, plasma distribution, and electric field in this region); c) emission mechanisms for high-frequency (radio-, optical, and X-ray) radiation in the circumpulsar plasma; and d) influence of the pulsar activity on regions which are farther away (in particular, on supernova shells).

We shall not consider here in detail the physics of neutron stars and the properties of their surface (see the survey articles by Ginzburg (1971), Kirzhnits (1970), and Pines (1970)) The part played by plasma processes is small for those properties. We restrict ourselves merely to a few well-known remarks.

It is, apparently, certain that pulsars are connected with dense, small but massive configurations. Up to now one knows two kinds of such objects: neutron stars with masses up to 1.2 to 1.4 solar masses and black holes. Black holes are stars contracted inside their Schwarzschild sphere. Black holes and neutron stars differ—apart from in other characteristics—also in that there is almost no magnetic field near the Schwarzschild sphere of the black hole while the magnetic field at the neutron sphere surface is very large. This difference leads to a significant change in the conditions of their emission. The situation is the following one.

Near dense objects there are strong gravitational fields; near black holes and neutron stars they are of the same order of magnitude, at least at distances of the order of a few radii of the object. Such fields lead to accretion of matter at the surface of the dense object and this is accompanied by the release of a large amount of energy, predominantly in the form of X-rays (Zel'dovich, 1964; Salpeter, 1964). Unimpeded accretion onto a black hole produces around it a dense atmosphere, most probably in the form of a disc, the emission of which may be thermal in character. The accretion is particularly important in tight binary systems where the dense star draws matter from the second component. It is therefore nowadays usual to assume that a large part of the recently observed X-ray sources occurring in binary systems might well be black holes (for a recent discussion of binary stars as X-ray sources we refer to a paper by Burbidge, 1972).

A slightly different situation occurs in the case when a neutron star with a strong magnetic field is a single star, that is, when there is in its vicinity no other star which can provide matter for accretion. Moreover, the magnetic field of the neutron star would also significantly prevent accretion. Instead of accretion we have here rather ejection: the eruption of matter from the surface of the object. The plasma density remains low and the conditions for the occurrence of various plasma processes turn out to be favourable. Pulsars, that is, single neutron stars with a strong magnetic field will therefore just be objects with a strong emission in the radio range which can be described by the methods of plasma astrophysics.

Transfer of rotational energy to the close circumpulsar space and its properties

What is the nature of the plasma in the vicinity of a pulsar? First of all, we shall show that this plasma is relativistic. We can indicate two observational criteria which enable us to estimate the magnitude of the average electron energy ε_{e*} and the electron density n_* in

the region where the emission is produced (Kaplan and Tsytovich, 1972c). Firstly, it is clear that the total energy flux per unit surface area of the emitting region, F, cannot be larger than the energy flux transferred by the particles, that is,

$$F < n_* \varepsilon_{e*} c. \tag{20.1}$$

Secondly, in the emitting region the total energy of all (electromagnetic and plasma) waves cannot exceed the total particle energy density, that is, $n_* \varepsilon_{e*}$ Hence we get a limit for the effective temperature of the radio-emission:

$$T_{\text{eff}} \leqslant N_{\text{D}} \varepsilon_{e*} \approx n_* \varepsilon_{e*} \left(\frac{c}{\omega_{\text{p}}} \right)^3 \approx \frac{\varepsilon_{e*}^{5/2}}{(4\pi e^2)^{3/2} n_*^{1/2}}, \tag{20.2}$$

as the minimum frequency of the radio-emission will be of the order of the plasma frequency, that is, ω_{p}. Using the data on the luminosities and effective temperatures of the radio-emission of pulsars which we gave in § 16 we get from equations (20.1) and (20.2) the following estimates for the average energy of the relativistic electrons and for their density: $5 \leqslant \varepsilon_{e*}/m_e c^2 \leqslant 10^4$, $10^8 \leqslant n_* \leqslant 10^{12}\,\text{cm}^{-3}$. One can assume $\varepsilon_{e*} \approx 10^3\, m_e c^2$ and $n_* \approx 10^{10}\,\text{cm}^{-3}$ in order to apply the theory and to make illustrative estimates. We note that Kaplan and Tsytovich (1971) and Levich and Syunyaev (1971) had earlier considered the relativistic character of the circumpulsar plasma, but now we have given estimates which are obtained directly from observational data, independent of the choice of some particular emission mechanism.

A rotating neutron star produces in the surrounding plasma a certain electromagnetic field configuration. Notwithstanding the appearance of many papers on the so-called "pulsar electrodynamics" problem we are still very far from having solved the problem of finding the structure of these fields. We shall restrict ourselves therefore to a few remarks some of which may be new.

One usually assumes that a neutron star possesses a magnetic field which to a first approximation we may assume to be a dipole field. The dipole moment of this field is $M \approx H_{\max} R^3$, where $H_{\max}$ is the maximum value of the field at the pulsar surface and R the pulsar radius. The rotational axis of the pulsar and the symmetry axis of the magnetic dipole field are probably not the same and it is quite possible that they are mutually perpendicular. The pulsar dipole moment produces outside a magnetic field with field strength $H \approx M/r^3$, which has both a static component (arising from the dipole moment component along the rotational axis), and a variable part (arising from the dipole moment component at right angles to the rotational axis). The frequency with which the variable part of the magnetic field changes is equal to the rotational angular velocity which we denote in what follows by Ω.

In turn the rotating magnetic field induces a macroscopic electrical field. Even when the dipole moment axis and the rotational axis are coincident a stationary electrical field will arise (unipolar induction). If we neglect the influence of the surrounding plasma, the strength of the stationary electrical field is $E \approx M\Omega R^2/cr^4$. The dipole moment component at right angles to the rotational axis produces a variable field of strength $E \approx M\Omega/cr^2$ which has a frequency Ω.

The existence of these electric fields leads to a number of far-reaching consequences (see,

for instance, Goldreich and Julian, 1969; Michel, 1969; Sturrock, 1971; Cohen and Toton, 1971; and Mestel, 1971). Firstly, the electrical fields will extract particles from the pulsar surface, producing around the pulsar a dense magnetosphere and leading to the appearance of a strong "pulsar wind". This outflow of matter is particularly strong in the region of the pulsar surface near the magnetic poles. It is true, as was pointed out by Ginzburg and Usov (1972), that a large work function may prevent the extraction of particles from the pulsar surface. The ionization of matter in a strong magnetic field also shows special properties (Kadomtsev, 1970). Secondly, charged particles might, in principle, be accelerated in the circumpulsar space by the electrical field up to very high energies of the order of $\varepsilon \approx eM\Omega/cR^2 \approx H_{\max}e\Omega R/c$. Thirdly, the circumpulsar plasma may be dragged along by the pulsar rotation. One could, for instance, assume that either up to the light cylinder, that is, up to a distance from the rotational axis equal to c/Ω, or up to a distance where $H^2/8\pi = n_*\varepsilon_{e*}$, that is, up to a distance $\sim R(H^2_{\max}/8\pi n_*\varepsilon_{e*})^{1/6}$, the whole of the circumpulsar plasma rotates rigidly. Of course, in that case the structures of the circumpulsar electrical and magnetic fields turn out to be completely different.

However, none of the conclusions stated a moment ago are certain. A total dragging along of the circumpulsar plasma is not at all probable. Firstly, the assumption that the field is frozen in is here not justified, as it usually does not occur in a relativistic plasma (Tsytovich, 1961a). Secondly, the role of the anomalous resistivity is here also large (Coppi and Ferrari, 1970). Let us consider this problem in some detail. We showed in § 18 that inevitably in an ultra-relativistic plasma in a strong magnetic field an instability for Alfvén and longitudinal waves with a frequency of the order ω_p arises. The energy density of this turbulence can even be comparable to the particle energy density.

It is clear that the scattering of particles by the waves of the plasma turbulence, and not at all the rare collisions between the relativistic particles, determines the conductivity of the relativistic plasma. For large turbulent energy densities the conductivity turns out to be anomalously low. The existence of strong induced electrical fields in the circumpulsar plasma leads to the appearance of a beam instability, that is, to an additional source for plasma turbulence. It seems to us in general very probable that there will always be plasma turbulence in the cirumpulsar plasma and that the energy density of this turbulence will also always be comparable with the particle energy density. In that case the characteristic free flight time of the particles τ will be comparable to $\omega_p^{-1}(W/n_*\varepsilon_{e*})^{-2}$ (see (17.13)) and when $W/n_*\varepsilon_{e*} \lesssim 1$ reaches the value $1/\omega_p$. The minimum value of the anomalous conductivity of the ultra-relativistic plasma then is

$$\sigma \approx \frac{n_* e^2 c^2}{\varepsilon_{e*}} \tau \approx \frac{n_* e^2 c^2}{\varepsilon_{e*}\omega_p} \approx \frac{\omega_p}{4\pi}. \tag{20.3}$$

The particles then acquire in the electrical field E an energy which does not exceed

$$\varepsilon \approx ecE\tau \approx \frac{ecE}{\omega_p} \approx \varepsilon_{e*}\left[\frac{E^2}{4\pi n_*\varepsilon_{e*}}\right]^{1/2}. \tag{20.4}$$

When the turbulent state is established it is possible that $W^1 \approx n_*\varepsilon_{e*} \approx E^2/4\pi$. The current density in a medium with the conductivity (20.3) in a state with an established plasma turbuleuce is $j \approx \omega_p E/4\pi \approx n_* ec$.

Equation (20.3) is valid in a relatively weak magnetic field, $H^2/8\pi \ll n_* \varepsilon_{e*}$. In the opposite case, the effective collision frequency lies close to ω_{H*} so that

$$\sigma \approx \frac{\omega_p^2}{4\pi\omega_{H*}}. \tag{20.5}$$

It is essential here that the conductivity becomes very low and decreases significantly with distance from the pulsar. One might assume that the condition $H^2/8\pi \approx n_* \varepsilon_{e*}$ divides the co-rotation and non-co-rotation regions near the pulsar. If this were the case, the co-rotation region would lie inside the light cylinder. As the anomalous conductivity shows an essential drop in the layer between the co-rotation and the non-co-rotation regions, one could consider the plasma where $H^2/8\pi < n_* \varepsilon_{e*}$ to be a good conductor, as compared to the plasma in the co-rotation region. This might lead to a closing of the magnetic field lines in the non-co-rotating region of the pulsar, and thus significantly reduce that part of the magnetic moment which varies with time—due to the pulsar rotation. However, even the conductivity (20.3) is very low as compared to the collisional one, and leads to a state where the magnetic field lines are not frozen in.

Thus, the absence of the freezing-in and the anomalously low conductivity of the circumpulsar plasma makes it possible for us to assume that it is impossible that the plasma is completely dragged along by the pulsar rotation. On the other hand, the assumption that the circumpulsar plasma is immobile is also very improbable. If this were, indeed, the case, the stationary part of the magnetic and electrical field would also become appreciable in the region of the non-moving plasma. There would again occur a strong toroidal static magnetic field of strength $H \approx 4\pi\sigma M\Omega R^2/c^2 r^3$, while the variable part of both the electrical and the magnetic field would penetrate into the non-moving plasma only down to the skin depth $c/\sqrt{(4\pi\sigma\Omega)} \approx c/\sqrt{(\omega_p \Omega)}$. Both the stationary and the variable part of the electrical and the magnetic field of the pulsar will exert a rotating action on the circumpulsar plasma with a torque

$$N = \frac{1}{c}\int [r \wedge [j \wedge H]]\, d^3r \approx \frac{\sigma}{c}\int [r \wedge [E \wedge H]]\, d^3r. \tag{20.6}$$

The magnitude of the torque per unit volume in the skin-layer region is about the same as in the region of the stationary fields and equal to $H_{max}^2 R^2 \sigma\Omega/c^2$. Unfortunately, the problem of the dragging along of the circumpulsar plasma by this torque, taking into account inertia and the viscosity of the plasma, has so far not yet been considered.

Another aspect of this problem —the emission of low-frequency waves by the rotating magnetic moment of the pulsar—has attracted appreciably more attention; we refer for a discussion of the electromagnetic radiation to papers by Pacini (1967, 1968), Gunn and Ostriker (1969), Bertotti, Cavaliere, and Pacini (1969), and Kaplan and Eĭdman (1969, 1971b), while the emission of Alfvén waves has, for instance, been calculated in papers by Dokuchaev (1970) and Kulsrud (1971).

If we assume that the plasma surrounding the pulsar does not prevent such an electromagnetic emission and that the phase velocity of such low-frequency waves (of frequency Ω) is equal to c, the emission power is equal to

$$L \approx \frac{M^2\Omega^4}{c^3}. \tag{20.7}$$

As the energy of the emission is taken from the rotational energy of the pulsar, which is proportional to Ω^2, the decrease in the angular velocity of the pulsar will be given by

$$\frac{d\Omega}{dt} \propto \frac{L}{\Omega} \propto \Omega^3. \tag{20.8}$$

It is well known (see, for instance, ter Haar, 1972) that this dependence is observed to be weaker: $(d\Omega/dt)_{\text{obs}} \propto \Omega^{2.2 \text{ to } 2.6}$.

It is, however, unclear in how far the circumpulsar plasma will change the characteristic properties of this emission. We must here bear in mind that the electromagnetic waves considered are formed in the wave zone, that is, beyond the limits of the light cylinder of radius c/Ω. It is possible that there the plasma is swept away by this radiation and that one can thus assume there to be a vacuum, so that equation (20.7) is valid.

To estimate the emission of Alfvén waves we must bear in mind that their phase velocity depends strongly on the angle ϑ between the external magnetic field and their wavevector. Taking into account that according to the data in § 18 the phase velocity of Alfvén waves in an ultra-relativistic plasma lies close to $c \cos \vartheta$, we find for the total power of the dipole emission of Alfvén waves:

$$L_{\text{A}} \approx \frac{M^2\Omega^4}{c^3} \left\langle \frac{1}{\cos^3 \vartheta} \right\rangle, \tag{20.9}$$

where the averaging over directions, that is $< \cos^{-3} \vartheta >$, is performed taking the directionality diagram into account. It is clear that the largest contribution to the Alfvén emission power comes from waves with the smallest phase velocity $c \cos \vartheta_{\text{min}}$, that is, with the smallest wavelength, $\lambda_{\text{min}} = 2\pi c \cos \vartheta_{\text{min}}/\Omega$. If we assume that λ_{min} is independent of the frequency (for instance, that it is determined by the size of the emitter, as was assumed by Dokuchaev (1970) and by Kulsrud (1971)), we get from (20.9) that $L_{\text{A}} \propto \Omega$, and $d\Omega/dt \sim$ constant. The Alfvén emission power then turns out to be anomalously large. In fact, however, the minimum wavelength of the low-frequency Alfvén waves emitted by the pulsar is determined by the conditions in the circumpulsar plasma and in particular by its conductivity. The actual frequency dependence of the emission power may thus differ from the limiting cases given a moment ago and, in particular, one could probably in this way explain the observed frequency dependence of the quantity $d\Omega/dt$.

Summarizing what we have said so far, one can reach the following conclusions. The complete picture of the distribution of magnetic and electrical fields in the circumpulsar region is so far unclear. However, it is almost unavoidable to conclude that the pulsar rotation produces there a region with strong plasma turbulence. Particles are not accelerated to very high energies, but a quasi-stationary state is produced in which the average energy density of the particles, of the turbulent oscillations, and of the electrical field are comparable. The energy density of the magnetic field is probably appreciably larger. The circumpulsar plasma is partly dragged along by the pulsar rotation and, moreover, moves outwards going from a region of strong fields to one of weaker fields. The energy to heat the circumpulsar plasma and to set it in motion is taken from the pulsar rotational energy.

Somewhere, relatively close to the pulsar surface, probably at a distance of a few radii (10^6 to 10^7) cm there are the regions where the generation of high-frequency radio, optical, and X-ray emission is possible. It is clear that these regions participate in the pulsar rotation.

The fact that the emitting region is close to the pulsar surface follows from the absence of an appreciable delay of the radio-pulses in relation to the optical pulses (Drake, 1971). Another strong argument (ter Haar, 1972) for emission close to the pulsar surface (at least for all type C pulsars) lies in the fact that the complex pulse envelope is repeated faithfully over long periods. If the emission took place well away from the surface, either the pulse envelope would be the same for all pulsars, or it would not reproduce over long periods: the existence of a characteristic pulse envelope for individual pulsars seems to indicate some permanent features of the central body which determines the intensity of the emission at different values of the phase in the period.

The total energy of the high-frequency emission of the pulsar in the Crab Nebula is of the order of the loss of rotational energy, which can easily be determined from the slowing-down of the rotation; it is probable that the same is also true for other pulsars (Tsytovich, Buckee, and ter Haar, 1970a, b).

Synchrotron hypothesis of pulsar emission

As in the case of studies of many other powerful cosmic sources, the first mechanisms to be invoked for an explanation of the pulsar emission was the synchrotron one (Shklovskiĭ, 1969, 1970; Zheleznyakov, 1970b). We described in § 15 the method for analyzing the emission of objects using the synchrotron mechanism, and we shall therefore at once give the results here.

Using this mechanism we can explain the optical and X-ray emission of the Crab pulsar with the following choice of parameters. Assuming that the maximum intensity occurs at a frequency $\omega_* \approx 10^{15}\,\mathrm{s}^{-1}$, we get: $\omega_{He} \approx 10^{10}$ to $10^{12}\,\mathrm{s}^{-1}$ ($H \approx 10^3$ to 10^5 Oe); density of the radiating electrons $n_* \approx 10^{13}$ to $10^{14}\,\mathrm{cm}^{-3}$; electron energy at the maximum of the distribution function $\varepsilon_*/m_e c^2 \approx 10$ to 100. The plasma frequency of the relativistic electrons is then $\omega_p \approx 10^{11}\,\mathrm{s}^{-1}$, that is, comparable with ω_{He}, but larger than $\omega_H \approx eHc/\varepsilon_*$. The size of the emitting region is $\sim 10^7$ cm. We must emphasize that all these estimates are more or less unambiguous and that it is impossible to find another self-consistent solution if one changes one of the parameters, say, n_*, strongly.

The estimates obtained are in contradiction to possible conditions in the circumpulsar space but there are also some other serious difficulties. Firstly, the electrons emit only during a very short time. Even electrons with $\varepsilon \approx \varepsilon_*$ lose their energy in $\sim 10^{-2}$ s, and electrons responsible for the X-ray emission lose their energy in about 10^{-5} s. This means that inside the emission region there must be a powerful acceleration mechanism which will also strongly change the emission conditions.

Secondly, the plasma frequency ω_p is appreciably higher than the radio-frequencies so that one must now assume that the region of the radio-emission lies much farther away —by a factor of 10 or more—than the region of the optical emission and this, apparently, contradicts the observational data, that is, the coincidence of the pulses.

Thirdly, for the directionality of the whole radiation we must assume an anisotropy in the electron velocity distribution. For instance, it has been assumed that either the electrons move preferentially along the magnetic field (pencil diagram) or close to the equatorial plane of the magnetic dipole (fan diagram). However, these and other distributions of relativistic electrons are unstable against the excitation of plasma and electromagnetic

waves at frequencies $\sim 10^{11}$ to 10^{12} s^{-1}. On the one hand, this means that it is difficult to realize such anisotropic distributions, and, on the other hand, one should have observed bursts of intensive radiation at those frequencies at which just such a gap in the spectral curve of the pulsars has been observed.

Even larger difficulties are encountered, if one tries to explain the radio-emission of the pulsars in the framework of the synchrotron hypothesis. One needs here either an amplification of the electromagnetic radiation or its coherent generation in bunches consisting of a large number of charges. Ginzburg (1971) has called the last kind of mechanism antenna mechanisms.

One can obtain amplification through synchrotron instability due to the influence of the index of refraction (Zheleznyakov, 1967b; Sazonov, 1969, 1970). This effect is, however, unimportant in the actual circumstances of the pulsars as it either requires the presence of a cold plasma or is possible only in a very weak magnetic field.

Another possibility for amplification in the synchrotron mechanism (Sazonov and Tsytovich, 1968) occurs when all electrons move almost strictly in the plane at right angles to the magnetic field ($|\theta - \frac{1}{2}\pi|) \ll m_e c^2/\varepsilon$). However, such distributions are rather unstable against other plasma modes, such as Alfvén waves (Lerche, 1966). Moreover, the fast heating of the plasma so rapidly increases the spread in the directions of the velocities that the synchrotron instability disappears.

Several authors (Pacini and Rees, 1970; Gold, 1971) have proposed that the radio-emission occurs in bunches of relativistic electrons where they emit coherently. One can, however, show that such bunches spread out rapidly due to the synchrotron emission itself (Bonch-Osmolovskiĭ, Perel'shtein, and Tsytovich, 1969; Bonch-Osmolovskiĭ, Makhan'kov, Tsytovich, and Shchinov, 1971). The mean free path of a bunch is determined by the formula

$$l \approx \frac{c}{\omega_H} \left(\frac{\varepsilon}{e^2 n_* \lambda^2} \right)^{1/2}, \tag{20.10}$$

where λ is the size of the bunch and at the same time of the order of the wavelength at which the coherent emission takes place. Equation (20.10) is valid for $l \gg \lambda$ and $l \gg c/\omega_H$. In the conditions of the circumpulsar space the mean free path of a bunch is in all variants only a few times larger than its size. Kaplan and Éĭdman (1971a) have proposed a radio-emission mechanism based on the coherent acceleration of electrons, up to relativistic energies, in the field of a very strong wave.

Other variants of antenna and synchrotron mechanisms, and instabilities of a very dense plasma in a very strong magnetic field (Chiu and Canuto, 1969, 1971) have also been proposed, but we shall not discuss them here.

Compton mechanism hypothesis for the emission in a weak magnetic field

We noted earlier that a strong plasma turbulence must be excited in the emission region. Even if we ignore other causes, the Compton scattering of electromagnetic waves itself efficiently transfers their energy to plasma waves. Indeed, if we evaluate the growth rate for the transfer of energy from transverse waves (using the observed intensity) to longitudinal plasmons, it turns out that an appreciable exchange of energy takes place during a charac-

teristic time of a pulse length. It is, however, very likely that the generation of turbulence in the plasma in the circumpulsar space is directly connected with the mechanism for the transfer of energy from the pulsar surface to the emission region.

The presence of plasma turbulence in the emission region makes it possible for the Compton emission mechanism to operate. To obtain an estimate for PSR 0531, say, we shall start from the following conditions (Kaplan and Tsytovich, 1970b, 1971). As the pulsar spectra drop off for $\omega \lesssim 10^8$ to 10^9 s^{-1} one can take that value to lie close to the plasma frequency,although part of the drop will be due to the pulse broadening at lower frequencies (compare Higgins, Komesaroff, and Slee, 1971). Assuming that the maximum of the spectrum of the infra-red and optical emission at $\omega_* \approx 10^{15}$ s^{-1} is due to reabsorption we get from equation (19.4) and (19.7) the estimate $\varepsilon_* \approx 3\times 10^3 m_e c^2$, where we have assumed that $W^t \approx 0{\cdot}1 n_* \varepsilon_*$. The density of the relativistic electrons is now $n_* \approx 10^{11}$ cm^{-3}.

The intensity of the radiation in the maximum of the optical spectrum, $I_\omega \approx 10^4$ erg/cm^2 is even somewhat larger than the observed value. The Compton mechanism thus explains the optical and X-ray emission not worse, but rather better than the synchrotron mechanism (less rigid requirements on the particle density and the possibility that the radio-emission occurs also in the region where the optical emission originates).

We note, by the way, that another interpretation of the maximum in the emission spectrum is also possible which can now be connected with the maximum in the particle distribution function, that is, with the quantity $\omega_p(\varepsilon_*/m_e c^2)^2$. In the case of the synchrotron mechanism this was not possible to do as then at low frequencies $I_\omega \propto \omega^{1/3}$ which is clearly in contradiction to the observations. Here, however, $I_\omega \propto \omega$ (see equation (13.22)) which agrees better with the observational data. Moreover, this interpretation can also explain a certain discrepancy which we mentioned earlier, between the estimate of I_ω using observations and the relations for an optically thick plasma.

The dominance of the Compton mechanism over the synchrotron mechanism is a noticeable feature of the explanation of the radio-emission of pulsars. The Compton mechanism achieves amplification of the electromagnetic radiation field much more easily. Indeed, the radio-emission of pulsars occurs in a frequency band from $\sim\omega_p$ to $\sim 10\omega_p$ in which a very strong interaction between longitudinal and transverse plasmons takes place. We remind ourselves, incidentally, that the frequency range of longitudinal waves in a relativistic plasma is of the order of $5\omega_p$. The presence of anisotropy increases the width of the region of frequencies through stimulated transfer. The transfer growth rate at frequencies below $\omega_p(\varepsilon_*/m_e c^2)^2$, that is, in the radio-emission region, is determined by equation (17.14) which shows that in the radio-emission region of pulsars the optical depth for the amplification of electromagnetic waves must be of the order of unity up to frequencies of $\sim 10^{11}$ to 10^{12} s^{-1}.

We note that a direct amplification of electromagnetic waves through Compton scattering of high-frequency electromagnetic waves with a lowering of the frequency is also possible (see § 13 and also a paper by Kaplan, Tsytovich, and Chikhachev, 1970). To obtain an estimate we must now rewrite equation (13.57) to be applicable to the case of a relativistic plasma:

$$\frac{\gamma(\mathrm{t}+\mathrm{e} \rightleftharpoons \mathrm{t}'+\mathrm{e}')}{\omega_p} \approx \frac{\omega_p}{\omega}\frac{m_e c^2}{\varepsilon_*}\frac{W_*^t}{n_*\varepsilon_*}. \tag{20.11}$$

Substituting here the above mentioned values of the parameters we find that the characteristic time for the transfer is of the order of 10^{-3} s. The amplification of the pulsar radio-emission could thus be explained by the direct transfer of energy from high-frequency optical and X-ray waves to radio-waves through Compton scattering with a lowering of the frequency.

All energic estimates also turn out to be reasonable. For instance, we have for the effective temperature of the radio-emission: $T_{\text{eff}} \approx 2\pi^2 W^l c^3/\omega_p^3 \approx 10^{31}$ °K. The characteristic growth rate of the turbulent heating is rather large (according to (17.13) $\tau \approx 1/\gamma$ is of the order of 10^{-6} s). Already close to the pulsar surface the plasma is thus strongly heated. We can use equation (19.4) obtained in the theory of turbulent plasma reactors for an independent estimate of ε_*, and we find from this $\varepsilon_* \approx 10^4 m_e c^2$, which is three times larger than the value obtained above when we interpreted the observations, but this difference is of little significance.

Many features of the pulsar emission are thus explained by the Compton mechanism, in particular, the coincidence of the radio-and optical emission. However, these ideas also encounter some difficulties. First of all, it is not clear how one can now explain the large directionality of the emission and its independence of frequency. Moreover, the maximum energy of the electrons producing X-ray emission cannot be obtained in the reactor described by equation (19.6) so that the problem of the particle acceleration to the very highest energies remains unsolved.

The hypothesis of cyclotron and magneto-brems emission of a plasma in a strong magnetic field

It is possible to explain the directionality of the emission, apparently, only by assuming that particles moving almost exactly along a strong magnetic field, radiate. Recently, Buckee (1972) and Miranda (1972; see also ter Haar, Buckee, and Miranda, 1973) have shown that in the circumpulsar plasma in which there is a beam of relativistic particles present, radio-emission with the observed power may occur. They use a plasma density of about 10^{10} cm^{-3}, a plasma temperature of about 10^4 °K, a beam density of about 10^8 cm^{-3}, and a beam velocity of about 10^9 cm/s. Compton scattering by plasmons at a distance of a few pulsar radii from the surface where the magnetic field is of the order of 10^9 Oe will produce radio-emission predominantly at right angles to the magnetic field and with linear polarization. Moreover, Grounds and ter Haar (1973) have shown that the polarization will not be affected by its passage through the pulsar magnetosphere. It is important to take into account that the emission of the particles itself leads to a compression of their motion along the field (see § 18). Tsytovich, Buckee, and ter Haar (1970a, b) were the first to apply this effect to models of synchrotron emission of pulsars.

The second peculiarity of the behaviour of a plasma in a strong magnetic field which is important for the interpretation of pulsars consists in that the developed high-frequency instabilities (§ 18) lead to the formation of a self-consistent picture of anisotropic particle distributions with angular spreads which are independent of the particle energies.

All we have said above occurs in a plasma with $\omega_H \gg \omega_p$. The theory of processes in such a plasma was developed in previous sections of the present chapter and we shall use the relations obtained there. We shall, to begin with, start from the assumption that the frequ-

ency of the drop in the radio-emission ($\sim 10^8$ s^{-1}) corresponds to the frequency ω_p. The case where the radio-emission frequency falls into the region of the gyro-frequency of a relativistic plasma, that is, eHc/ε_*, is, however, also possible.

In the first case the radio-emission is probably connected with the Alfvén instability and the excitation of Alfvén and Langmuir waves while the optical emission is explained by the high-frequency emission mechanism considered in § 18. Here also different variants are possible. The maximum in the optical radiation spectrum can be explained either as the influence of reabsorption or by the presence of a maximum in the spontaneous emission spectrum. We must also take into account the different ways the frequency depends on energy in these mechanisms.

If we assume that the maximum in the optical radiation spectrum corresponds to the effect of reabsorption we find that the two emission mechanisms that is, both the magneto-brems emission and the cyclotron scattering, make comparable contributions at frequencies $\omega_{He}\varepsilon/m_ec^2$. It turns out that the maximum at the frequency $\omega_{He}\varepsilon_{e*}/m_ec^2$ lies close to the maximum due to reabsorption. We note that the drop of the spectrum in the low-frequency region, connected with reabsorption, is very steep, approximately as $I_\omega \propto \omega^5$ (see (19.10)). If the maximum is determined by the energy ε_* we have in the case of magneto-brems emission at low frequencies $I_\omega \propto \omega^2$.

Comparison of the observational date with theoretical relations leads to the following estimates for the parameters in this case: $\omega_p \approx 10^8$ s^{-1}, $\omega_{He} \approx 3\times 10^{12}$ s^{-1}, $H \approx 10^5$ Oe, $\varepsilon_{e*}/m_ec^2 \approx 3\times 10^2$ to 10^3, $\omega_H \approx 10^{10}$ s^{-1}, that is, indeed, much larger than ω_p. The density of the relativistic electrons is of the order of 3×10^9 cm^{-3}. We can obtain another estimate for ε_* from equations (19.32) and (17.32): $\varepsilon_{e*}/m_ec^2 \approx 3\times 10^3$ which on the whole does not contradict the first estimate. The plasma heating, estimated using the scattering effects and (19.1) and (17.13) is less effective.

An estimate of the effective temperature of the radio-emission, using the formula $T_{\text{eff}} \approx 2\pi^2 W^1 c^3/\omega_p^3$ gives a value of the order of 10^{29} to 10^{30} °K. This value of is the order of the observed one but one should also take absorption into account. For other pulsars the effective temperature is much less but one needs other plasma parameters for them as the high-frequency radiation which so far has not been observed has a lower intensity and a maximum at lower frequencies.

Tsytovich, Buckee, and ter Haar (1970a, b) gave estimates for possible values of ω_* for a number of pulsars, based upon the assumption that the whole energy of the slowing down of the rotation is emitted in the high-frequency end of the spectrum. It turned out that these frequencies were of the order of 10^{13} to 10^{14} s^{-1}. If one takes into account that the intensity of the radiation in the maximum is proportional to $(\omega/\omega_{He})^3$ it follows at once from this that the high-frequency radiation of other pulsars not only lies in the infra-red region of the spectrum, but also has an intensity which is by 3 to 6 orders of magnitude lower than for the Crab pulsar. The estimates for ε_* and n_* also turn out to be lower by approximately one to two orders of magnitude which also leads to a lowering of the effective temperatures of the radio-emission by 2 to 5 orders of magnitude.

Such an interpretation enables us to explain, at least qualitatively, the polarization of the radiation. As we noted in § 18 magneto-brems emission and cyclotron scattering show within the confines of the directional diagram appreciable changes in polarization with maxima

at the edges of the diagram and a minimum in the centre. One can see from Fig. 73 on p. 225 that the polarization of the optical emission of the Crab pulsar shows just these properties.

As far as the polarization of the radio-emission is concerned, it is determined by the conditions for propagation of electromagnetic waves in a magneto-active plasma (Zheleznyakov, 1970b) or in a turbulent plasma. In the latter case the polarization is connected with the nature of the reabsorption of the radiation when it passes through higher layers. There the reabsorption is determined by the mechanisms described in § 18 and the polarization of the radio-emission has thus the same character as that of the high-frequency waves.

The directionality of the optical emission is determined by the fact that in the model considered the particles move almost strictly along the magnetic field lines. Using equations (18.24) and (18.25) one finds estimates for the radiative compression time in the emission region and they are of the order of seconds, but in the deeper layers, where the magnetic field is larger than $\sim 3\times 10^5$ Oe, the compression time is much shorter. This means that the longitudinal motion of the particles, that is, essentially, the directionality of the emission, is produced in the deeper layers. There is probably under those circumstances no emission as the particles collapse very rapidly towards the magnetic field lines. Passing into the region with a weaker field the particles obtain, on the one hand, smaller values of $p_\perp$ due to the conservation of the adiabatic invariant and, on the other hand, increase the angle due to the curvature of the magnetic field lines. As the radiative compression is here insufficient, stronger emission will occur. In other words, the emission region of a pulsar may be determined by the condition that the radiative compression is weakened. The pulse width, that is, the directionality of the total emission, is determined by the conditions of the divergence of the magnetic field lines.

In the variant considered Compton scattering at frequencies $\omega_p(\varepsilon/m_ec^2)^2$ does not determine the parameters of the emitting region, but may be responsible for the X-ray emission. A possible way to check this assumption would be the observation of 100 per cent polarization in the X-ray part of the spectrum.

In another variant of the interpretation of the observational data one can assume that the emission in the optical part of the spectrum is determined by Compton scattering at frequencies $\omega_p(\varepsilon/m_ec^2)^2$ while the maximum of the emission corresponds to the frequency $\omega_p(\varepsilon_*/m_ec^2)^2$. An optical depth of order unity is then reached only at appreciably lower frequencies. The radiation intensity for frequencies less than $\omega_p(\varepsilon_*/m_ec^2)^2$ then behaves as $I_\omega \propto \omega^{-1/2}$.

Estimates of the parameters for this variant give: $n_* \approx 3\times 10^{11}$ cm^{-3}, $H \approx 3\times 10^5$ Oe, $\varepsilon_*/m_ec^2 \approx 3\times 10^3$. We need here thus a larger relativistic electron density. The polarization of the radiation is now close to 100% and has no dip in the centre of the pulse.

Yet another variant of interpretation is based upon the assumption that the observed radio-frequencies correspond to the gyro-frequency eHc/ε_*. In that case the parameters of the emitting region are the following: $H \approx 3\times 10^4$ Oe, $n_* \approx 10^9$ cm^{-3}, $\varepsilon_*/m_ec^2 \approx 3\times 10^3$. The plasma frequency is of the order of 3×10^7 s^{-1}. In the optical part of the spectrum the system is transparent. This variant seems to be less preferable for the following reasons. A weak magnetic field does not guarantee compression and hence directionality of emission. The plasma frequency lies close to the gyro-frequency and one can therefore hardly expect a drop at frequencies less than 10^8 s^{-1}, but one must bear in mind that pulse broadening

may produce such a drop. The weak magnetic field obtained in this variant does not correspond to the whole scheme of the behaviour of the circumpulsar plasma in a strong magnetic field.

Turning now to the most favourable variant, considered a moment ago, we stress once again the peculiar features of the pulsar emission which could be understood qualitatively in the framework of modern plasma theory: a) the directionality, that is, the pulse character of the emission; b) its polarization; c) the possibility of high effective temperatures of the radio-emission, explained by instabilities in strong magnetic fields. As before, it remains ultimately obscure how one can get the acceleration of the particles to high energies—for the X-ray emission we need $\varepsilon/m_e c^2 \approx 10^5$.

The formation of turbulent reactors in an ultra-relativistic plasma in a strong magnetic field, of the kind considered in § 19, is possible; in those particle acceleration also occurs. However, estimates show that such reactors must be situated close to the pulsar surface. Fast particles accelerated in them could pass into the emitting region which lies further from the pulsar surface moving strictly along the magnetic field lines thanks to the radiative compression effect which dominates over the losses of the total energy.

It is possible that plasma turbulent reactors may exist also in circumcollapsar space. Indeed, the formation of strong shocks during the accretion of matter may generate plasma turbulence and maintain it permanently. In the case of a high plasma density of the order of 10^{18} to 10^{19} cm^{-3}, usually accepted for the accretion in binary systems, the turbulent reactor is set up. The frequency of the maximum in the emission from this reactor is at least of the order of 10^{17} s^{-1}.

Pulsars and the activity of supernova shells

It is well known that one has observed in the Crab Nebula perturbations propagating from the pulsar PSR 0531. They are probably connected with streams of particles ejected from its surface. This pulsar must supply energetic particles necessary to explain, for instance, the X-ray emission of the nebula. We meet thus with the problem of a "pulsar wind".

We showed above that for an explanation of the pulsar emission in the whole wavelength range we need only a relatively small number of particles moving at an angle to the magnetic field. It is, however, possible that a much larger number of particles either leaves strictly along the magnetic field lines or diffuses from the pulsar magnetosphere without producing appreciable radiation.

The minimum particle energy flux, estimated from the given parameters of the emitting region, is of the order of 10^{30} erg/s, or the particle flux is of the order of 10^{33} particles/s. This flux, and the radiation itself, produces at large distances around the pulsar a plasma turbulence region. The size of that region, estimated from the beam instability growth rate (12.60) and from the amplification growth rate of electromagnetic waves (4.59) is of the order of 10^{11} to 10^{13} cm.

In this region the number of particles of the main plasma is of the order of 10^{40}. Plasma turbulence heats and accelerates them so that such regions may be the source of sub-cosmic-ray particles. The density of the relativistic particles in this region is approximately two orders of magnitude lower.

It is possible that in the plasma turbulence region, excited by the pulsar radiation and the flux of particles ejected from the pulsar, "wisps" and other macroscopic formations, such as have been observed in the Crab Nebula, are formed. A discussion of these problems lies outside the framework of the present book.

Conclusion

(Problems for the further development of plasma astrophysics)

IN THIS book we have, more or less completely, considered a number of plasma astrophysics problems which were most important for us, on the one hand, and which at the present stage of the theory allowed us, on the other hand, to consider them within the framework of known properties of the cosmic plasma. Of course, in this book we have not studied anything like all aspects of the problems which we touched upon. In some cases we only discussed the statement of the problem and possible ways to solve it. However, in many cases we were able to obtain rather complete qualitative solutions.

For a number of other important problems not even the ways to solve them are clear. We shall now briefly discuss both unsolved problems of topics mentioned by us as well as possible approaches to new problems.

UNSOLVED PROBLEMS IN PLASMA ASTROPHYSICS

Problems connected with the theory of Langmuir turbulence have been studied rather fully in a well-defined approximation, but problems of ion-sound turbulence have not yet been worked out very far. All the same its consequences are important for an explanation of many non-stationary processes on the Sun, in the interplanetary medium, in the Earth's magnetosphere, and so on. It is also very important for the electrodynamics of the circumpulsar space. For the theory of chromospheric flares the transformation of ion-sound-turbulence into fast particles is, in particular, important. So far it has been studied in detail experimentally and one must therefore hope that a complex theoretical and experimental study of this effect will enable us to find a solution of the corresponding astrophysical problems. Apparently, a more detailed knowledge of the properties of Alfvén and magneto-sound turbulence will be of great value for the interpretation of phenomena occurring in the solar wind.

One also needs a detailed analysis of different aspects of the non-linear transfer of radiation in astrophysics and, in particular, in a turbulent plasma. Taking the non-linearity of the transfer into account may have an important effect on the characteristics of the observed radiation from powerful cosmic sources, such as quasars or pulsars. In the present book the transfer of radiation in a turbulent plasma was estimated only in first approximation, neglecting inhomogeneities in the plasma, refraction, and so on. Of course, a more refined analysis of this problem will be necessary in the future.

Conclusion

While the interaction of beams with a plasma is one of the well-developed problems in plasma physics, its astrophysical aspects contain many unsolved problems such as those connected with the ejection of beams from galactic nuclei. The macroscopic effects of plasma astrophysics which then occur are important for astrophysics.

We need also a refined theoretical consideration of turbulent plasma reactors and particularly of the problem of the escape of radiation and particles from a reactor. For this we need an analysis of the self-consistent diffusion of radiation and particles into the neighbouring space. We need a detailed development even of the theory of uniform reactors taking into account for instance, the anisotropy of particles and radiation in the case of a weak magnetic field or taking into account the high cyclotron resonances in the case of a strong field.

The problem of the cosmic-ray spectra

An important achievement of the theory of reactors is the prediction of particle power-law spectra. This enables us, in principle, to pose the problem of the cosmic ray spectra in the sources where they are generated on the basis of modern ideas about the nature of plasma turbulence. We remind ourselves that the observed ion spectra in cosmic rays have a clearly expressed power-law character with an index γ which is equal to the unique value 2·6 for the enormous energy range from 10^9 to 10^{15} eV. When $\varepsilon \approx 10^{15}$ eV the spectral index increases to $\gamma = 3{\cdot}2$ and when $\varepsilon \sim 10^{18}$ eV, it possibly turns out to be again equal to 2·7 (Fig. 80). There are no reliable data for energies from 10^{18} to 10^{20} eV. The cosmic ray electrons have the same energy power-law spectrum from 10^8 to 10^{12} eV with approxi-

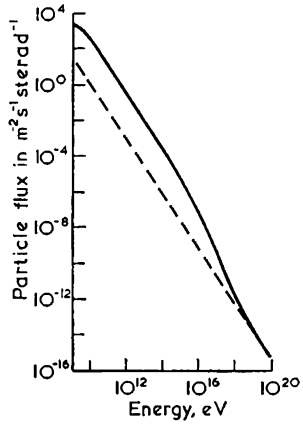

FIG. 80. Observed energy spectrum of the ion component of cosmic rays.

mately the same value of γ (Fig. 81). The high degree of isotropy of the cosmic rays is one of their important observational properties. Ginzburg and Syrovatskiĭ (1964a) describe in their book the cosmic ray properties in detail. Bulanov, Dogel', and Syrovatskiĭ (1971) have discussed the electron component.

For the formation of the observed spectrum the most important facts are, on the one hand, the formation of the spectrum directly in the sources and, on the other hand, its

deformation when the cosmic ray particles propagate through the interstellar and intergalactic space. In both problems plasma processes may play an important role. The problem of the propagation of cosmic rays through space is the simpler one. Firstly, one must note that the development of plasma instabilities can guarantee the isotropy of the cosmic rays.

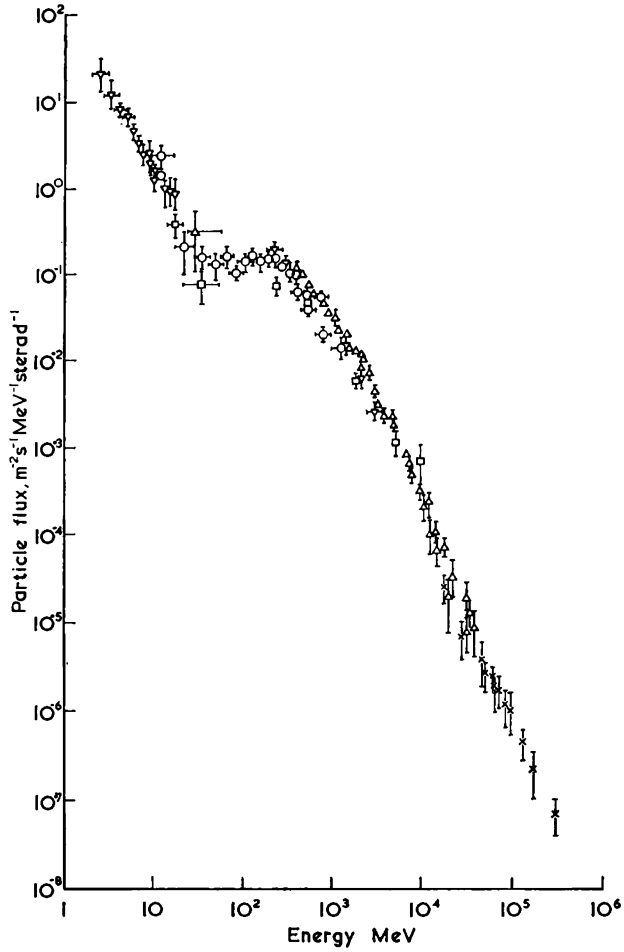

FIG. 81. Observed energy spectrum of the high-energy electron component of cosmic rays.

One may mention amongst such instabilities the Langmuir (Ginzburg, 1966; Tsytovich, 1966b; Pikel'ner, 1969) and Alfvén (Lerche, 1967; Pikel'ner, 1968; Pikel'ner and Tsytovich, 1969b) instabilities. Apparently, the combined action of these instabilities can guarantee a very high degree of isotropy of the cosmic rays. Non-linear stabilization effects are here important. A detailed analysis of the possible role of such instabilities for the isotropization and diffusion of cosmic rays in our Galaxy has been given by Ginzburg, Ptuskin, and Tsytovich (1973). According to present ideas the bend in the spectrum at energies between 10^{15} and 10^{18} eV (see Fig. 80) can be explained both by the combination of the galactic and the extragalactic cosmic ray components and by the diffusion of galactic cosmic rays from our Galaxy.

Conclusion

A more fundamental problem is the explanation of the conditions of the formation of power-law spectra in the sources (Ginzburg and Syrovatskiĭ, 1964a; Tsytovich, 1972a). Such spectra are "manufactured" in plasma turbulence reactors. While in the case of a non-relativistic plasma reactor (a small fraction of relativistic particles in a non-relativistic plasma) the power-law spectrum of the electrons can be formed in a relatively wide range of energies, the formation of such a spectrum for ions in such a reactor requires the realization of rather strict conditions. This is connected with the fact that the reabsorption coefficient for ions is much smaller than for electrons. It is true that if the injection conditions are such that the number of ions is three orders of magnitude larger than the number of electrons, the power-law spectrum of the ions can be "manufactured". One can say that, indeed, the injection of heavier particles is preferred when low-frequency turbulence is present (Melrose, 1968; Tsytovich, 1969a).

In relativistic turbulence reactors the conditions for the formation of a power-law spectrum for ions are considerably eased. Moreover, electrons and ions can then obtain an identical spectrum as the Compton scattering probabilities depend only on the particle energy. Ionization losses which strongly restrict the possible "manufacture" of a power-law spectrum in non-relativistic reactors do no longer play such a large role.

Turbulent reactors of a relativistic plasma are well possible near pulsars, but their size is small and they can therefore, apparently, not "manufacture" the power-law spectrum in a wide range of energies. To produce the power-law spectrum of the ion component of the cosmic rays one needs large-size objects with a rather dense relativistic plasma. It is possible that such objects are present in galactic nuclei. One must note that recently many researchers have shared the idea that cosmic rays are formed in galactic nuclei. Future developments in astrophysics may possibly enable us to detect new objects of this kind.

Appendix

Tables of averaged probabilities for plasma processes

THE introduction of emission, scattering, and decay process probabilities enables us to find easily the coefficients for emission, reabsorption, diffusive particle acceleration, scattering of electromagnetic radiation, and so on. Using these probabilities one can also easily write down kinetic equations for plasma waves and particles. In the original papers one usually finds complete expressions for these probabilities as functions of many variables. The use of such expressions in actual astrophysical applications requires additional calculations which are not always accessible to, say, an astrophysicist or experimentalist.

In the tables which follow we give an as far as possible complete summary of averaged expressions for the probabilities of those plasma processes which are most important under astrophysical conditions. A large part of these expressions has been evaluated anew. However, even in those cases where the probability for the processes was known we have averaged over those parameters, the dependence on which is unimportant for applications.

For instance, when evaluating the non-linear scattering of waves in isotropic plasma turbulence it is sufficient to know the probability averaged over the angle between the particle velocity vector and the plasma wave wavevector. Besides, even for the case of anisotropic turbulence we can fully use such an averaged probability as there is normally no particular value in calculating factors of order unity in astrophysical applications.

The averaged expressions for the probabilities are much simpler and physically translucent and can be used to obtain estimates in the overwhelming majority of cases of analyzing plasma processes in astrophysics. We have described in this book in detail the method for using the probabilities for various processes.

In the following tables we give expressions for the averaged probabilities $\bar{u}$ (in some cases retaining also the non-averaged expressions) in the notation common to the whole of the book. For each process we indicate the limits of applicability of the expressions obtained.

I. Probabilities for the emission of plasma waves

We give the probabilities for the emission of waves of type σ, averaged over the angle between the velocity vector of the emitting particle, $\boldsymbol{v}$, and the wavevector, $\boldsymbol{k}$, of the emitted wave.

For the case of isotropically distributed particles with a given momentum distribution function f_p these probabilities enable us to determine at once the emission coefficient per

unit wavenumber, J_k^σ and the damping rate $\gamma^\sigma(k)$ by a simple integration over the absolute magnitudes, p, of the momentum. We have

$$J_k^\sigma = = \frac{\omega^\sigma(k)k^2}{2\pi^2}\int_0^\infty \bar{u}^\sigma(k, v) f_p \, dp, \tag{A.1}$$

$$\frac{\gamma^\sigma(k)}{\omega^\sigma(k)} = \int_0^\infty \bar{u}^\sigma(k, v)\frac{d}{dp}\left(\frac{f_p}{p^2}\right)p^2 \, dp = -\frac{1}{T}\int \bar{u}^\sigma(k,v) f_p \, dp, \tag{A.2}$$

where T is the temperature and the last equality is obtained for a Maxwell distribution.

One can obtain exact values of J_k^σ and $\gamma^\sigma(k)$ using the averaged probabilities $\bar{u}^\sigma(k, v)$ only for isotropically distributed particles, but for obtaining estimates these quantities can be used also for arbitrary distributions. In particular, we find for the beam instability growth rate:

$$\frac{\gamma^\sigma}{\omega^\sigma} \approx \frac{\bar{u}^\sigma(\omega^\sigma/v_0, v_0)}{m_*(\Delta v_0)^2} n_*, \tag{A.3}$$

where v_0 and Δv_0 are the average velocity and the spread in particle velocities in the beam with density n_* and mass m_*. For relativistic beams with a large energy spread the quantity $m_*(\Delta v_0)^2$ must be replaced by the average energy ε_*.

We can similarly also evaluate other parameters. For instance, we find for the coefficient of diffusive acceleration in a strictly isotropic plasma turbulence

$$D_\parallel^\sigma(v) = \int \frac{\bar{u}^\sigma(k, v)}{v^2} W_k^\sigma[\omega^\sigma(k)]^2 \, dk, \tag{A.4}$$

where W_k^σ is the spectral energy density of the plasma waves.

In Table I we have given the probabilities for Cherenkov emission of all plasma waves (except electromagnetic waves) for the case when the wavelength is much smaller than the Larmor radius, that is, $k \gg \omega_H/v = eHc/\varepsilon v$ (unmagnetized particles) since only in that case is the Cherenkov emission probability not small. In what follows we must thus bear in mind this lower limit of the wavenumber for the applicability of the expressions given here.

TABLE I. *Probabilities for the emission of plasma waves*

Plasma wave mode	Averaged probability
1. Longitudinal plasmons ($\omega^l \approx \omega_{pe}$) in a weak magnetic field ($\omega_{pe} \gg \omega_{He}$).	$\bar{u}^l = \frac{\pi}{2}\frac{m_e}{n_e v}\frac{\omega_{pe}^3}{k^3}\begin{cases}1, & \omega_{pe}/k < v;\\ 0, & \omega_{pe}/k > v.\end{cases}$
2. Gyro-frequency plasmons ($\omega^h \approx \omega_{He}\lvert\cos\vartheta\rvert$) in a weak magnetic field ($\omega_{pe} \gg \omega_{He}$).	$\bar{u}^h = \frac{\pi}{8}\frac{m_e}{n_e v}\frac{\omega_{He}^3}{k^3}\begin{cases}1, & \omega_{He}/k < v;\\ 2\left(\frac{kv}{\omega_{He}}\right)^3, & \omega_{He}/k > v.\end{cases}$

Plasma wave mode	Averaged probability
Maximum probability for $k \approx \omega_{pe}/c$, $v < c\omega_{He}/\omega_{pe}$	$\bar{u}^{h}_{max} = \frac{\pi}{8}\frac{m_e c^3}{n_e v}\left(\frac{\omega_{He}}{\omega_{pe}}\right)^3.$
3. Longitudinal plasmons ($\omega^l \approx \approx \omega_{pe}\lvert\cos\vartheta\rvert$) in a strong magnetic field ($\omega_{pe} \ll \omega_{He}$).	$\bar{u}^{l} = \frac{\pi}{4}\frac{m_e}{n_e v}\frac{\omega^3_{pe}}{k^3}\begin{cases}1, & \omega_{pe}/k < v;\\ \left(\frac{kv}{\omega_{pe}}\right)^2, & \omega_{pe}/k > v.\end{cases}$
Maximum probability for non-relativistic electrons with $k \lesssim \omega_{He}/v$. For ions and relativistic electrons the value of $\bar{u}^{l}_{max}$ is $(\varepsilon/mc^2)^2$ times larger.	$\bar{u}^{l}_{max} = \frac{\pi}{4}\frac{m_e v^2}{n_e}\left(\frac{\omega_{pe}}{\omega_{He}}\right)^3.$
4. Gyro-frequency plasmons ($\omega^h \approx \omega_{He}$) in a strong magnetic field ($\omega_{pe} \ll \omega_{He}$).	$\bar{u}^{h} = \frac{2\pi}{3}\frac{m_e}{n_e v}\left(\frac{\omega_{pe}}{\omega_{He}}\right)^4\frac{\omega^3_{He}}{k^3}\begin{cases}1, & \omega_{He}/k < v;\\ 0, & \omega_{He}/k > v.\end{cases}$
5. Whistlers ($\omega^w = \omega_{He}c^2k^2\times \times\lvert\cos\vartheta\rvert/\omega^2_{pe}$) in a weak magnetic field ($\omega_{pe} \gg \omega_{He}$).	$\bar{u}^{w} = \frac{\pi}{4}\frac{m_e}{n_e v}\frac{\omega^3_{He}}{k^3}\begin{cases}\ln 2\left(\frac{kv}{\omega_{He}}\right)^2+(3\ln 2-2)\left(\frac{kc}{\omega_{pe}}\right)^4, & \frac{\omega_{He}c^2k}{\omega^2_{pe}} < v;\\ \left(\frac{kc}{\omega_{pe}}\right)^4\left(\frac{\omega^2_{pe}v}{\omega_{He}c^2k}\right)^2, & \frac{\omega_{eH}c^2k}{\omega^2_{pe}} > v.\end{cases}$
6. Ion-sound waves ($\omega^s = = v_s k/(1+k^2 d_e^2)^{1/2}$ at frequencies $\omega > \omega_{Hi}$.	$\bar{u}^{s} = \frac{\pi}{2}\frac{m_i}{n_e v}\frac{(\omega^s)^3}{k^3}\begin{cases}1\\ 0\end{cases} \approx \frac{\pi}{2}\frac{m_i v_s^2}{n_e v}\begin{cases}1\\ 0\end{cases}$, if $k \ll (d_e)^{-1}$, $\begin{matrix}v_s < v;\\ v_s > v.\end{matrix}$
7. Alfvén waves ($\omega^a \approx v_A k\lvert\cos\vartheta\rvert$) in a magnetic field satisfying the condition $c \gg v_A \gg v_s$. The condition for the applicability of the equations is $\omega_H/v < k < \omega_{pi}/c$.	$\bar{u}^{a} = \frac{\pi}{16}\frac{m_i}{n_e v}\left(\frac{\omega_{pi}}{ck}\right)^2 v_A(v_A^2+2v^2)\begin{cases}1, & v_A < v;\\ \frac{4v^2}{v_A^2}, & v_A > v.\end{cases}$
Maximum probability for particles with $v \gg v_A$.	$\bar{u}^{a}_{max} = \frac{\pi}{8}\frac{m_i v_A v^3}{n_e c^2}\left(\frac{\omega_{pi}}{\omega_H}\right)^2.$
8. Fast magneto-sound waves ($\omega^m = v_A k$) in a magnetic field satisfying the condition $c \gg v_A \gg v_s$. The limits of applicability of the formulae are: $\omega_H/v < k < \omega_{pi}/c$.	$\bar{u}^{m} = \frac{\pi}{8}\frac{m_i}{n_e v}\left(\frac{\omega_{pi}}{ck}\right)^2 v_A(v^2-v_A^2)\begin{cases}1, & v_A < v;\\ 0, & v_A > v.\end{cases}$
9. Slow magneto-sound waves ($\omega^{ms} \approx v_s k\lvert\cos\vartheta\rvert$) in a magnetic field satisfying the condition $c \gg v_A \gg v_s$. The limits of applicability of the formulae are: $\omega_H/v < k < \omega_{Hi}/v_s$.	$\bar{u}^{ms} = \frac{\pi}{4}\frac{m_i v_s^3}{n_e v}\begin{cases}1, & v_s < v;\\ \left(\frac{v}{v_s}\right)^2, & v_s > v.\end{cases}$
Maximum probability for particles with $v \approx v_s$.	$\bar{u}^{ms}_{max} \approx \frac{m_i v_s^2}{n_e} \approx \frac{T_e}{n_e}.$

Appendix

II. Probabilities for non-linear scattering of plasma waves by epithermal particles leading to the generation of electromagnetic waves

When Langmuir and gyro-frequency plasmons, and also ion-sound waves, are scattered by epithermal particles high-frequency radiation is produced with frequencies $\omega \gg \omega_{pe}$ or $\omega \gg \omega_{He}$. The particles are assumed to be unmagnetized, $v \gg \omega_H/k$. We give in Table II the probabilities averaged over all angular variables, that is, both over the angle between the velocity vector and the wavevector of the scattered plasma wave, and over the angle between the velocity vector and the wavevector of the emitted electromagnetic wave. In the case of isotropic distributions of the particle velocities v and of the plasma wave wavevectors $\boldsymbol{k}_1$ the emission coefficient per unit frequency can be evaluated using the formula

$$J^t_\omega = \frac{\omega^3}{\pi^2 c^3} \int_0^\infty \int_0^\infty \bar{u}^t_\sigma(\omega, k_1, v) f_p W^\sigma_{k_1} \frac{dk_1\, dp}{\omega^\sigma(k_1)} . \tag{A.5}$$

We have used here for the dispersion relation for the electromagnetic waves $\omega = ck$; k_1 is the wavenumber of the plasma waves with spectral energy density $W^\sigma_{k_1}$.

TABLE II. *Probabilities for non-linear scattering of plasma waves by epithermal particles*

Conversion process	Averaged probability
1. Langmuir wave scattering in a weak magnetic field ($\omega^l \approx \omega_{pe}$). In a strong magnetic field ($\omega^l \approx \omega_{pe}\lvert\cos\vartheta\rvert$) the conversion probability is smaller by a factor two.	electrons (l+e ⇌ t+e′) $\bar{u}^t_l = \frac{\pi}{10}\frac{\omega_{pe}}{n_e^2}\left(\frac{\omega_{pe}}{\omega}\right)^3 \frac{\omega_{pe}}{k_1 v_e c^2}\left(v_e^2 - \frac{\omega^2}{k_1^2}\right)$ $\begin{cases} 1, \omega/k_1 < v_e; \\ 0, \omega/k_1 > v_e. \end{cases}$ ions (l+i ⇌ t+i′) $\bar{u}^t_l = \frac{\pi}{6}\frac{\omega_{pe}}{n_e^2}\left(\frac{\omega_{pe}}{\omega}\right)^3 \frac{\omega_{pe}}{k_1 v_i}$ $\begin{cases} 1, \omega/k_1 < v_i; \\ 0, \omega/k_1 > v_i. \end{cases}$
2. Gyro-frequency plasmon scattering ($\omega^h \approx \omega_{He}\lvert\cos\vartheta\rvert$) in a weak magnetic field ($\omega_{pe} \gg \omega_{He}$). In the case of a strong magnetic field ($\omega^h \approx \omega_{He}$) the probability must be multiplied by $(\omega_{pe}/\omega_{He})^4$.	electrons (h+e ⇌ t+e′) $\bar{u}^t_h = \frac{\pi}{15}\frac{\omega_{pe}}{n_e^2}\left(\frac{\omega_{He}}{\omega}\right)^3 \frac{\omega_{pe}}{k_1 v_e c^2}\left(v_e^2 - \frac{\omega^2}{k_1^2}\right)$ $\begin{cases} 1, \omega/k_1 < v_e; \\ 0, \omega/k_1 > v_e. \end{cases}$ ions (h+i ⇌ t+i′) $\bar{u}^t_h = \frac{\pi}{24}\frac{\omega_{pe}}{n_e^2}\left(\frac{\omega_{He}}{\omega}\right)^3 \frac{\omega_{pe}}{k_1 v_i}$ $\begin{cases} 1, \omega/k_1 < v_i; \\ 0, \omega/k_1 > v_i. \end{cases}$
3. Scattering of ion-sound waves ($\omega^s = v_s k_1 (1 + k_1^2 d_e^2)^{-1/2}$).	electrons (s+e ⇌ t+e′) $\bar{u}^t_s = \frac{\pi}{10}\frac{\omega_{pe}}{n_e^2}\left(\frac{\omega^s}{\omega}\right)^3 \frac{m_i}{m_e}\frac{\omega_{pe}}{k_1 v_e c^2}\left(v_e^2 - \frac{\omega^2}{k_1^2}\right)$ $\begin{cases} 1, \omega/k_1 < v_e; \\ 0, \omega/k_1 > v_e. \end{cases}$ ions (s+i ⇌ t+i′) $\bar{u}^t_s = \frac{\pi}{6}\frac{\omega_{pe}}{n_e^2}\left(\frac{\omega^s}{\omega}\right)^3 \frac{m_i}{m_e}\frac{\omega_{pe}}{k_1 v_i}$ $\begin{cases} 1, \omega/k_1 > v_i; \\ 0, \omega/k_1 > v_i. \end{cases}$

Similarly we have for the damping rate (due to the inverse conversion of electromagnetic waves in plasma waves of type σ):

$$\frac{\gamma^t(\omega)}{\omega} = \int_0^\infty \int_0^\infty \vec{u}_o^t(\omega, k_1, v) \frac{W_{k_1}^\sigma}{\omega^\sigma(k_1)} p^2 dp \frac{d}{dp^2} \frac{f_p}{p^2} = -\frac{1}{T} \int_0^\infty \int_0^\infty \vec{u}_o^t(\omega, k_1, v) f_p W_{k_1}^\sigma \frac{dk_1 dp}{\omega^\sigma(k_1)}. \tag{A.6}$$

Similarly we can obtain approximate formulae also for anisotropic distributions.

III. Probabilities for Compton scattering of plasma waves by relativistic electrons accompanied by their conversion into high-frequency electromagnetic waves

The scattering probabilities, averaged over all angles and polarizations can be expressed in terms of a dimensionless function $\Phi^\sigma(q)$ of a dimensionless parameter q so that we have for the averaged probability itself

$$\vec{u}_o^t(\omega, k_1, \varepsilon) = \frac{(2\pi)^3 e^4 \omega_{pe} \varepsilon^2}{4 m_e^4 c^4 \omega^4} \Phi^\sigma(q) = \frac{\pi}{8} \frac{\omega_{pe}}{n_e^2} \left(\frac{\omega_{pe}}{\omega}\right)^4 \left(\frac{\varepsilon}{m_e c^2}\right)^2 \Phi^\sigma(q). \tag{A.7}$$

The functions $\Phi^\sigma(q)$ are given in Table III. The parameter q is in all cases, except 2, 5, and 7 equal to $q_1 = (\omega/2k_1 v)(m_e c^2/\varepsilon)^2$. In cases 2 and 5 we have $q = q_2 = (\omega/2\omega_{pe})(m_e c^2/\varepsilon)^2$, and in case 7 : $q = q_3 = (\omega/2\omega_{He})(m_e c^2/\varepsilon)^2$.

Using (A.7) we can evaluate the emission and reabsorption coefficients using the equations given in the introduction to II. In particularly, we have for the emission coefficient per unit frequency:

$$J_\omega^t = \frac{2\pi e^4}{m_e^4 c^7} \frac{\omega_{pe}}{\omega} \int_0^\infty \int_0^\infty \Phi^\sigma(q) W_{k_1}^\sigma f_\varepsilon \frac{dk_1 \varepsilon^2 d\varepsilon}{\omega^\sigma(k_1)}. \tag{A.8}$$

Other parameters of the electromagnetic radiation can be evaluated similarly.

TABLE III. *Probabilities for Compton scattering by relativistic electrons*

Mode of converted wave	Function $\Phi^\sigma(q)$
1. Longitudinal plasmons ($\omega^l \approx \omega_{pe}$) in a weak magnetic field ($\omega_{pe} \gg \omega_{He}$). Case of small phase velocities ($v_p = \omega_{pe}/k_1 \ll c$).	$\Phi^l = \Phi_1 = \frac{8}{3} q[(1-q)^3 - 3q^2(1-q+\ln q)], \quad q = q_1 \ll 1.$
2. Longitudinal plasmons ($\omega^l \approx \omega_{pe}$) in a weak magnetic field ($\omega_{pe} \gg \omega_{He}$). Case of large phase velocities ($v_p = \omega_{pe}/k_1 \gg c$).	$\Phi^l = \Phi_2 = \frac{8}{3} q^2(1-2q+2q^2), \quad q = q_2 \ll 1.$

Mode of converted wave	Function $\Phi^{\sigma}(q)$
3. Gyro-frequency plasmons ($\omega^{h} \approx \omega_{He}\lvert\cos\vartheta\rvert$) in a weak magnetic field ($\omega_{pe} \gg \omega_{He}$). Here the phase velocities ω_{He}/k_1 are always smaller than the velocity of light.	$\Phi^{h} = \frac{1}{4}(\omega_{He}/\omega_{pe})^3\Phi_1, \quad q = q_1 \ll 1.$
4. Longitudinal plasmons ($\omega^{l} \approx \approx \omega_{pe}\lvert\cos\vartheta\rvert$) in a strong magnetic field ($\omega_{pe} \ll \omega_{He}$). Case of small phase velocities ($v_p \ll c$).	$\Phi^{l} = \frac{1}{2}\Phi_1, \quad q = q_1 \ll 1.$
5. Longitudinal plasmons ($\omega^{l} \approx \approx \omega_{pe}$) in a strong magnetic field ($\omega_{pe} \ll \omega_{He}$). Case of large phase velocities ($v_p \gg c$).	$\Phi^{l} = \Phi_2, \quad q = q_2 \ll 1.$
6. Gyro-frequency plasmons ($\omega^{h} \approx \omega_{He}$) in a strong magnetic field ($\omega_{pe} \ll \omega_{He}$). Case of small phase velocities ($v_p \ll c$).	$\Phi^{h} = (2_{pe}\omega/3\omega_{He})\Phi_1, \quad q = q_1 \ll 1.$
7. Gyro-frequency plasmons ($\omega^{h} \approx \omega_{He}$) in a strong magnetic field ($\omega_{pe} \ll \omega_{He}$). Case of large phase velocities ($v_p \gg c$).	$\Phi^{h} = (2\omega_{pe}/3\omega_{He})\Phi_2, \quad q = q_3 \ll 1.$
8. Whistlers ($\omega^{w} = \omega_{He}\lvert\cos\vartheta\rvert \times \times c^2k_1^2/\omega_{pe}^2$) under the condition $v_A \ll c\sqrt{(m_e/m_i)}$. Wavenumbers in the range $\omega_{pi}/c < k_1 < \omega_{pe}/c$.	$\Phi^{w} = \frac{4}{3}q(10+9q^2-19q^3+30q^2\ln q)\omega_{He}k_1^2c^2/\omega_{pe}^3, \quad q = q_1 \ll 1.$
9. Ion-sound waves ($v_s \approx \approx v_sk_1(1+k_1^2d_e^2)^{-1/2}$) under the condition $v_s \ll c$.	$\Phi_s = [(\omega^s)^3/\omega_{pi}^2\omega_{pe}]\Phi_1 \begin{cases} 1, & \omega_{Hi}/v_s < k_1, \\ \frac{1}{2}, & \omega_{Hi}/v_s > k_1, \end{cases} \quad q = q_1 \ll 1.$
10. Alfvén waves ($\omega^{a} \approx v_Ak_1 \times \times\lvert\cos\vartheta\rvert$) in the case of a weak magnetic field ($\omega_{Hi} \ll \omega_{pi}$; $v_s \ll v_A \ll c$).	$\Phi^{a} = \frac{4}{3}q(1+3q^2-4q^3+3q^2\ln q)(v_Ak_1/\omega_{pe}), \quad q = q_1 \ll 1.$
11. Fast magneto-sound waves ($\omega^{m} \approx v_Ak_1$) in the case of a weak magnetic field ($v_s \ll v_A \ll c$).	$\Phi^{m} = \frac{4}{3}q(8+3q^2-11q^3+24q^2\ln q)(v_Ak_1/\omega_{pe}), \quad q = q_1 \ll 1.$
12. Alfvén waves ($\omega^{a} = v_Ack_1 \times \times\lvert\cos\vartheta\rvert\,(v_A^2+c^2)^{-1/2} \approx ck_1\lvert\cos\vartheta\rvert$) in the case of a strong magnetic field ($v_A \gg c$).	$\Phi^{a} = \frac{4}{3}q\left[-2(1+2\ln 2)q^3+\frac{32}{5}\left(\frac{16}{5}+3\ln 2+\frac{3}{8}\ln q\right)q^2-\left(\frac{77}{24}+36\ln 2 + \frac{11}{2}\ln q\right)q+\frac{40}{21}\right](ck_1/\omega_{pe}), \quad < q = q_1 \ll 1;$ $\Phi^{a} = \frac{4}{3}q\left(\frac{8}{525}q^7-\frac{7}{8}q^5+\frac{16}{3}q^4-\frac{}{3}q^3+4q^3\ln\frac{q}{2}+\frac{664}{25}q^2 + \frac{140}{5}q^2\ln\frac{q}{2}+\frac{80}{21}q+\frac{14}{3}\right)(ck_1/\omega_{pe}), \quad 1 \ll q = q_1 \ll 2.$
13. Fast magneto-sound waves ($\omega^{m} = cv_Ak_1(v_A^2+c^2)^{-1/2} \approx ck_1$) in the case of a strong magnetic field ($v_A \gg c$).	$\Phi^{m} = \frac{4}{3}q^2\left[58-15q+3q^2-5q^3+6(6q-1)\ln\frac{q}{2}\right]\left(\frac{ck_1}{\omega_{pe}}\right), \quad q = q_1 \ll 2.$

IV. Probabilities for the conversion of plasma waves into electromagnetic waves through non-linear scattering by relativistic ions

The scattering probabilities, averaged over all angles can be expressed in terms of a dimensionless function $\Phi^\sigma(q)$ of a dimensionless parameter q in such a way that the averaged probability itself is given by the formula

$$\bar{u}^{\mathrm{t}}_\sigma(\omega, k_1, \varepsilon) = \frac{\pi^3 Z^2 e^4 \omega_{\mathrm{pe}}}{m_{\mathrm{e}}^2 \omega^5} k_1 c \, \Phi^\sigma(q). \tag{A.9}$$

Here Z is the ion charge. The functions $\Phi^\sigma(q)$ are given in Table IV. The parameter q is in cases 1, 3 to 5, and 7 to 9 equal to $q_4 = (\omega/2k_1 v)(m_{\mathrm{i}}c^2/\varepsilon)^2$; in case 2 we have $q = q_5 = (\omega/2\omega_{\mathrm{pe}})(m_{\mathrm{i}}c^2)/\varepsilon^2$, and in case 6 $q = q_6 = (\omega/2\omega_{He})(m_{\mathrm{i}}c^2/\varepsilon)^2$. The formulae are valid for the following range of ion energies:

$$m_{\mathrm{i}}c^2 \ll \varepsilon \ll (m_{\mathrm{i}}/m_{\mathrm{e}})m_{\mathrm{i}}c^2. \tag{A.10}$$

The spectral intensity of the electromagnetic radiation for the scattering of waves in isotropic turbulence by isotropically distributed particles can be evaluated, using the formula

$$J^{\mathrm{t}}_\omega = \frac{\pi Z^2 e^4 \omega_{\mathrm{pe}}}{m_{\mathrm{e}}^2 c^2 \omega^2} \int_0^\infty \int_0^\infty \Phi^\sigma(q) W^\sigma_{k_1} f_\varepsilon \frac{k_1 \, dk_1 \, d\varepsilon}{\omega^\sigma(k_1)}. \tag{A.11}$$

Other characteristics of the conversion can be determined similarly.

We have not given the conversion probabilities for Alfvén and fast magneto-sound waves as they are very small because of a number of cancellations in the matrix elements.

TABLE IV. *Probabilities for the conversion of plasma waves by relativistic ions*

Mode of the converted wave	Function $\Phi^\sigma(q)$
1. Longitudinal plasmons ($\omega^{\mathrm{l}} \approx \approx \omega_{\mathrm{pe}}$) in a weak magnetic field ($\omega_{\mathrm{pe}} \gg \omega_{He}$). Case of small phase velocities ($v_{\mathrm{p}} \ll c$.)	$\Phi^{\mathrm{l}} = \Phi_3 = q - 1 - \ln q$, $\quad q = q_4 \ll 1$. Maximum intensity occurs for $q_{\min} \approx (m_{\mathrm{i}}c^2/\varepsilon)^2$.
2. Longitudinal plasmons ($\omega^{\mathrm{l}} \approx \approx \omega_{\mathrm{pe}}$) in a weak magnetic field ($\omega_{\mathrm{pe}} \gg \omega_{He}$). Case of large phase velocities ($v_{\mathrm{p}} \gg c$).	$\Phi^{\mathrm{l}} = (1-q)(2k_1 c/\omega_{\mathrm{pe}})$, $\quad q = q_5 \ll 1$.
3. Gyro-frequency plasmons ($\omega^{\mathrm{h}} \approx \omega_{He}\lvert\cos\vartheta\rvert$) in a weak magnetic field ($\omega_{\mathrm{pe}} \gg \omega_{He}$). Phase velocities less than the velocity of light ($v_{\mathrm{p}} \ll c$).	$\Phi^{\mathrm{h}} = \frac{1}{4}(\omega_{He}/\omega_{\mathrm{pe}})^3 \Phi_3$, $\quad q = q_4 \ll 1$.
4. Longitudinal plasmons ($\omega^{\mathrm{l}} \approx \approx \omega_{\mathrm{pe}}\lvert\cos\vartheta\rvert$) in a strong magnetic field ($\omega_{\mathrm{pe}} \ll \omega_{He}$). Case of small phase velocities ($v_{\mathrm{p}} \ll c$).	$\Phi^{\mathrm{l}} = \frac{1}{2}\Phi_3$, $\quad q = q_4 \ll 1$.

Mode of the converted wave	Function $\Phi^{\sigma}(q)$	
5. Gyro-frequency plasmons ($\omega^{h} \approx \omega_{He}$) in a strong magnetic field ($\omega_{pe} \ll \omega_{He}$). Case of small phase velocities ($v_p \ll c$).	$\Phi^{h} = \frac{2}{3}(\omega_{pe}/\omega_{He})\Phi_3,$	$q = q_4 \leqslant 1.$
6. Gyro-frequency plasmons ($\omega^{h} \approx \omega_{He}$) in a strong magnetic field ($\omega_{pe} \ll \omega_{He}$). Case of large phase velocities ($v_p \gg c$).	$\Phi^{h} = \frac{2}{3}(k_1 c/\omega_{He})(1-q),$	$q = q_6 \leqslant 1.$
7. Whistlers ($\omega^{w} = \omega_{He}\lvert\cos\vartheta\rvert \times$ $\times c^2 k_1^2/\omega_{pe}^2$) in a weak magnetic field ($\omega_{pe} \gg \omega_{He}$).	$\Phi^{w} = \frac{1}{4}(\omega_{He}/\omega_{pe})^3 (k_1 c/\omega_{pe})^4 \Phi_3,$	$q = q_4 \leqslant 1.$
8. Ion-sound waves ($\omega^{s} = \omega_s k_1 \times$ $(1+k_1^2 d_e^2)^{-1/2}$) under the conditions $\omega^{s} \gg \omega_{Hi}$, $v_s \ll c$.	$\Phi^{s} = (m_e/m_i)^{1/2}(k_1 d_e)^3 (1+k_1^2 d_e^2)^{-3/2}\Phi_3,$	$q = q_4 \leqslant 1.$
9. Slow magneto-sound waves ($\omega^{ms} = v_s k_1 \lvert\cos\vartheta\rvert$) under the conditions $\omega^{ms} \ll \omega_{Hi}$, $v_s \ll c$.	$\Phi^{ms} = \frac{1}{2}(m_e/m_i)^{1/2} k_1^3 d_e^3 \Phi_3,$	$q = q_4 \leqslant 1.$

V. Coefficients for the differential transfer of plasma waves within one mode or between different modes through non-linear scattering by thermal ions and through fusion processes

Averaging the scattering probabilities over the angles and assuming all plasma turbulence modes to be isotropic we can obtain the differential expressions to determine the change with time of the spectral density of plasma waves for the case of differential transfers.

TABLE v. *Coefficients for differential transfer of plasma waves*

Scattering or fusion processes	Formula for the change in the spectral density	Scattering and conversion coefficient
1. Transfer of longitudinal plasmons ($\omega^{l} \approx \omega_{pe}$) within one mode ($l+i \rightleftarrows l'+i'$, $l+s \rightleftarrows l'$).	$\frac{\partial W_k^{l}}{\partial t} = \alpha^{ll} W_k^{l} \frac{\partial W_k^{l}}{\partial k},$	$\alpha^{ll} = \alpha^{l} = \frac{\pi}{27}\frac{\omega_{pe}^3}{n_e m_i v_{Te}^4} \begin{cases} \frac{1}{(1+T_e/T_i)^2}, & T_e < 3T_i; \\ 1, & T_e \gg T_i. \end{cases}$
2. Transfer of longitudinal plasmons ($\omega^{l} \approx \omega_{pe}$) into transverse plasmons ($\omega^{p} \approx \omega_{pe}$) ($l+i \rightleftarrows p'+i'$, $l+s \rightleftarrows p'$).	$\frac{\partial W_k^{p}}{\partial t} = \alpha^{lp} W_k^{p} \frac{1}{k}\frac{\partial}{\partial k}(k W_k^{l}),$	$\alpha^{lp} = \alpha^{l}.$
3. Transfer of transverse into longitudinal plasmons ($p+i \rightleftarrows l'+i'$, $p+s \rightleftarrows l'$).	$\frac{\partial W_k^{l}}{\partial t} = \alpha^{pl} W_k^{l} k \frac{\partial}{\partial k}\left(\frac{W_k^{p}}{k}\right),$	$\alpha^{pl} = \alpha^{l}.$

Scattering or fusion processes	Formula for the change in the spectral density	Scattering and conversion coefficient
4. Transfer of transverse into transverse plasmons (p+i ⇄ p′+i′, p+s ⇄ p′).	$\frac{\partial W_k^{\mathrm{p}}}{\partial t} = \alpha\ W_k^{\mathrm{p}} \frac{\partial W_k^{\mathrm{p}}}{\partial k},$	$\alpha^{\mathrm{pp}} = 9 \frac{v_{Te}^2}{c^2} \alpha^{\mathrm{l}}.$
5. Transfer of high-frequency transverse waves ($\omega \gg \omega_{\mathrm{pe}}$) (t+e ⇄ ⇄ t′+e′, t+l ⇄ t′).	$\frac{\partial W_\omega^{\mathrm{t}}}{\partial t} = \alpha^{\mathrm{tt}} W_\omega^{\mathrm{t}} \frac{\partial}{\partial \omega}\left(\frac{W_\omega^{\mathrm{t}}}{\omega}\right),$	$\alpha^{\mathrm{tt}} = \frac{\pi}{6} \frac{\omega_{\mathrm{pe}}^3}{n_e m_e c^2}.$
6. Transfer of whistlers into whistlers (w+i ⇄ w′+i′).	$\frac{\partial W_\omega^{\mathrm{w}}}{\partial t} = W_\omega^{\mathrm{w}} \times\left(\alpha_1^{\mathrm{ww}} \omega^2 \frac{\partial W_\omega^{\mathrm{w}}}{\partial \omega} + \alpha_2^{\mathrm{ww}} \omega W_\omega^{\mathrm{w}}\right),$	$\alpha_1^{\mathrm{ww}} \approx \alpha_2^{\mathrm{ww}} \approx \approx \frac{\pi}{4} \frac{\omega_{\mathrm{pe}}^2}{(1+T_e/T_i)^2 \omega_{He} n_e m_i c^2}.$
7. Transfer of whistlers into ion-sound waves (w+i ⇄ s+i′).	$\frac{\partial W_\omega^{\mathrm{s}}}{\partial t} = \alpha^{\mathrm{ws}} W_\omega^{\mathrm{s}} \omega^3 \frac{\partial W_\omega^{\mathrm{w}}}{\partial \omega},$	$\alpha^{\mathrm{ws}} = \frac{\pi}{2} \frac{1}{(1+T_e/T_i)^2 n_e m_i v_A^2}.$
8. Transfer of ion-sound waves into whistlers (s+i ⇄ w+i′).	$\frac{\partial W_\omega^{\mathrm{w}}}{\partial t} \approx \alpha^{\mathrm{sw}} W_\omega^{\mathrm{w}} \omega \frac{\partial}{\partial \omega}(\omega^2 W_\omega^{\mathrm{s}}),$	$\alpha^{\mathrm{sw}} = \alpha^{\mathrm{ws}}.$
9. Transfer of ion-sound into ion-sound waves (s+i ⇄ s′+i′) and of slow magneto-sound waves into slow magneto-sound waves (ms+i ⇄ ⇄ ms′+i′).	$\frac{\partial W_k^{\mathrm{s}}}{\partial t} = \alpha^{\mathrm{s}} W_k^{\mathrm{s}} k^2 \frac{\partial}{\partial k}(k W_k^{\mathrm{s}}),$	$\alpha^{\mathrm{s}} = \frac{4\pi}{15} \frac{T_i}{n_e m_i^2 v_s^3},$ $\alpha^{\mathrm{ms}} = \alpha^{\mathrm{s}}.$
10. Transfer of fast magneto-sound and Alfvén waves both within one mode and from one mode to another (a+i ⇄ a′+i′; m+i ⇄ m′+i′; m+i ⇄ a′+i′).	$\frac{\partial W_\omega^{\mathrm{a,m}}}{\partial t} = \alpha^{\mathrm{am}} W_\omega^{\mathrm{a,m}} \omega^2 \times\left(\omega \frac{\partial W_\omega^{\mathrm{a,m}}}{\partial \omega} + W_\omega^{\mathrm{a,m}}\right),$	$\alpha^{\mathrm{am}} \approx \alpha^{\mathrm{ms}} \approx \alpha^{\mathrm{aa}} \approx \alpha^{\mathrm{mm}} \approx \frac{\pi}{4} \frac{1}{(1+v_A^2/c^2) n_e m_i v_A^2}.$
	$\frac{\partial W_\omega^{\mathrm{ms}}}{\partial t} = \alpha^{\mathrm{m,ms}} \omega^4 W_\omega^{\mathrm{ms}} \times\left(\omega \frac{\partial W_\omega^{\mathrm{m}}}{\partial \omega} + 2 W_\omega^{\mathrm{m}}\right),$	
11. Transfer between the Alfvén, fast and slow magneto-sound wave modes (m+i ⇄ ms+i′) (a+i ⇄ ms+i′) (ms+i ⇄ m+i′) (ms+i ⇄ a+i′)	$\frac{\partial W_\omega^{\mathrm{ms}}}{\partial t} = \alpha^{\mathrm{a,ms}} \omega^4 W_\omega^{\mathrm{ms}} \times\left(\omega \frac{\partial W_\omega^{\mathrm{a}}}{\partial \omega} + 4 W_\omega^{\mathrm{a}}\right),$	$\alpha^{\mathrm{m,ms}} \approx \alpha^{\mathrm{a,ms}} \approx \alpha^{\mathrm{ms,m}} \approx \approx \alpha^{\mathrm{ms,a}} \approx \alpha^{\mathrm{ms}} \approx \approx \frac{\pi}{4} \frac{(T_i/T_e)^2}{\omega_{Hi}^2 n_e v_i m_A^2}.$
	$\frac{\partial W_\omega^{\mathrm{m}}}{\partial t} = \alpha^{\mathrm{ms,m}} \omega^4 W_\omega^{\mathrm{m}} \times\left(\omega \frac{\partial W_\omega^{\mathrm{ms}}}{\partial \omega} + 2 W_\omega^{\mathrm{ms}}\right),$	
	$\frac{\partial W_\omega^{\mathrm{a}}}{\partial t} = \alpha^{\mathrm{ms,a}} \omega^4 W_\omega^{\mathrm{a}} \times\left(\omega \frac{\partial W_\omega^{\mathrm{ms}}}{\partial \omega} - 2 W_\omega^{\mathrm{ms}}\right),$	

Appendix

When evaluating the transfer between transverse and longitudinal plasma waves we have used in Table V the "longitudinal" wavenumber scale (see § 3).

VI. Coefficients for fusion and decay of waves, leading to the generation and absorption of electromagnetic radiation near the frequencies ω_{pe}, $2\omega_{pe}$, and $2\omega_{He}$

Averaging the fusion and decay probabilities over the angles and assuming all plasma turbulence modes to be isotropic we can obtain expressions to determine the change in time of the spectral densities of plasma waves in fusion and decay processes of different modes (Table VI).

TABLE VI. *Coefficients for the generation of radiation through the fusion of plasma waves*

Fusion and decay processes	Formula for the change in spectral density	Fusion coefficient
1. Fusion of longitudinal plasmons into an electromagnetic wave with frequency $\omega^t \approx 2\omega_{pe}(l+l' \to t)$. The phase velocities of the longitudinal waves are smaller than the velocity of light.	$\frac{\partial W^t}{\partial t} = \beta^l \int \left(\frac{W^l_{k_1}}{k_1}\right)^2 dk_1,$	$\beta^l = \frac{4\sqrt{3}\pi}{5} \frac{\omega_{pe}^4}{n_e m_e c^5}.$
2. Fusion of two gyro-frequency plasmons into an electromagnetic wave with frequency $\omega^t \lesssim 2\omega_{He}(h+h' \to t)$.	$\frac{\partial W^t}{\partial t} = \beta^h \int \left(\frac{W^h_{k_1}}{k_1}\right)^2 dk_1,$	$\beta^h = \frac{32\pi}{5} \frac{\omega_{pe}^2 \omega_{He}^2}{n_e m_e c^5}.$
3. Fusion of a longitudinal plasmon and an ion-sound wave $(l+s \to p)$.	$\frac{\partial W^p_k}{\partial t} = \beta^{ls} \frac{W^l_k W^s_k}{k},$	$\beta^{ls} = \frac{\pi}{2\sqrt{3}} \frac{\omega_{pe}^3}{n_e m_e v_{Te} c^3}.$
4. Decay of a transverse plasmon, involving an ion-sound wave $(p \to l+s)$.	$\frac{\partial W^p_k}{\partial t} = -\beta^{ps} \frac{W^p_k W^s_k}{k},$	$\beta^{ps} = \frac{\pi}{18} \frac{\omega_{pe}^3}{n_e m_e v_{Te}^4}.$
5. Fusion of a longitudinal plasmon and a whistler $(l+w \to p)$.	$\frac{\partial W^p_k}{\partial t} = \beta^{lw} \frac{W^l_k W^w_k}{k},$	$\beta^{lw} = \sqrt{2} \frac{\omega_{pe}^{9/2}}{\omega_{He}^{3/2} n_e m_e c^4}.$
6. Decay of a transverse plasmon, involving a whistler $(p \to l+w)$.	$\frac{\partial W^p_k}{\partial t} = -\beta^{pw} \frac{W^p_k W^w_k}{k},$	$\beta^{pw} = \frac{\pi}{8} \frac{\omega_{pe}^6}{\omega_{He}^3 n_e m_e c^4}.$
7. Fusion of a longitudinal plasmon and a fast magneto-sound wave $(l+m \to p)$.	$\frac{\partial W^p_k}{\partial t} = \beta^{lm} \frac{W^l_k W^m_k}{k^{3/2}},$	$\beta^{lm} = \frac{1}{\sqrt{2}} \frac{\omega_{pe}^{3/2} \omega_{pl}^2}{n_e m_l v_A^{3/2} c^3}.$
8. Decay of a transverse plasmon involving a magneto-sound wave $(p \to l+m)$.	$\frac{\partial W^p_k}{\partial t} = -\beta^{pm} W^p_k W^m_k,$	$\beta^{pm} = \frac{\pi}{8} \frac{\omega_{pl}^2}{n_e m_l v_A^3}.$

VII. Probabilities for fusion and decay of high-frequency electromagnetic waves with plasma waves

We give in Table VII the probabilities for the fusion and decay of electromagnetic waves with frequencies $\omega \gg \omega_{pe}$ and $\omega \gg \omega_{He}$ involving all possible plasma turbulence waves with wavevector k_1. Only the coefficients are averaged while the delta-functions take into account the conservation laws during the fusion and decay processes so that the probability u_{k,k',k_1} for the process can be expressed as follows in terms of the quantities $\bar{u}(\omega, k_1)$ given in the table:

$$u_{k,k',k_1} = (2\pi)^6 \bar{u}(\omega, k_1)\, \delta^3(k-k'-k_1)\, \delta(\omega^t(k)-\omega^t(k')-\omega^\sigma(k_1)). \tag{A.12}$$

One must substitute expression (A.12) into equations (5.20) to (5.22) after which one can in particular, obtain the subsequent formulae from §5.

To determine the cross-section for the elastic scattering of electromagnetic waves with $\omega \gg ck_1$ one can use the simple approximate formula

$$\sigma_\perp = \frac{(2\pi)^3}{\omega^2} \int \bar{u}(\omega, k_1) W^\sigma_{k_1} \frac{ck_1}{\omega_\sigma(k_1)}\, dk_1. \tag{A.13}$$

TABLE VII. *Probabilities for the decay of high-frequency electromagnetic waves*

Fusion and decay processes	Averaged probability coefficients
1. Fusion and decay of electromagnetic waves involving longitudinal plasmons (t ⇄ t′+l). In the case of a process in a strong magnetic field, where $\omega^l \approx \omega_{pe}\lvert\cos\vartheta\rvert$, there appears a factor $\frac{1}{2}$ in the coefficient.	$\frac{1}{12\pi}\frac{e^2}{m_e^2}\frac{\omega_{pe}k_1^2}{\omega^2} = \frac{1}{48\pi^2}\frac{\omega_{pe}k_1^2}{n_e m_e}\left(\frac{\omega_{pe}}{\omega}\right)^2$.
2. Fusion and decay involving gyro-frequency plasmons in a strong magnetic field (t ⇄ t′+h).	$\frac{1}{18\pi}\frac{e^2}{m_e^2}\frac{\omega_{pe}^2 k_1^2}{\omega_{He}\omega^2}$.
3. Fusion and decay involving whistlers (t ⇄ t′+w).	$\frac{1}{48\pi}\frac{e^2}{m_e^2}\frac{\omega_{pe}^2 k_1^2}{\omega_{He}\omega^2}$.
4. Fusion and decay involving ion-sound waves (t ⇄ t′+s).	$\frac{\sqrt{(m_e/m_i)}}{12\pi}\frac{e^2}{m_e^2}\frac{\omega_{pe}^2\omega_s}{v_{Te}^2\omega^2}$.
5. Fusion and decay involving Alfvén waves (t ⇄ t′+a).	$\frac{1}{48\pi}\frac{e^2}{m_e^2}\frac{c^2k_1^2}{v_A\omega^2}$.
6. Fusion and decay involving fast magneto-sound waves (t ⇄ t′+m).	$\frac{1}{18\pi}\frac{e^2}{m_e^2}\frac{\omega_{pi}^2 k_1}{v_A\omega^2}$.

VIII. Probabilities for the emission of electromagnetic waves and the conversion of plasma waves into electromagnetic waves through scattering by particles in an ultra-relativistic plasma in a very strong magnetic field (Chapter IV)

We shall give here both the non-averaged expressions for the various probabilities as well as the averaged expressions within the limits of the directional diagram of the radiation. We assume that the particles move at small angles to the magnetic field, $\theta \ll mc^2/\varepsilon$. The magnetic field is so strong that the following condition holds:

$$\omega_{H_*} = \frac{eHc}{\varepsilon_*} \gg \omega_p \approx \sqrt{\left(\frac{4\pi n_* e^2 c^2}{\varepsilon_*}\right)}, \tag{A.14}$$

where ε_* is the average particle energy. In the formulae given below $\omega_H = eHc/\varepsilon$ and $\omega_{He} = eH/m_e c$. We have described in chapter IV the method for using the emission probabilities in a very strong magnetic field.

1. Probability for magneto-brems emission at an angle ϑ to the magnetic field lines:

$$u_k^t(\omega, \vartheta, \varepsilon) = \frac{(2\pi)^3}{8\pi} \frac{e^2c^4p_\perp^2}{\varepsilon^2\omega} \left\{1+\left[1-\frac{\omega}{\omega_H}\left(\frac{m_ec^2}{\varepsilon}\right)^2\right]^2\right\} \delta\left\{\frac{\omega}{2}\left[\left(\frac{m_ec^2}{\varepsilon}\right)^2+\vartheta^2\right]-\omega_H\right\}.$$

Averaging within a solid angle $\Omega = \pi(\omega_{He}/\omega)^2$ gives:

$$\bar{u}^t(\omega, \varepsilon) = \frac{(2\pi)^3}{4\pi} \frac{e^2c^4p_\perp^2}{\varepsilon^2\omega_{He}^2} \left\{1+\left[1-\frac{\omega}{\omega_H}\left(\frac{m_ec^2}{\varepsilon}\right)^2\right]^2\right\} \approx$$
$$\frac{\pi}{2}\frac{\varepsilon_*}{n_e}\left(\frac{\omega_p}{\omega_{He}}\right)^2\left(\frac{cp_\perp}{\varepsilon}\right)^2\left\{1+\left[1-\frac{\omega}{\omega_H}\left(\frac{m_ec^2}{\varepsilon}\right)^2\right]^2\right\}.$$

2. Probability for scattering at an angle ϑ to the magnetic field for a longitudinal plasmon with frequency ω_{pe} during which it changes into an electromagnetic wave; case when the transverse momentum component is unchanged:

$$u_k^t(\omega, \vartheta, \varepsilon) = (2\pi)^3 \frac{e^4c^4\omega W^l}{\varepsilon^2\omega_p^4}\left(\frac{m_ec^2}{\varepsilon}\right)^4 \vartheta^2 \delta\left\{\frac{\omega}{2}\left[\left(\frac{m_ec^2}{\varepsilon}\right)^2+\vartheta^2\right]-\omega_p\right\}.$$

Averaging over the solid angle $\Omega = 2\pi\omega_p/\omega$ gives:

$$\bar{u}^t(\omega, \varepsilon) = (2\pi)^3 \frac{2e^4c^4W^l}{\varepsilon^2\omega_{pe}^4}\left(\frac{m_ec^2}{\varepsilon}\right)^4\left[1-\frac{\omega}{2\omega_{pe}}\left(\frac{m_ec^2}{\varepsilon}\right)^2\right]$$
$$\approx \frac{\pi}{2}\frac{W^l}{n_e^2}\left(\frac{\varepsilon_*}{\varepsilon}\right)^2\left(\frac{m_ec^2}{\varepsilon}\right)^4\left[1-\frac{\omega}{2\omega_{pe}}\left(\frac{m_ec^2}{\varepsilon^2}\right)^2\right].$$

3. Probability for cyclotron scattering of plasma waves—with a change in the transverse component of the particle momentum; case $\omega_N \gg \omega_p$:

$$u_k^t(\omega, \vartheta, \varepsilon) = \frac{(2\pi)^3}{4} \frac{e^4c^4W^l\omega_H^2c^2p_\perp^2}{\varepsilon^4\omega_p^4\omega}\left[1-\frac{\omega}{\omega_H}\left(\frac{m_ec^2}{\varepsilon}\right)^2\right]^2\left\{1+\left[1-\frac{\omega}{\omega_H}\left(\frac{m_ec^2}{\varepsilon}\right)^2\right]\right\}$$
$$\times\delta\left\{\frac{\omega}{2}\left[\left(\frac{m_ec^2}{\varepsilon}\right)^2+\vartheta^2\right]-\omega_H\right\}.$$

Averaging over the solid angle $\Omega = \pi(\omega_{He}/\omega)^2$ gives:

$$\vec{u}^{t}(\omega, \varepsilon) = \frac{(2\pi)^3}{2} \frac{e^4 c^6 p_\perp^2 W^1}{\varepsilon^4 \omega_p^4} \left(\frac{m_e c^2}{\varepsilon}\right)^2 \left[1 - \frac{\omega}{\omega_H}\left(\frac{m_e c^2}{\varepsilon}\right)^2\right]^2 \left\{1 + \left[1 - \frac{\omega}{\omega_H}\left(\frac{m_e c^2}{\varepsilon}\right)^2\right]^2\right\}$$
$$\approx \frac{\pi}{4} \frac{W^1}{n_e^2} \left(\frac{\varepsilon_*}{\varepsilon}\right)^2 \left(\frac{c p_\perp}{\varepsilon}\right)^2 \left(\frac{m_e c^2}{\varepsilon}\right)^2 \left[1 - \frac{\omega}{\omega_H}\left(\frac{m_e c^2}{\varepsilon}\right)^2\right]^2 \left\{1 + \left[1 - \frac{\omega}{\omega_H}\left(\frac{m_e c^2}{\varepsilon}\right)^2\right]^2\right\}.$$

4. The probability for cyclotron scattering of plasma waves—with a change in the transverse component of the particle momentum; case when $\omega_p \ll \omega_H$:

$$u_k^t(\omega, \vartheta, \varepsilon) = \frac{(2\pi)^3}{4} \frac{e^4 c^6 p_\perp^2 W^1 \omega^3}{\varepsilon^4 \omega_H^2 \omega_p^4} \left(\frac{m_e c^2}{\varepsilon}\right)^4 \vartheta^4 \, \delta(\omega - \omega_p - k_z v_z \pm \omega_H).$$

Averaging over the solid angle $\Omega = 2\pi\omega_p/\omega$ gives:

$$\vec{u}^{t}(\omega, \varepsilon) \approx \pi \frac{W^1}{n_e^2} \left(\frac{\omega_p}{\omega_H}\right)^2 \left(\frac{\varepsilon_*}{\varepsilon}\right)^2 \left(\frac{c p_\perp}{\varepsilon}\right)^2 \left(\frac{m_e c^2}{\varepsilon}\right)^4 \frac{\omega}{\omega_p} \left[1 - \frac{\omega}{2\omega_{pe}}\left(\frac{m_e c^2}{\varepsilon}\right)^2\right]^2.$$

IX. Probabilities for some plasma processes in which quantum effects are important

In a very dense plasma the frequencies turn out to be so large that quantum effects become appreciable. We give here the probabilities for some processes of this kind, averaged as in Table I.

1. Cherenkov emission of any longitudinal wave—dielectric constant ε^l, dispersion relation $\omega^l(k)$—by a charge with momentum $\boldsymbol{p}$ and energy $\varepsilon(\boldsymbol{p})$: process $e \to e' + l$. Quantum effects appear when $\hbar k \geqslant mc$, where m is the particle mass. The probability, averaged over the angles is:

$$\vec{u}_e^l(k, \varepsilon) = \frac{4\pi^2}{k^3} \frac{e^2 \varepsilon(p)}{\hbar p c^2 \left.\frac{\partial \varepsilon^l}{\partial \omega}\right|_{\omega^l(k)}} \left[1 - \frac{\hbar\omega^l(k)}{\varepsilon(p)} + \frac{\hbar^2[\omega^l(k)]^2 - \hbar^2 c^2 k^2}{4[\varepsilon(p)]^2}\right].$$

2. Cherenkov emission of a plasma wave by a neutrino of momentum p_ν: process $\nu \to \nu' + l$. The averaged probability is:

$$\vec{u}_\nu^l(k, \varepsilon) = \frac{2}{k^3} \frac{g^2}{e^2} \frac{\hbar[\omega(k)]^4}{c^3 \left.\frac{\partial \varepsilon^l}{\partial \omega}\right|_{\omega^l(k)}} \left[\left(\frac{kc}{\omega^l(k)^2}\right)^2 - 1\right]^2 \left\{1 - \frac{\hbar k}{2p_\nu}\left(\frac{\omega^l(k)}{kc}\right)^2 - \frac{\hbar^2 k^2}{4c^2 p_\nu^2}\left[1 - \left(\frac{\omega^l(k)}{kc}\right)^2\right]\right\},$$

where $g = 3 \times 10^{-12}(\hbar/mc)^2$ is the coupling constant for the weak $(e\nu)(e\nu)$ interaction. The formula is valid when $\omega^l(k) < ck$, $\hbar k < p_\nu$. The ratio of the emission probabilities for a neutrino and an electron for the case when $\omega^l(k) \approx kc \approx p_\nu c/\hbar$, is given by the equation

$$\frac{\vec{u}_\nu^l}{\vec{u}_e^l} = 10^{-20}\left(\frac{p_\nu}{m_e c}\right)^4.$$

Appendix

3. Electron-positron pair production by a longitudinal plasma wave; process $l \to e + \tilde{e}$. The total probability is

$$\bar{u}_l^{e,\tilde{e}}(k) = \frac{e^2}{3\hbar c} \frac{1}{\left.\frac{\partial \varepsilon^l}{\partial \omega}\right|_{\omega^l(k)}} \left[1 + \frac{2m_e^2c^4}{\hbar^2[\omega^l(k)]^2 - \hbar^2k^2c^2}\right] \sqrt{\left(1 - \frac{4m_e^2c^4}{\hbar^2[\omega^l(k)]^2 - \hbar^2k^2c^2}\right)}.$$

The threshold for this effect is $\hbar\omega^l(k) > (4m_e^2c^4 + \hbar^2c^2k^2)^{1/2}$, that is, for Langmuir waves $\omega_{pe} > 2m_ec^2/\hbar$.

4. Neutrino-antineutrino pair production by a longitudinal plasma wave; process $l \to \nu + \tilde{\nu}$. The total probability is:

$$\bar{u}_l^{\nu,\tilde{\nu}}(k) = \frac{g^2}{6\pi^2e^2} \frac{\hbar[\omega^l(k)]^4}{c^3 \left.\frac{\partial \varepsilon^l}{\partial \omega}\right|_{\omega^l(k)}} \left[1 - \left(\frac{ck}{\omega^l(k)}\right)^2\right]^2.$$

The total loss through pair production has a power:

$$Q^{\nu,\tilde{\nu}} = \int u_l^{\nu,\tilde{\nu}}(\varepsilon, k)\hbar\omega(k)N_k \frac{d^3k}{(2\pi)^3} \approx 10^{-23}W^l\omega_{pe}\left(\frac{\hbar\omega_{pe}}{m_ec^2}\right)^4.$$

There is now no threshold, as $m_\nu = 0$.

We refer to papers by Adams, Ruderman, and Woo (1963), and by Tsytovich (1961b, 1962, 1964) for a discussion of the use, in astrophysics, of the effects, described by these probabilities.

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